

## **A MATHEMATICAL MODELLING APPROACH FOR IRRIGATION WATER MANAGEMENT UNDER WATER SHORTAGE AND SALINITY CONDITIONS: THE WAVE\_MS MODEL**

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### **ABSTRACT**

Computer models based on numerical solution of the equations describing water and solute transport in the vadose zone can be used to assist in assessing the impact of water and salinity stress on crops, and to establish sustainable irrigation and drainage practices. Many models have been developed and used to simulate water and solute flux in the crop rootzone. This paper describes the development of the WAVE model to simulate water and solute transport in the vadose zone and their effect on crop transpiration and yield. The WAVE model was modified to include the effect of salinity on crop transpiration, and used to simulate soil water balances, to investigate long-term salinity build-up in the root zone, and in conjunction with a crop yield response model to assess their effect on crop yield. The modified model is now capable of dealing with the combined effect of water stress and salinity on crop transpiration and yield.

**Keywords:** Salinity, Transpiration, Irrigation, Crop yield, Mathematical Modelling

### **INTRODUCTION**

Water is essential for plant growth. It plays an important role in most physiological processes and transports nutrients to the plant from the soil through the root system. When soil moisture in the rootzone is insufficient to meet crop water requirements, crop transpiration and yield decrease below their potential values and the crop is said to be under water stress. In areas of limited water resources, where annual precipitation is not sufficient to meet crop water requirements, irrigation is needed to provide the soil rootzone with adequate moisture to avoid physiological water stress in the crop and to achieve acceptable yield.

The situation becomes more critical when water shortage is combined with salinity problems. Soil salinity is usually caused by irrigating with low quality water or by the upward movement of saline water from a shallow watertable as a result of inadequate drainage. Sustainable crop production can only be achieved if rootzone salinity does not exceed a threshold value, which varies according to crop type. Water shortage and salinity are the major limiting factors to agricultural production under arid and semi-

arid conditions. The combined effects of water stress and salinity reduce crop transpiration and result in low yield.

Under these conditions, it is necessary to establish sustainable irrigation and drainage management practices. Establishment of sustainable irrigation and drainage management is complicated by the complex interactions that exist between the crop, soil water, and hydroclimatic conditions. Mathematical model that deal with water and solute transport in the unsaturated or variably saturated zone, along with crop yield response models, can be useful in assessing the impact of different irrigation and drainage practices on crop yield.

A technical review was carried out of six codes for water and solute transport in the vadose zone. The WAVE model was selected for use in this research, primarily because of source code access that would permit further model development with respect to incorporating the combined effects of water and salinity stress on actual evapotranspiration, and crop yield. In addition, it provides physics based modelling of soil moisture movement, solute transport, and root development. It is able to assess the contribution from the watertable by capillary flux in partially meeting crop water requirements. It is also able to assess the influence of watertable on the salinity levels in the root zone. The usefulness of WAVE is limited by weakness in the calculation of crop evapotranspiration as it does not take into account the combined effect of soil water stress and salinity on crop transpiration, which makes it less applicable under salinity conditions.

The aim of this research is to develop and apply the WAVE model to simulate soil water balances and to investigate long-term salinity build up in the soil root zone under different management practices and to assess their effect on crop yield.

## **THE WAVE MODEL DESCRIPTION**

The WAVE model (**W**ater and **A**grochemicals in soil and **V**adose **E**nvironment, Vanclouster *et al.*, 1994) is a one-dimensional mathematical model developed by researchers of the Institute for Land and Water Management at the Katholieke University of Leuven in Belgium. WAVE is a deterministic numerical model that simulates the transport and transformations of water, heat and solute in the soil, crop and vadose environment. The model consists of five modules; a water transport module, a solute transport module, a heat transport module, a crop growth module, and a nitrogen fate module. A finite-difference technique is used to solve the differential equations describing the water and solute transport processes.

One of the advantages of the WAVE model is that it is programmed in a modular way, which makes it relatively easy to incorporate new concepts and routines without the need to significantly change the model structure or its input files (Vanclouster *et al.*, 1994).

## THE WATER TRANSPORT MODULE (WAT)

In the water transport module, the vertical flow of water in the unsaturated and saturated zones is described by the well-known Richards' equation (Vanclooster *et al.*, 1994).

