

FLOW RATE MEASUREMENTS BY FLUMES

W. Dąbrowski¹ and U. Polak²

¹ Department of Environmental Engineering, Cracow University of Technology,
Warszawska 24, 31-135 Kraków, Poland, E-mail: wdabrow@pk.edu.pl

² Institute of Environmental Engineering, National Higher East-European School,
Tymona Terleckiego 4, 37-700 Przemyśl, Poland, E-mail: inz-srod@pwsw.pl

ABSTRACT

Shortcomings of the ISO Standard 4359 flumes have been pointed out, focused on the concept of a boundary layer, applied for computing flow rate through Venturi and Palmer–Bowlus flumes. Two different approaches to flow rate measurements by flumes have been compared. The first one requires and the second approach does not require crossing of critical depth by the liquid free surface somewhere in a flume throat. Computing the critical depth position in a flume throat was described and a method of improving flow rate computations accuracy suggested. This method differs from the ISO Standard 4359 in computing the distance by which depth and width of liquid stream in a throat should be modified according to the boundary layer concept. While the ISO Standard 4359 prescribes in calculating this distance for the end of a flume throat, the authors suggest calculating it for a cross section of critical depth. Some examples of flow rate computations are presented, showing a visible improvement in flow rate computations after applying this suggestion.

Keywords: Flume, Boundary layer, Venturi flume, Palmer–Bowlus flume

1. INTRODUCTION

Water management requires precise measurements of amount of water flowing through streams and rivers, especially in arid regions. Even more important is measurement of flow rate in sewerage for: predicting which sewers are overloaded, adjusting wastewater treatment plants operation parameters to an actual situation, calculating financial compensations for pollution load to the environment and calibrating sewerage mathematical models.

Professional monitoring is expensive so it is strongly advised to investigate the situation in advance. Before installing a flume in a channel it is absolutely necessary to recognize a range of flow rates which may happen. This can be done simply by measuring channel flow depth and slope, or by using one of several available nonstationary monitoring methods. In sewerage other inaccurate, but cheap and unsophisticated methods of monitoring can be used, such as installing wooden black boards covered by chalk, to predict maximum flow depths, installing floats carried out to opposite side of combined sewerage overflows during each spill, calculating flow

rates from depth of flow, interpreting trajectory of free outflow streams to manholes, using vertical floats, Pitot tubes and tracers are used to estimate flow rate in gravitational sewers.

In general measurements are done on stationary or temporary bases. Different kinds of flumes are relatively inexpensive and quite accurate solutions to monitoring requirements in part depth flow surface water channels and wastewater sewers. Flumes are devices having constricted cross section of free surface flow through channels. This constriction results in an increase of flow depth above the flume. Flow rate is calculated from measured depth of flow if an equation relating this depth with flow rate is known. Accuracy of flow rate measurements by flumes is a little lower than offered by sharp crest weirs, but operation much more reliable, and much lower demands for cleansing. In conclusion, flumes offer quite precise measurements of surface water and wastewater flow rate for a reasonable total price of construction and operation.

The paper focuses on mistakes done in the ISO4359 standard [1], still being in use for interpretation of flow rate measurements, by rectangular, trapezoidal and U-shaped flumes.

2. TWO DIFFERENT APPROACHES

A Palmer-Bowlus flume is presented in Figures 1, 2 and will be used to discuss two different approaches for measurements of flow rate. To calculate flow rate through a flume from the Bernoulli equation (1) and from two mass balance equations (2), (3) it is enough to measure flow depth h_1 before the flume and flow depth h_2 somewhere inside of the flume throat. The following notation is applied:

h_1 – flow depth in the cross section 1,

h_2 – flow depth in the cross section 2,

$A_1(h_1)$ – wetted area in the cross section 1,

$A_2(h_2)$ – wetted area in the cross section 2,

Q – flow rate,

α_1, α_2 – energy (Saint–Venant) coefficients of sections 1 and 2, respectively,

g – acceleration due to gravity,

v_1, v_2 – average velocities of flow in the cross sections 1 and 2, respectively

Δh_{loss} – energy dissipation caused by friction to flow, assumed here to be negligible.

$$h_1 + \frac{\alpha_1 v_1^2}{2g} = p + h_2 + \frac{\alpha_2 v_2^2}{2g} + \Delta h_{\text{loss}} \quad (1)$$

$$A_1 v_1 = A_2 v_2 \quad (2)$$

$$Q = A_1 v_1 \quad (3)$$

As the geometry of a given flume is known, so both wetted areas A_1 , A_2 are known functions of flow depths h_1 , h_2 . Assuming for a moment that the energy (Saint-Venant) coefficients α_1 , α_2 are known the unknown variables are v_1 , v_2 and Q , so the number of unknowns equals to the number of equations. In this approach it is not possible to create a chart $Q(h_1)$, or $Q(h_2)$. Moreover, errors of both flow depths h_1 , h_2 measurements contribute to an inaccuracy of the flow rate indirect measurement. However, the method of measurement is reliable, range of flows rather large, and problems with sediment are usually minor.

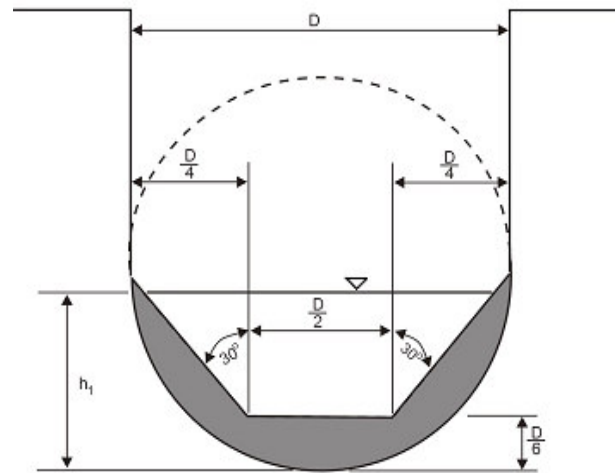


Fig. 1 Geometry of a cross section through a Palmer-Bowlus flume

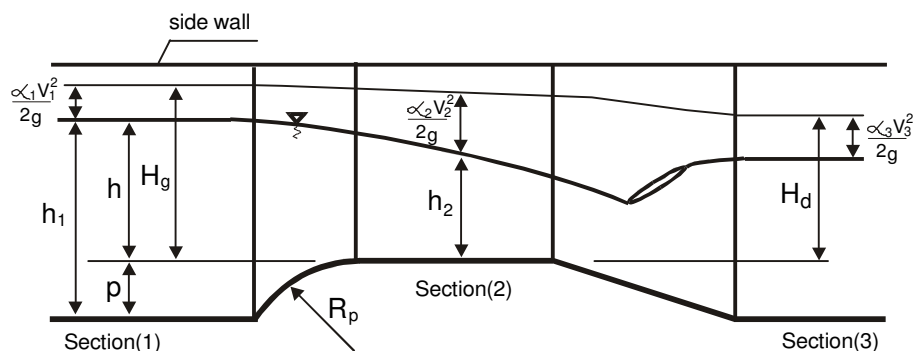


Fig. 2 A longitudinal section through a Palmer-Bowlus flume, as an illustration to the set of equations (1), (2), (3), written for the cross section 1 before the flume and for the cross section 2 in the flume throat

International standards for flumes ISO 4359, 9826, 8368 [1-3], ASTM D1941-91 [4] and Polish [5] Standards prefer another approach, based solely on the assumption that somewhere in a flume throat the free wastewater surface goes through the critical depth, $h_2 = h_{cr}$. Due to one additional equation for the critical depth one more unknown variable can be calculated and it is possible to omit any measurements in the flume throat. In this case a graph $Q(h_1)$ may be constructed and only the error of h_1 measurement contributes to the error of predicting Q . However, if the free liquid

stream surface does not cross out the critical depth h_{cr} , because of flow rate exceeds the proper range, the result of measurements is misleading, the error is unknown and likely really substantial. In some way the choice between these two approaches depends on a preference between high accuracy but low reliability versus lower accuracy but higher reliability of measurements. Unfortunately flumes require crossing critical depth by free wastewater surface somewhere in the throat, are operated under higher impoundment and may be applied for more narrow range of flow rates in comparison with the other kind of flumes. Someone may also consider the cost of monitoring as in the second approach it is enough to install only one ultrasonic device to measure flow depth, while in the first approach two such devices are necessary. However, the cost of flow depth measurements is low.

3. ISO 4359 INACCURACY

In the previous paragraph it was assumed temporarily that the energy (Saint–Venant or Coriolis) coefficients α_1 , α_2 are known, which is not the case. To overcome this difficulty a concept of a boundary layer was applied in the ISO 4359 standard. In this concept it was assumed that inside the flume the flow is turbulent and in long period of time, it can be characterized by the same value of an average flow velocity across the whole cross section perpendicular to flow. On the contrary velocity of flow starts from zero at the wall of a flume and rises visibly in a layer of a thickness y_1 , as presented in Figure 3. In the boundary layer concept instead of predicting α_1 , α_2 both depth and width of wastewater stream are modified by a calculated value to ensure the same mass balance, or the same momentum, or the same energy, by assuming that the average velocity in the liquid stream of the modified dimensions equals to the average velocity of flow in the turbulent part of the stream. For example, if Figure 3 describes a velocity profile close to the bottom of a sewer to satisfy the mass balance the flow depth should be decreased by δ^* , for which the area “a” equals the surface area “b”. Then the coefficients α_1 , α_2 are assumed to be equal to 1.