To solve Richards equation, the soil moisture retention (*MRC*) and hydraulic conductivity (*HCC*) characteristics must be specified for each layer. The WAVE model uses the function proposed by van Genuchten (1980) to describe the moisture retention curve, which is the relationship between soil moisture content and  $pF$ , the pressure head as  $\log_{10}$  cm of water. The van Genuchten (1980) function can be written as (Vanclooster *et al.*, 1994):

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{\left(1 + (\alpha|h|)^n\right)^m} \quad (1)$$

where,  $\theta_s$  is the saturated volumetric soil water content ( $m^3 m^{-3}$ ),  $\theta_r$  is the residual volumetric soil water content ( $m^3 m^{-3}$ ),  $\alpha$  is the inverse of the air entry ( $m^{-1}$ ) and  $n, m$  are shape parameters. These parameters can be based on pedo transfer functions presented in the WAVE manual (Vanclooster *et al.*, 1994). The soil moisture retention parameters must be input for each layer. These parameters are extremely important in the model calibration, and can be refined to improve the model fit to observed soil moisture data. In this study, Gardner's model (Gardner, 1958) was used to describe the relationship between hydraulic conductivity and soil water pressure head.

It is necessary to define upper and lower boundary conditions in order to calculate the flux between soil compartments. The upper boundary condition is defined by a flux through the soil surface. The lower boundary condition can be defined in a number of ways: a groundwater level boundary condition, a pressure head boundary condition, a flux through the bottom of the model or free outflow at the bottom of the model (Vanclooster *et al.*, 1994).

## Evapotranspiration

The potential crop evapotranspiration ( $ET_C$ ) is calculated in the WAVE model by multiplying the reference crop evapotranspiration ( $ET_o$ ) by a crop coefficient ( $K_C$ ) as described in the FAO Irrigation and Drainage Paper No. 24 (Doorenbos and Pruitt, 1977).

The WAVE model considers only the effect of water stress on crop transpiration according to the model proposed by Feddes *et al.*, (1978). This function depends only on the effect of soil water pressure head:

$$\alpha(h) = \frac{h - h_4}{h_3 - h_4} \quad (2)$$

where  $\alpha(h)$  is a dimensionless transpiration reduction function of the soil water pressure head ( $0 \leq \alpha \leq 1$ ),  $h$  is soil water pressure head,  $h_3$  is soil water pressure head at the threshold of moisture stress,  $h_4$  is soil water pressure head at wilting.

For each soil compartment, the root water uptake is computed by multiplying the maximum root water uptake ( $S_{\max}$ ) with the reduction factor  $\alpha(h)$ , which is based on the pressure head ( $h$ ). The WAVE model provides linear and hyperbolic relationship between  $\alpha(h)$  and the pressure head. The wilting point value ( $h_3 = -16000 \text{ cm}$ ,  $pF = 4.2$ ) is often taken as the lower limit of this relationship. Root water uptake at different depths is determined by integrating the root water uptake term from the soil surface to an increasing depth less or equal to the rooting depth, until the integral becomes equal to the potential transpiration rate. If the integration over the complete rooting depth is insufficient to explain the potential transpiration rate, water stress is considered to occur.

## THE SOLUTE TRANSPORT MODULE (SOL)

The solute transport module is based on the coupled convection-diffusion equation. So it assumes the existence of both mobile and immobile soil water regions.

### Model Inputs

The WAVE model requires four data files (Vanclooster *et al.*, 1994):

- CLIMDATA.IN: contains daily climatic data: precipitation, evapotranspiration and water applications (irrigation and leaching);
- GENDATA.IN: contains general information: number of soil layers with different characteristics, number of soil compartment in each layer and the bulk density of each layer;
- SOLDATA.IN: contains additional input for modelling solute transport in soil-plant system;
- WATDATA.IN: contains input required for modelling soil water flow: moisture retention characteristics and hydraulic conductivity in each layer.

## THE MODIFIED WAVE MODEL – WAVE\_MS

The WAVE model is capable of dealing with the effect of soil water stress on crop transpiration. Fernandez *et al.*, (2002) report that in terms of crop transpiration, the approach adopted by the original model works well under water shortage conditions.

However, it is not able to take into account the effect of salinity on potential evapotranspiration. This restricts WAVE model application in arid and semi-arid conditions where water shortage and salinity are significant problems. Considering the effects of salinity is important, as the combined effect of water and salinity stress reduces crop transpiration and result in yield reduction. The higher the water stress and salinity, the lower the crop transpiration and yield. Where salinity stress exists and crop transpiration is reduced as a result of this, there will be impacts on soil moisture storage also, and water stress may be lower than would occur in the absence of salinity stress. It is also unable to assess the combined effect of irrigation water applications, leaching amounts, and drainage rates on crop yield since it does not include any crop yield response function; its' crop growth module calculates the time course of the leaf area index, the accumulation of the dry matter of the different plant organs and the root length and root density extension rate and requires a lot of parameters.