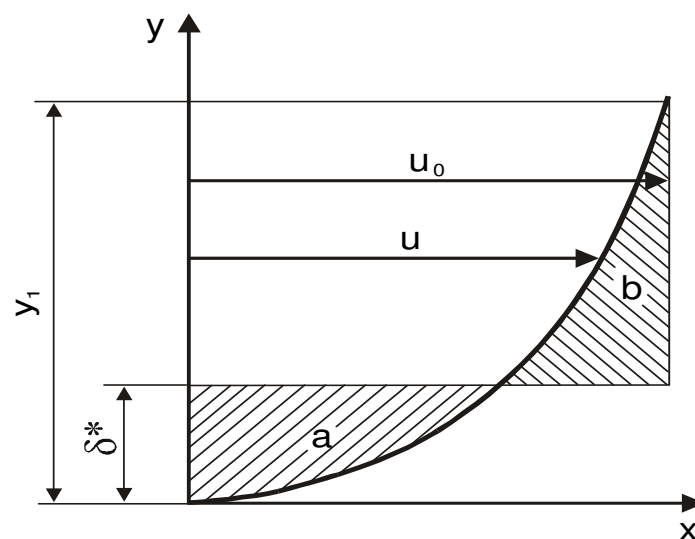


Fig. 3 An idea of a boundary layer concept in a reference to measurements done by flumes [1]

In the ISO 4359 Standard the modification of wastewater stream was calculated in such a way to ensure the same Q , but in fact only equations (2), (3) express the mass balance law, while equation (1) is the energy conservation equation. It means that dimensions of the wastewater stream in a flume should be modified differently for equation (1) and differently for both equations (2) and (3).

Moreover concept of the boundary layer was developed for objects inside of a liquid stream and it is not well applicable for flumes as the flume starts from a sewer wall, where velocity field is deformed anyway.

Finally, according to the ISO 4359 standard the thickness δ^* is to be calculated at the end of the flume throat, while it should be predicted for the cross section 2 from Figure 2, so for the distance where the free liquid surface crosses critical depth of flow.

4. ATTEMPT OF AN IMPROVEMENT

If only some mistakes are corrected it is difficult to guess in advance whether the result of calculations improves or not. In the case of ISO 4359 Standard existing literature does not deliver information on calculation of the flow depth and width modifications δ^{***} , to ensure the same energy, depending on a kind of a flume. In this situation only one of the ISO 4359 Standard mistakes was corrected by the authors. In our calculations the liquid stream depth and width was modified by δ^* at the cross section of the critical flow depth, instead of at the end of a flume throat, as suggested by the ISO 4359 Standard. For the Reynolds number $Re^* < 3 \times 10^5$ (calculated for the characteristic dimension x , so different than Re) the distance δ^* value was calculated from the Blasius formula, as suggested by the ISO 4359 Standard. To make the improvement it was necessary to compute first the position of the cross section 2 for which $h_2 = h_{cr}$. This was done solving the differential equation (4) for steady gradually varied flow (Chow [6], Smith [7], Sawicki [8], Yen and Wenzel [9]).

$$\frac{dh}{dx} = \frac{S_0 - S_f + \frac{\alpha Q^2}{gA^3} \frac{\partial A}{\partial x}}{1 - \frac{\alpha Q^2 B}{gA^3}} \quad (4)$$

in which the following notations are used:

- B - width of stream free surface along the flow,
- x - distance measured from the beginning of a flume,
- S_0 - bottom sewer slope,
- S_f - energy line slope,
- A - wetted surface area,
- α - energy (Saint-Venant) coefficient, and
- g - acceleration due to gravity.

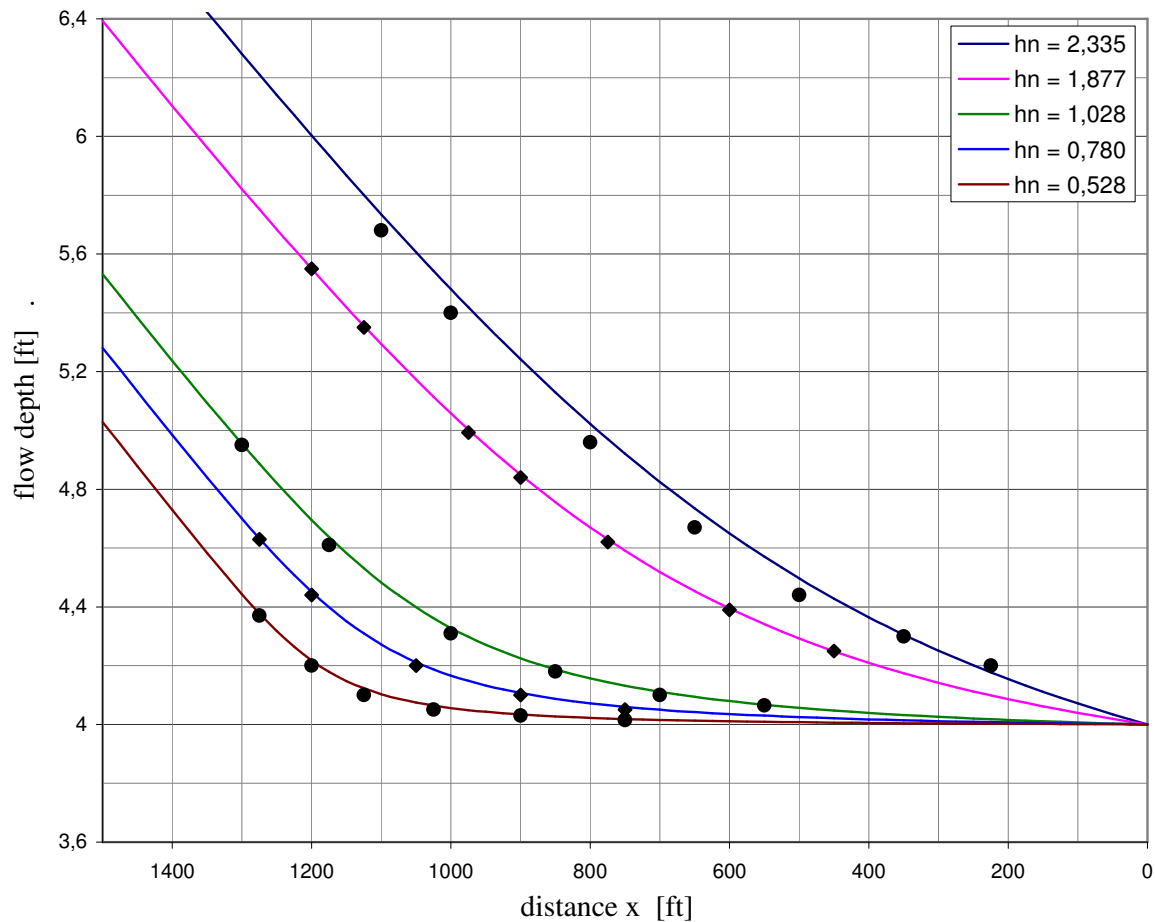


Fig. 4 An example of several software tests, this time using free surface water profiles measured (points) in a prismatic channel of a partially blocked outflow [10], [11]. Lines represent results of computations.

In computations the Manning formula was mostly used for predicting energy slope S_f with the value of Manning's roughness coefficients a little higher than expected for several materials covered by biofilm. Assuming slightly higher roughness in equation (4), when applied for quite fast changes in flow depth $h(x)$ is suggested in the literature, because this partially compensates the effect of using equation (4) for non-gradually changes in flow velocity along the flume. The coefficient α in equation (4) was predicted by a trial and error method for each of Venturi and Palmer-Bowlus flume sizes separately, in such a way, to reach the upper and the lower limits of the Q measurement ranges for critical depths close to the beginning and to the end of flumes throats.

Correctness of the numerical solution to equation (4) was carefully verified in numerical experiments carried out for data known from the literature. An example of such a test is presented in Figure 4, in which points show depth of flow measured by Lansford and Mitchell [10] and reported by Prasad [11] in a prismatic sewer of

partially blocked outflow and lines represent values predicted from computations. The software was also tested on the measurements published by Chow [6] for a nonprismatic sewer. The results of computations are compared with the measurements in Table 1. Original US units are used both in Figure 4 and in Table 1.

Table 1 Comparison between computed depths and reported ones by Chow [6] in a noncircular channel

| Number of the cross section (i) | Distance between the following cross sections [ft] | Width of the sewer [ft] | Depth of flow [ft] reported by Chow [6] | Computed depth of flow [ft] |
|--|--|-------------------------|---|-----------------------------|
| For Q = 6200 [cfs] n = 0.014 S₀ = 0.08 | | | | |
| I | | 130 | 3.54 | |
| II | 7.0 | 120 | 3.48 | 3.48 |
| III | 7.0 | 110 | 3.58 | 3.57 |
| IV | 8.0 | 100 | 3.71 | 3.71 |
| V | 10.0 | 90 | 3.87 | 3.86 |
| VI | 12.0 | 80 | 4.08 | 4.07 |
| VII | 15.0 | 70 | 4.35 | 4.34 |
| VIII | 12.0 | 65 | 4.41 | 4.41 |
| IX | 23.0 | 60 | 4.28 | 4.27 |
| For Q = 6200 [cfs] n = 0.014 S₀ = 0.1788 | | | | |
| X | 62.5 | 55 | 3.123 | 3.122 |
| XI | 62.5 | 50 | 2.855 | 2.854 |
| XII | 40 | 46.8 | 2.817 | 2.815 |
| For Q = 6200 [cfs] n = 0.014 S₀ = 0.033 | | | | |
| XIII | 22.5 | 45 | 2.958 | 2.957 |
| XIV | 62.5 | 40 | 3.399 | 3.399 |
| XV | 62.5 | 35 | 3.948 | 3.947 |

After receiving positive results of numerical tests the software was used to compute localization of critical depths in flumes throats. An example of such calculations is presented in Figure 5 for a Palmer-Bowlus flume, installed in a sewer of the inside diameter 0.381m. This time the energy line slope S_f in equation (4) was calculated from the Colebrook–White equation for the sand roughness coefficient $k_s = 0.8\text{mm}$.