Other limitation is that, the maximum simulation period is limited to one year; this is a limitation particularly in application to salinity management problems as it can take several years for the impacts of any particular water management practice to be observable, or to significantly affect crop yield (Mott MacDonald, 2000).

Accordingly, it was considered important that the WAVE model be improved to deal with the combined effect of soil water and salinity stress on crop transpiration.

## **THE WAVE\_MS MODEL INTERFACES AND UTILITIES**

Mott MacDonald (2000) developed a number of programs to operate with the WAVE model that permitted it to be used for multi-year simulations, and provided outputs that were more suitable for evaluating sustainable crop production under different water management strategies. They developed a crop yield response model to run with the output data on evapotranspiration and salinity produced by the WAVE model, and also developed a user interface that made the model easier to use, and errors in input data less likely. Graphical post-processing routines were also developed.

## **THE C\_YIELD MODEL**

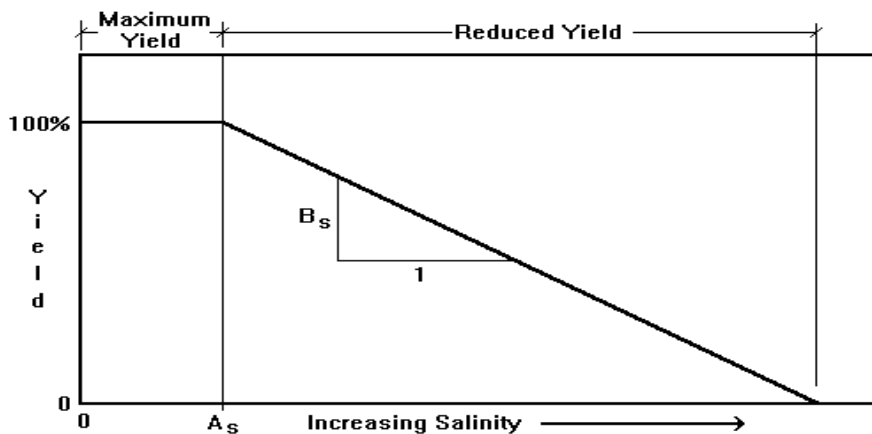
The WAVE model is unable to quantify the impact of water supply, rootzone salinity, and root zone water logging on crop yield response, as it does not include any crop yield response function. The model was improved by linking a crop yield response function to it through a computer programme called C\_YIELD. C\_YIELD (Mott MacDonald, 2000) was developed at the University of Edinburgh to read actual crop transpiration and soil salinity data from the WAVE model outputs and to calculate the impact of water shortage, root zone salinity and root zone water logging on crop yield response. It prepares a table of time series results, as well as summary results.

The C\_YIELD program calculates the relative yield response to water stress, soil salinity and water logging in each growth stage using separate functions for each. The total relative yield is the product of the relative yields from the three functions.

Many water production functions have been developed and used to predict crop yield response to water stress. C\_YIELD uses the multiplicative function proposed by Rao *et al.*, (1988).

Under standard conditions, crop yield can be obtained at its potential level until some threshold electrical conductivity  $EC_e$  (electrical conductivity of saturated soil extract) is reached. Threshold  $EC_e$  values vary with crop type and variety. Very few yield response to salinity models are described in literature. The yield response to salinity in the C\_YIELD program is based on the approach presented by Maas and Hoffman (1977) in which the crop yield linearly decrease as the  $EC_e$  value increase above the threshold soil electrical conductivity ( $A_s$ ). The approach also requires the rate at which relative crop yield declines with increasing salinity ( $B_s$ ).

The Maas and Hoffman approach is recommended in FAO Irrigation and Drainage Paper 56 (Allen *et al.*, 1998). The relationship between yield and salinity is shown in Figure 1 below:



**Figure 1: Simple relationship between yield and salinity (Mass and Hoffman, 1977)**

Water logging occurs when the soil pores in the root zone are filled with water and the roots become asphyxiated. It can severely restrict crop growth; in extreme cases crops die due to lack of oxygen in the root system. C\_YIELD uses a very simple relationship in which potential yield is reduced by the proportion of the rooting depth that is saturated. It is also multiplicative between time steps.

The C\_YIELD program requires two data files in addition to the files either used or created by WAVE. These are CROP\_CHAR.DAT, which contains the crop characteristics, and SOIL\_STRUCT.DAT.

## PRE-PROCESSING

The WAVE model is limited to application to a single calendar year at a time. This is a significant limitation when evaluating the effect of irrigation and drainage management practices on salinity build up, where it can take many years for the effect of any particular water management practise to become apparent, or to significantly reduce crop yield. To address this limitation a number of computer programs were written that permit it to be used in time series runs of up to 26 years. The storage levels and variable values from the end of one year's simulation are in effect input as starting conditions for the next year (Mott MacDonald, 2003b).

The data input files WATDATA.IN, and SOLDATA.IN (described above) need to be updated with configuration data for any particular model run. The GENDATA.IN and CLIMDATA.IN files are configured for individual years and base files are created initially for each year of the run. Further data on crop characteristics ( $K_c$  factors,  $LAI$  values and rooting depths during the growing season) are contained crop files called COTTON\_CHAR.DAT and ALFALFA\_CHAR.DAT. Appropriate values are copied from these files into the WATDATA.IN file (Mott MacDonald, 2003c). The file manipulations are managed through a series of DOS command files that are created at run time from the pre-processing interface.

Pre- and post-processing routines were developed that enable the WAVE model and C\_YIELD to be from a Windows based menu system. This simplified operation of the model and made parameter modification much simpler and less error prone. Input data and parameters that are varied between model runs (irrigation and leaching applications, drainage conditions, soil and crop characteristics etc) are displayed through the interface and are stored in a data file with the extension CFG (for configuration).

The data required by the WAVE model can be considered under three categories: soil data, groundwater data, and cropping and irrigation data.

## POST-PROCESSING

Post processing routines for the model were developed to extract data from the WAVE model outputs and prepare tables of time series results in a file called WAVE\_PLOT.OUT containing the following daily data: Rainfall ( $mm$ ), Irrigation ( $mm$ ), Rootzone moisture content (%), Root depth ( $mm$ ), Rootzone salinity (% dry solids), Potential evapotranspiration ( $mm$ ), Actual evapotranspiration ( $mm$ ), Cumulative evapotranspiration deficit ( $mm$ ), Groundwater level ( $mm$ ), Soil moisture

by layer (%), Soil salinity by layer (% dry solids), Water flux at the bottom of the profile (mm), and Solute flux at the bottom of the profile ( $mg\ m^{-1}$ ).

C\_YIELD produces from this a table of yield reductions due to water stress, salinity stress and water logging in a file called YIELD.OUT, which contains the following data: Stage of growth, Crop reduction factor, actual crop evapotranspiration by stage, potential crop evapotranspiration by stage, yield response to water, yield response to salinity, yield response to water logging and the overall crop yield.

The model interface provides a series of options in the view results screen. The view result screen is shown in Figure 6. Graphical presentations of the following are available: Salinity in the root zone, Salinity by soil layer, Soil moisture in the root zone, Soil moisture by layer, Depth to water table and  $ET_C$  and  $ET_a$ .

## ACTUAL CROP EVAPOTRANSPIRATION

As part of this research the calculation of the actual crop transpiration (actual root water extraction rate) in the water transport module was modified to incorporate the effect of salinity on crop transpiration following the approach outlined in the FAO Irrigation and Drainage Paper No. 56 (Allen *et al.*, 1998). This included modifications the subroutine WAT\_UPT.FOR to calculate crop transpiration reduction factors. The FAO approach can be described as (Allen *et al.*, 1998):