The range of computations h_1/D was established based on the ISO 4359 Standard. By L_{cr} a distance from the beginning of the flume to the cross section of critical depth was denoted. As previously, h_1 is the depth of flow measured before the flume (cross section 1 in Figure 2).

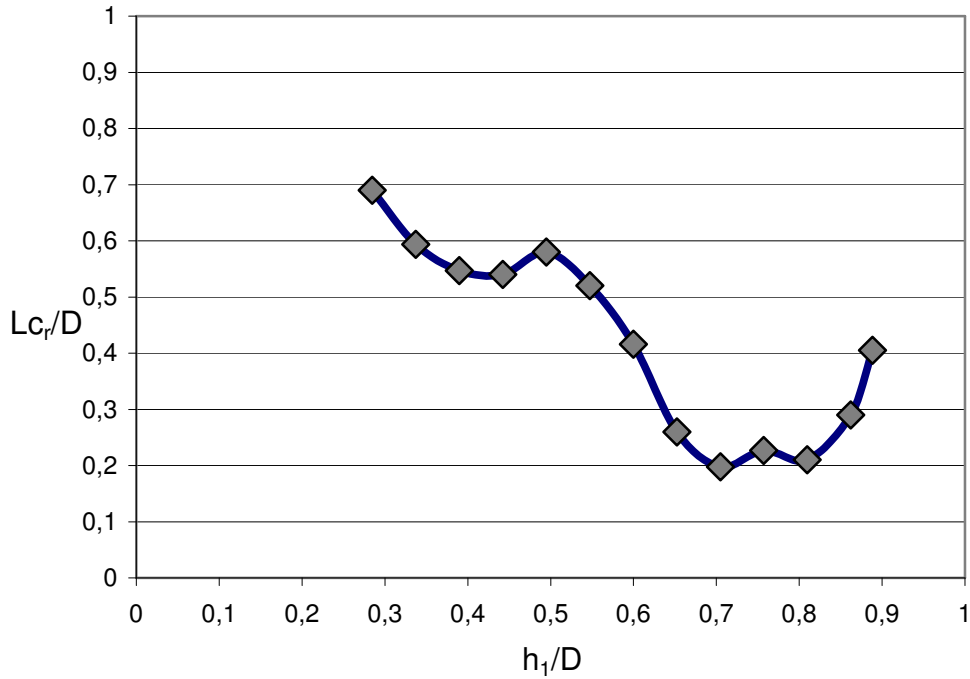


Fig. 5 Position of critical depth in a Palmer–Bowlus flume, installed in a sewer of inside diameter 0.381m and of sand roughness coefficient k_s equal to 0.8 mm

Similar computations were done for several sewer diameters. Then the value of δ^* was calculated from the Blasius formula [1] for the distance L_{cr} . All other calculations were done exactly according to the ISO 4359 Standard.

5. RESULTS

In Figures 6 and 7 an example of one of the tests is presented. Assuming, somewhat unrealistically, that the experimental data Q_{exp} published by Grant and Dawson [12], are in 100% precise, and Q_c is the computed value of flow rate, the relative error of calculations ΔQ_c was defined by equation (5):

$$\Delta Q_c = \{(Q_c - Q_{exp}) / Q_{exp}\} * 100\% \quad (5)$$

The error ΔQ_c resulting from calculations done exactly according to the ISO 4359 Standard is presented in Figure 6, the error ΔQ_t received from computations done according to the modified method by calculating δ^* in the cross section of $h_2 = h_{cr}$ in

Figure 7. For Venturi flumes, a simple algebraic equation for the position of h_{cr} in a flume throat was developed but it may not be published here, as it is a part of another manuscript. However, looking for such a simple arithmetical equation for positioning of critical depth in the Palmer-Bowlus flumes was unsuccessful. As can be noticed from Figures 6 and 7 correcting the computations of Q , by calculating δ^* at the distance where flow depth $h_2 = h_{cr}$, significantly improved the results. Similar improvements were received for all other Venturi flumes from the Uniklar 77 [5] catalogue.