$$ET_C = K_S \cdot K_C \cdot ET_o \quad (3)$$

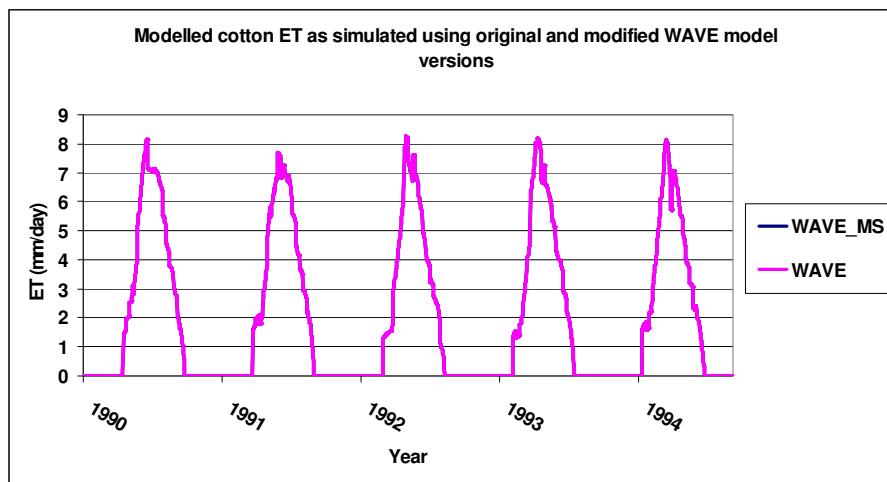
$$K_S = 1 - \frac{b}{K_y 100} (EC_e - EC_{threshold}) \quad (4)$$

Where  $EC_e$  is the mean electrical conductivity of the saturation extract for the root zone ( $dS\ m^{-1}$ ),  $EC_{threshold}$  is the electrical conductivity of the saturation extract when yield starts to become affected by salinity,  $K_y$  is a yield response factor,  $b$  is the reduction in yield per increase in  $EC_e$  ( $\%/(dS\ m^{-1})$ )

There were no field data on actual crop transpiration that would permit direct evaluation of the new functions in WAVE\_MS. In addition, no experimental data on crop transpiration under soil water and salinity stress could be found in the literature with which to evaluate the performance of WAVE\_MS. Qualitative evaluation of the revised model could only be made by comparing results produced by WAVE\_MS with those produced by the original model. Actual crop transpiration computed using the WAVE\_MS model should be lower than computed using the original version of WAVE when the  $EC_e$  exceeds the threshold value. This in turn affects the simulated soil moisture in the rootzone as well as simulated groundwater level, which should be higher than those simulated by the original model. In order to test the modifications, 11-year simulation runs were carried out with the original and modified versions of the

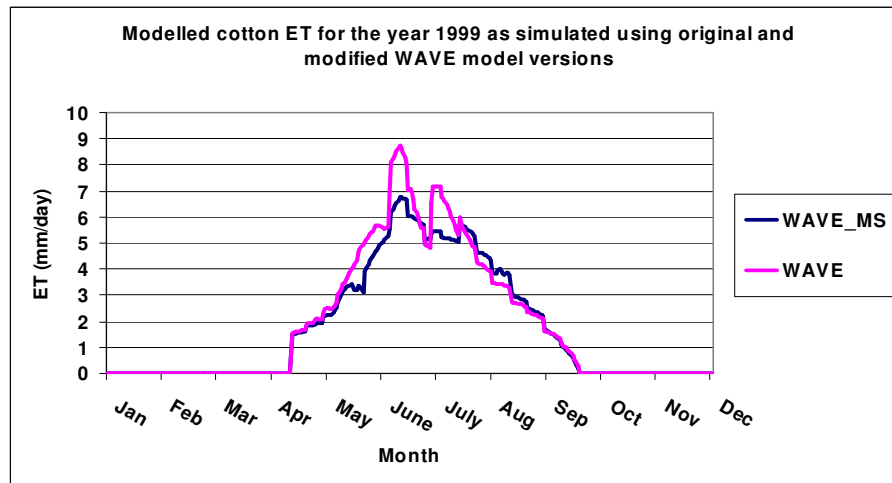


model. A cotton crop was used, with inadequate irrigation and drainage. The model was set up with three irrigation applications of 100 mm each in May, June and July, and two leaching applications of 100 mm each in January and February. The irrigation and leaching water were assumed to have a solute concentration of 1360 mg/l. There was no groundwater or solute flux out of the lower boundary of the model by deep percolation. The salinity build up was quite dramatic over the simulation period (1990-2000) but it did not exceed the threshold value (7.7 dS/m for cotton) for the first five years of simulation (1990-1994). As a result, actual daily cotton transpiration as simulated using WAVE\_MS and WAVE was exactly the same for this period (Figure 2).

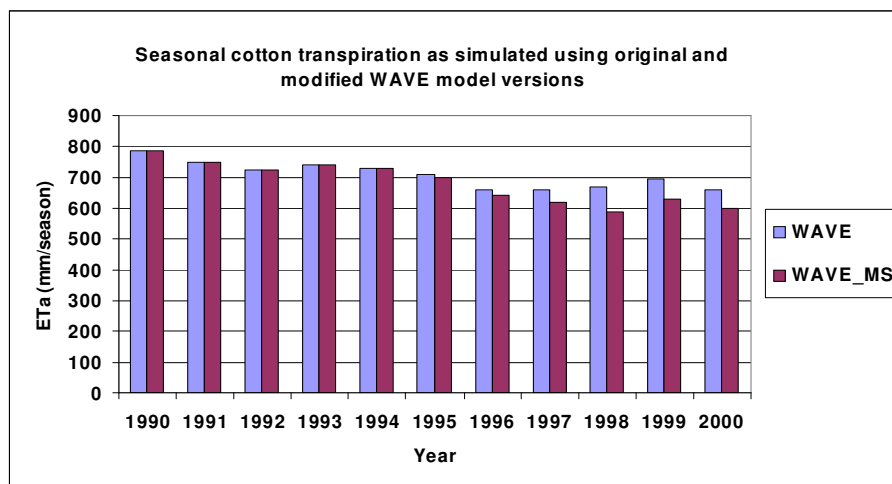


**Figure 2: Modelled cotton ET using original and modified WAVE model versions (1990-1994)**

The daily  $ET_a$  simulated using WAVE\_MS was lower than the  $ET_a$  simulated using the original model for the period 1995-2000 because the soil salinity exceeded the threshold value in this period. Figure 3 shows the reduction in cotton transpiration due to salinity stress for the 1999 calendar year. The time series of annual cotton transpiration simulated using both versions are shown in Figure 4. Clearly, the cumulative salinity build up is influencing actual evapotranspiration.