Visible improvements of Q computing accuracy were also obtained for calculating δ^* ($h_2 = h_{cr}$) instead of δ^* (L_2), where L_2 is the length of the Palmer-Bowlus flume throat. For example relative errors ΔQ_i in Figure 9 are all below 1% in contrast to ΔQ_c from Figure 8. Because solving equation (5) by everyone designing his own flume is unrealistic, so instead of calculating δ^* ($h_2 = h_{cr}$) an attempt was made to improve the computations accuracy by applying in the ISO 4359 standard δ^* ($h_2 = 0.5 L_2$), where L_2 is the length of the Palmer-Bowlus flume throat. In most cases a visible improvement was reached.

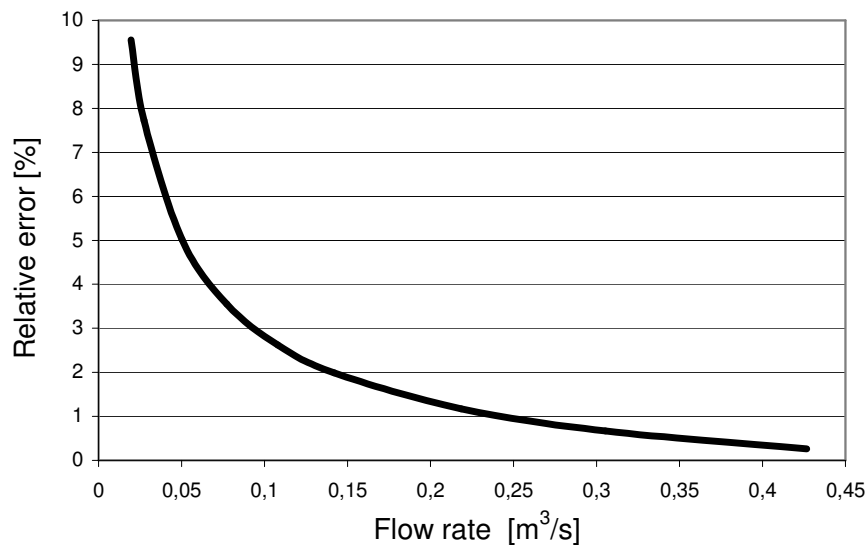


Fig. 6 Relative error ΔQ_c defined by Eqn. (5) for computations done exactly according to the ISO 4359 Standard for a rectangular Venturi flume (Uniklar 77 [5]) installed in a channel of a 1 m width

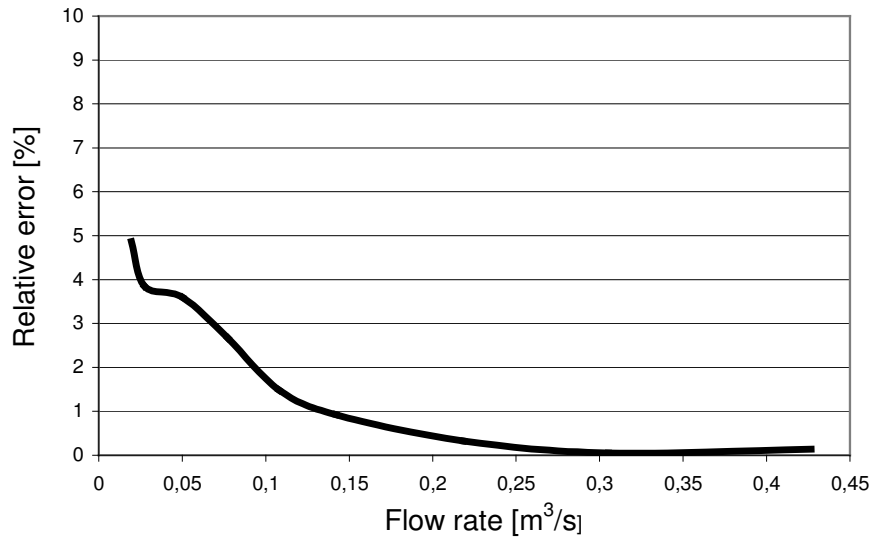


Fig. 7 Relative error ΔQ_t defined by Eqn. (5) for computations done according to the modification of the ISO 4359 Standard, including computing of δ^* in the distance for which $h_2 = h_{cr}$. Computations refer to a rectangular Venturi flume (Uniklar 77 [5]), installed in a channel of a 1 m width