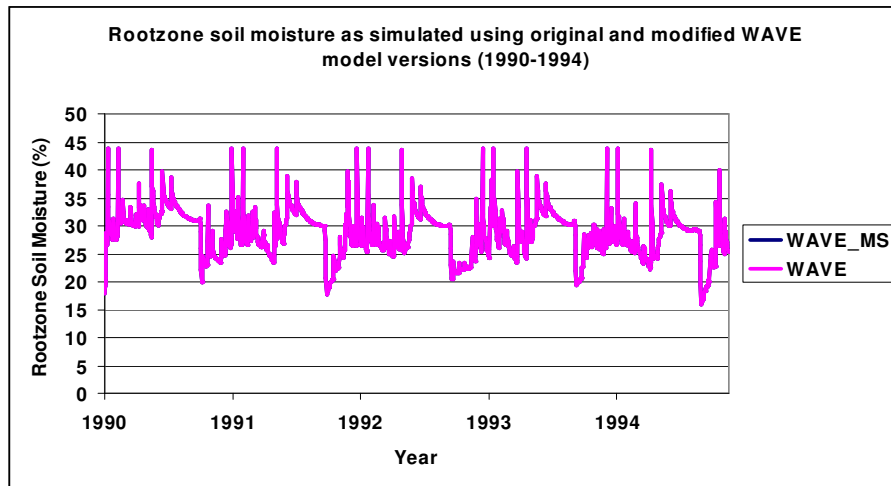


**Figure 3: Modelled cotton ET using original and modified WAVE model versions (1999)**

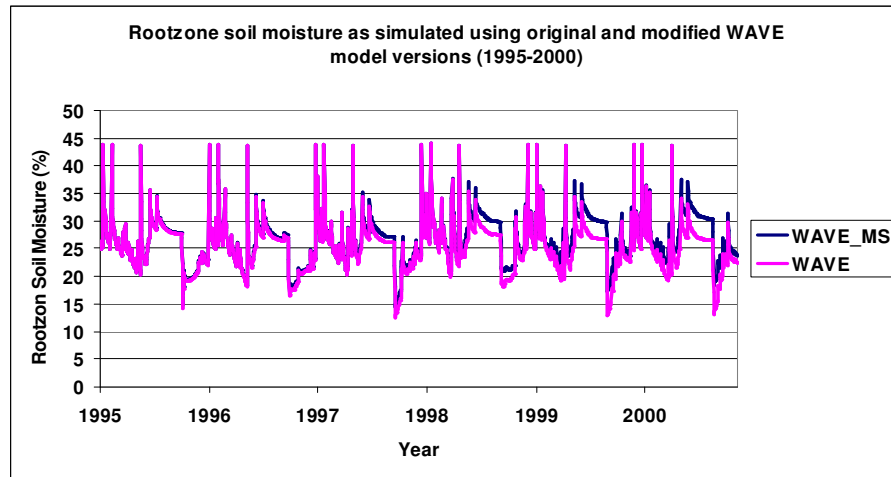


**Figure 4: Seasonal cotton transpiration using original and modified WAVE model versions (1990-2000)**

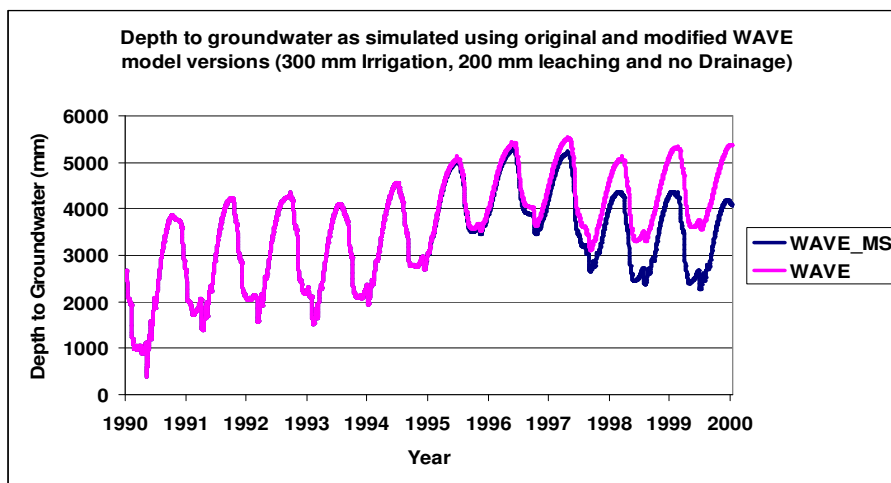
With a reduction in simulated actual evapotranspiration, root zone soil moisture content simulated by the WAVE\_MS model was higher than that simulated using the original version. The difference in the root zone soil moisture between the two versions increased as the effect of salinity on transpiration in the modified version increased. For the period 1990-1994 there was no difference in root zone soil moisture between the two versions (Figure 5). Figure 6 shows simulated soil moisture for the 1995-2000 period. Simulated groundwater levels are also affected by changes in actual evapotranspiration. With reduced evapotranspiration there is increased groundwater recharge and higher groundwater levels. Figure 7 shows groundwater depths simulated with both versions of the model. Higher groundwater levels (lower depth to water table) result in the 1995-2000 period.



**Figure 5: Soil moisture content in the rootzone using original and modified WAVE model versions (1990-1994)**



**Figure 6: Soil moisture content in the rootzone using original and modified WAVE model versions (1995-2000)**



**Figure 7: Groundwater depth using original and modified WAVE model versions (1995-2000)**

These test results verify that the WAVE\_MS model is capable of dealing with the combined effects of water and salinity stress. The results produced were satisfactory and as would have been expected. Full verification of the modified model would require experimental lysimeter data that were not available for this research. An impact of incorporating salinity stress is that water stress is reduced, as actual evapotranspiration is reduced through salinity stress. As mentioned above, the WAVE model calculates crop yield based on a yield reduction factor that represents a proportion of maximum yield. The final yield reduction factor is based on the product of three separate yield reduction factors, those due to water logging, water stress, and salinity stress. In the WAVE\_MS model, the potential evapotranspiration is reduced to a lower level when soil salinity in the rootzone exceeds certain value. This is not considered in the WAVE model. A comparison of the crop yield response produced from WAVE\_MS and from WAVE results is presented in Table 1. The relative influence of soil salinity on crop yield over 11 years simulation period using both WAVE and WAVE\_MS shows that crop yield as simulated using both versions was exactly the same for the period 1990-1994 as soil salinity did not exceed the threshold value because crop evapotranspiration remains the same over this period. Crop yield simulated using WAVE\_MS starts to decrease below that simulated using WAVE for the period 1995-2000 because the soil salinity exceeds the threshold value above which the evapotranspiration decreases below its potential value.

**Table 1: Simulated cotton yield using original and revised WAVE model versions**

<b>Year</b>	<b>1990</b>	<b>1991</b>	<b>1992</b>	<b>1993</b>	<b>1994</b>	<b>1995</b>	<b>1996</b>	<b>1997</b>	<b>1998</b>
<b>WAVE</b>	0.94	0.91	0.9	0.91	0.91	0.84	0.81	0.8	0.85
<b>WAVE_MS</b>	0.94	0.91	0.9	0.91	0.91	0.82	0.79	0.73	0.72

## CONCLUSIONS

In this paper the theory behind the WAVE model has been described. As part of this research, the model has been modified by incorporating the effect of salinity in addition to the effect of water stress on crop transpiration. The new version of the model is called WAVE\_MS. The modifications have been successful. The crop transpiration calculated by the revised model was lower than that calculated using the original version once soil salinity exceeded the threshold value for salinity stress. The root zone soil moisture and the ground water level were higher with the revised model because of the decrease in the crop transpiration. Full verification against experimental results has not been possible; however, Crop yield is also affected by the modifications in WAVE\_MS. Crop yields simulated by WAVE\_MS were lower than those simulated using the original version, once soil salinity exceeded the threshold value. The decrease in the crop transpiration due to salinity stress simulated using WAVE\_MS resulted in further yield reduction.

## REFERENCES

- Allen, R. L., Raes, D., Smith, M., 1998. Crop Evapotranspiration: Guidelines for computing crop water requirements. Irrigation and Drainage Paper No. 56, FAO, Rome, Italy.
- Doorenbos, J., Pruitt, W. O., 1977. Crop Water Requirements. Irrigation and Drainage Paper No. 24, Food and Agriculture Organisation, Rome, Italy.
- Feddes, R. A., Kowalik, P. J., Zaradny, H., 1978. Simulation of field water use and crop yield. Simulation Monographs. Pudoc, Wageningen.
- Fernandez, J. E., Slawinski, C., Moreno, F., Walczak, R. T., Vanclooster, N., 2002. Simulation the fate of water in a soil-crop system of a semi-arid Mediterranean area with the WAVE 2.1 and the EURO-ACCESS-II models. Agric. Water Management. 1738, 1-17.
- Gardner, W., 1958. Some steady state solutions of the unsaturated moisture flow equation with application to evaporation from a water table. Soil Sci. 85, 228-232.
- Maas, E. V., Hoffman, G. J., 1977. Crop salt tolerance: current assessment. Irrigation and Drainage Engineering, ASCE. 103(IR2), 115-134.
- Mott MacDonald, 2000. Water Resources and Land Improvement Project (WRMLIP). Working Paper No. 11b. Status Report. Govt. of Kazakhstan.
- Mott MacDonald, 2003a. Water Resources and Land Improvement Project (WRMLIP). Working Paper No. 27. Soil Monitoring Programme Results. Govt. of Kazakhstan.
- Mott MacDonald, 2003b. Water Resources and Land Improvement Project (WRMLIP). WRMLLLIP\_WAVE model user's manual. Govt. of Kazakhstan.
- Mott MacDonald, 2003c. Water Resources and Land Improvement Project (WRMLIP). Working Paper No. 29. WAVE Set-up, Calibration and Prediction. Govt. of Kazakhstan.
- Mott MacDonald, 2003d. Water Resources and Land Improvement Project (WRMLIP). Scientific Report on Organisation Works and Field Investigation on Pilot Areas (2000-2002). Govt. of Kazakhstan.
- Ragab, R., 2002. A holistic generic integrated approach for irrigation, crop and field management: the SALTMED model. Environmental Modelling & Software. 17, 345-361.
- Rao, N. H., Sarma, P. B., Chander, S., 1988. A simple dated water production function for use in irrigated agriculture. Agric. Water Management. 13, 25-32.
- Simunek, J., Huang, K., and van Genuchten, M. Th., 1999. The HYDRUS-2D software package. IGWMC-TPS-53. International Ground Water Modelling Centre, Colorado School of Mines, Golden, Colorado.
- Simunek, J., Suarez, D. L., and Sejna, M., 1996. The UNSATCHEM Software package Version 2.0. Research Report No. 141. USDA, ARS, U.S. Salinity Laboratory, Riverside, California.
- Van Dam, J. C., Huygen, J., Wesseling, J. G., Feddes, R. A., Kabat, P., van Waslum, P. E. V., Groenendijk, P., Van Diepen, C. A., 1997. Theory of SWAP version 2.0. Technical Document 45. Wageningen Agricultural University, Wageningen, The Netherlands.
- van Genuchten, M. Th., 1980. A close-form equation for predicting the hydraulic conductivity of soil. Soil Sci. Soc. Am. J. 44, 892-898.

Vanclooster, M., Viaene, P., Diels, J., Christiaens K., 1994. WAVE a mathematical model for simulating water and agrochemicals in the soil and vadose environment. Institute for Land and Water Management, Katholieke University Leuven, Leuven, Belgium.