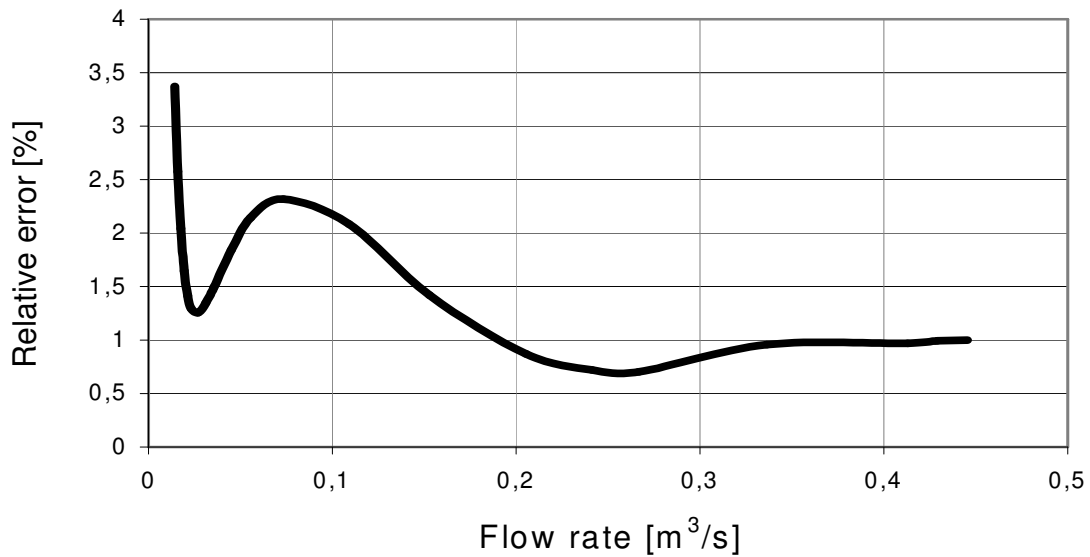


Fig. 8 Relative error ΔQ_c defined by Eqn. (5) of calculating flow rate Q , from measurements of flow depth h_1 , using exactly the ISO 4359 procedure for the Palmer-Bowlus flume. The Palmer-Bowlus flume was installed in a circular sewer of inside diameter $D = 0.762$ m

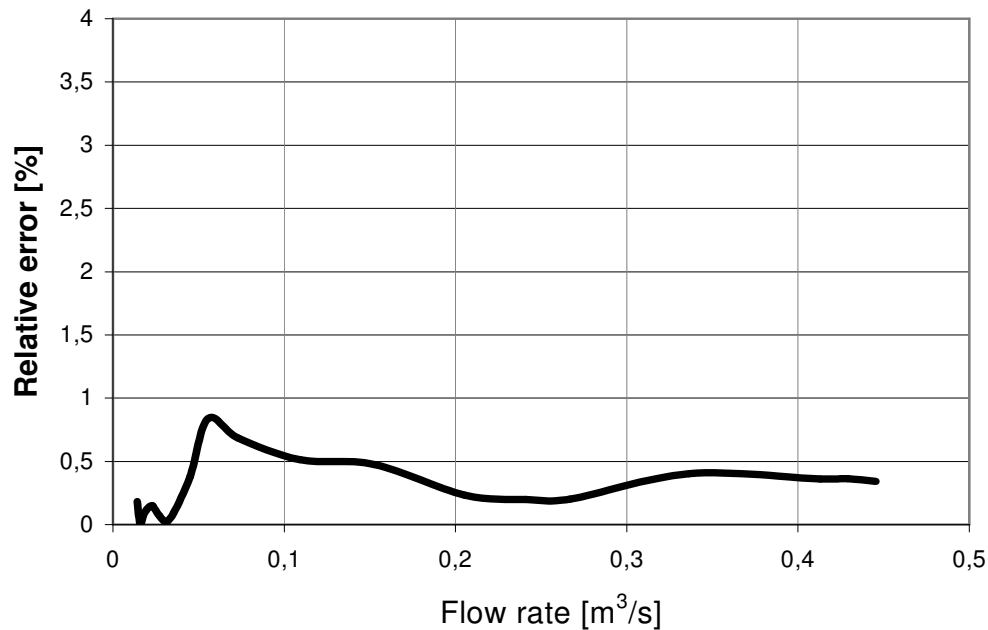


Fig. 9 Relative error ΔQ_t defined by Eqn. (5) of calculating flow rate Q , from measurements of flow depth h_1 , using a modification of the ISO 4359 procedure, based on calculation of δ^* in the cross section of the Palmer–Bowlus flume for which $h_2 = h_{cr}$. The Palmer–Bowlus flume was installed in a circular sewer of inside diameter $D = 0.762$ m

6. DISCUSSION AND CONCLUSIONS

Flumes are economically reasonable and quite precise tools for predicting flow rates in channels and free surface sewers. Not always a suitable flume is available on the market so methods of flow rate measurements based on simple theory are important. The concept of the boundary layer applied in the ISO 4359 standard is unfortunate because it was thought mostly for describing a velocity field around an obstacle inserted inside a liquid stream. Instead it is applied for computing flow rate through Venturi, Palmer–Bowlus and U-shaped flumes. Moreover the same modification of the stream depth and width was suggested by the Standard for equation (1) and equations (2), (3). This is incorrect, because equation (1) describes the energy balance and equations (2),(3) the mass conservation, so if the boundary layer concept is applied different modifications of stream dimensions should be made for equation (1) and for equations (2), (3). Moreover this modification should be calculated for the cross section of the flume throat, for which $h_2 = h_{cr}$, while in the ISO 4359 Standard it was suggested to be done for the end of the throat. Removing this mistake alone resulted in visible improvements in computation of Q accuracy for all flumes investigated here.

ACKNOWLEDGEMENTS

The authors are grateful to Prof. J. Sawicki from the Technical University of Gdańsk for his suggestions to conducting the research reported here. The Polish National Foundation (KBN) has been sponsoring the manuscript from the project 5633/B/T02/2010/38.

NOMENCLATURE

| | |
|----------------------|--|
| A | - wetted surface area |
| $A_1(h_1)$ | - wetted area in the cross section 1 of Figure 2 |
| $A_2(h_2)$ | - wetted area in the cross section 2 of Figure 2 |
| B | - width of stream free surface along the flow |
| D | - inside pipe diameter |
| g | - acceleration due to gravity |
| h_1 | - flow depth in the cross section 1 of Figure 2 |
| h | - part of flow depth h_1 in the cross section 1 of Figure 2 which refers to is measured from the level of horizontal flume throat bottom |
| h_{cr} | - critical depth of flow |
| h_2 | - flow depth in the cross section 2 of Figure 2 |
| H_d | - height of energy line after the flume, measured in respect to the flume throat bottom |
| H_g | - height of energy line before the flume, measured in respect to the flume throat bottom |
| h_n | - normal depth of flow |
| L_{cr} | - distance from the beginning of the throat to the cross section of a critical depth |
| L_2 | - length of a flume throat |
| P | - height of the horizontal flume throat bottom measured in respect to the upflow sewer bottom, at the beginning of the flume |
| Q | - flow rate |
| Q_c | - computed value of flow rate |
| Q_{exp} | - measured value of flow rate |
| R_p | - radius of a flume inlet converging section bottom |
| S_f | - energy line slope |
| S_o | - bottom channel slope |
| v_1, v_2 | - average velocities of flow respectively in the cross section 1 and in the cross section 2 of Figure 2 |
| x | - distance measured from the beginning of a flume |
| α | - energy (de Saint–Venant) coefficient |
| α_1, α_2 | - energy coefficients for the cross sections 1 and 2 in Figure 2 respectively, |
| Δh_{loss} | - energy dissipation caused by friction to flow |
| ΔQ_c | - error in predicting flow rate, defined by Equation (5) |
| ΔQ_t | - error in predicting flow rate, defined analogously to ΔQ_c from Equation (5), but for δ^* calculated for the cross section of $h_2 = h_{cr}$, instead of at the end of |

the flume throat

δ^* , δ^{**} , δ^{***} - three different distances by which depth and width of a stream should be decreased to receive the same mass, or momentum, or energy, assuming everywhere the same velocity of flow as in the middle of the stream

REFERENCES

- [1] ISO 4359, "Liquid flow measurement in open channel. Rectangular, trapezoidal and U-shaped flumes". International Organization for Standardization, 1983, Geneva, Switzerland.
- [2] ISO 9826, Measurement of liquid flow in open channels – Parshall and SANIIRI flumes, 1992, Geneva, Switzerland.
- [3] ISO 8368, Hydrometric determinations – Flow measurements in open channels using structures, guidelines for selection of structure, 1999, Geneva, Switzerland.
- [4] ASTM D 5640, Standard guide for selection of weirs and flumes for open – channel flow measurement of water, American Society for Testing and Materials, 2008.
- [5] Uniklar 77, Venturi flumes KPV, Center for Communal Technology, Warsaw, 1981 (in Polish).
- [6] Chow V. Te., "Open Channel Hydraulics", McGraw-Hill Book Company, 1959.
- [7] Smith K.V.H., Computer determination of critical depth control points in open channel flow, Proc. Instn. Civ. Engrs., 1972, 461-470
- [8] Sawicki J., Flow with unconstrained liquid surface, Wydawnictwo Naukowe PWN, Warsaw, 1998 (in Polish).
- [9] Yen B., Wenzel H., Dynamic equations for steady spatially varied flow, Journal of the Hydraulics Division, ASCE, 96(HY3), 1970, 801-814.
- [10] Lansford W.M., Mitchell W.D., An investigation of the backwater profiles for steady flow in prismatic channels, University of Illinois Engineering Experiment Station Bulletin No. 381, Urbana, Illinois - USA, March 1949.
- [11] Prasad R., Numerical method of computing flow profiles, Journal of the Hydraulics Division, ASCE, 1970, 96, HY1, 75-86.
- [12] Grant D.M., Dawson D.B., Isco open channel flow measurement handbook, Isco Inc., 1995.