AN INTEGRATED FINITE DIFFERENCE MODEL FOR WATER RESOURCES MANAGEMENT

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ABSTRACT

In this paper it is proposed to study by means of a numerical model the available ground water resources near Benghazi region (Libya). In fractured media where the water flows mainly through fractures, the major concern of the modeller is to follow the principal directions of motion. For this reason, the integrated finite difference scheme is used as the solution method because it is capable of projecting the problem on the flow lines. The model consists of two phases, that of calibration and verification of the past records, and that of prediction of the future performance of the aquifer. Simulation of steady state conditions and historical groundwater withdrawals were consistent with observed data. The model determines the hydro-geologic parameters of the aquifer, estimates the net recharge, and simulates the response of the aquifer by testing various management schemes of irrigation-development plans in advance of their adoption. The numerical results of the transmissivity values (ranging from $1 \times 10^{-4}$ to $2 \times 10^{-2}$ m$^2$/s) are irregularly distributed in the project area confirming the great heterogeneity of the basin. The storage coefficient values obtained from the available data (i.e. $S = 9\%$) is found to be high. A better matching between the simulated and the observed water levels is achieved when reducing its values to a range from 2 to 3.3%. The average capacity of the aquifer in supporting the future domestic and agricultural abstractions is confirmed at wadi Ahmar area where a gradual piezometric drawdown, which is acceptable, is observed. On the other hand, at wadi Al Bab the drawdown exceeds the exploitability limits.

Keywords: Finite Difference, numerical model, water management, aquifers

1. INTRODUCTION

Due to the importance of ground water resources in Benghazi region, Khan et al. [1], it is proposed to study by means of a mathematical model the available ground water resources and analyse the effects of increased pumping in the wadi Al Bab and wadi Ahmar ground water basins. The study area is located on the south-western flank of the Jable Akhdar mountain range and extends east approximately 30 to 70 km southeast of Benghazi City. It comprises the wadi Al Bab catchment area (1035 km$^2$), the wadi Ahmar catchment area (307 km$^2$), the coastal wadis catchment area (150 km$^2$).
and a strip of the plain at the eastern edge of the coastal plain (360 km²). The total study area amounts to approximately 2000 km².

2. OBJECTIVES OF THE STUDY

Based on the integrated finite difference method, a mathematical model for wadi Albab and wadi Ahmar groundwater basins is developed. Calibrating the model under steady and transient-state conditions using the available data, thus obtaining the aquifer parameters. The above model is used to predict the future response of the basins for different management schemes. The developed model will be useful to study the behaviour of the aquifers under the increased abstractions for the newly developed farms in this area.

3. GOVERNING EQUATION

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t} - \frac{q(x, y, t)}{T}$$  \hspace{1cm} (1)

Although strictly valid only for confined aquifers, Equation (1) can also be used for unconfined aquifers by allowing the transmissivity to vary with time as the saturated thickness changes, Wange and Anderson [2]. It can also be used to solve for steady-state conditions by specifying a large time step, Prickett and Lonnquist [3], or by setting the storage coefficient equal to zero, Trescott and Larson [4].

4. INTEGRATED FINITE DIFFERENCE METHOD

The central concept to the integrated finite difference method is to discretize the total flow domain into conveniently small sub-domains or elements and evaluate the mass balance in each element, Narasimhan and Witherspoon [5]. The final result is an algebraic equation for each node in the grid system. As with other finite difference methods, it is assumed that all recharge or withdrawal to and from the nodal area occurs at the node point and that water level in the entire nodal area is the same as at the node point. The difference differential equation for a node B connected to adjoining elements $i = 1$ to $n$ can be written as:

$$\sum_i \frac{T_{ib}}{D_{ib}} W_{ib} (h_i^{t+1} - h_B^{t+1}) + A_B q_B^{t+1} = \frac{A_B}{\Delta t} S_B (h_B^{t+1} - h_B^t)$$  \hspace{1cm} (2)

where $i$ and $B$ are the adjacent contiguous node and node in question, $h_i$ and $h_B$ are the water table elevations at nodes $i$ and $B$, $A_B$ is the area of polygon associated with node $B$, $W_{ib}$ is the length of perpendicular bisector associated with nodes $i$ and $B$,
\( T_{ib} \) is the transmissivity at mid point between nodes \( i \) and \( B \), \( D_{ib} \) is the distance between nodes \( i \) and \( B \), \( Y_{ib} = T_{ib} W_{ib} / D_{ib} \) is the conductance of the interface separating nodes \( i \) and \( B \), \( S_B \) is the storage coefficient of node \( B \), and \( q_B \) is the net volumetric flow rate per unit area at node \( B \). The superscripts \( t \) and \( t+1 \) represent continuous points along the time axis (i.e., \( t+1 = t+\Delta t \)). Equation (2) states that the summation of sub-surface flow between a given area and its surrounding areas plus surface inflow or outflow rate to or from the given area must equal the rate of change of storage in the given area. With the zonal configuration defining values of \( W \), \( D \), and \( A \), and estimates from hydrogeological data for \( S \), \( T \), and \( q \), time variation of \( h \) over the aquifer can then be computed from solution of the system of simultaneous equations. The reasons for adopting the integrated finite difference method are as follows: (1) it allows for easy boundary approximations, (2) in fractured media, the water flows mainly through fractures and the major concern of the modeller is to follow the principal directions of motion and this method is capable of projecting the problem on the flow lines, and (3) the relatively small number of meshes to be used does not require the use of finite element automatic meshes generation. Equation (2) represents the heart of the computer program used in this study. The computer program consists of subroutines that reads aquifer parameters, prepares the system matrix, solves for the aquifer equations, compute the residuals required for the iterative method, and print out the results.

5. HYDRO-GEOLOGICAL CHARACTERISTICS OF STUDY AREA

In the western part of this study area, there is a long straight and continuous scarp called the first Escarpment running from North-Northwest to South-Southeast. The first Escarpment divides the study area into a flat plain called Saluq plain to the west and Table land to the east. Wadi Albab and wadi Ahmar were created by the erosion of Tableland. Of the two wadis in the study area, wadi Ahmar flows almost directly due west while wadi Albab is nearly flowing in the southwest direction. From the ground water levels, the ground water contour lines, and the hydrogeological cross sections, geology of main aquifer is the Derna formation in the uplift area, the Faidiyah formation in the downstream basin of wadi Albab and in the Saluq plain, and the Alabraq formation and the Derna formation in the intermediary areas thereof; however, it is the Rajmah formation and the Faidiyah formation in the fan zone of wadi Albab, which uniquely possess such geology. The Derna formation consists of calca-calcilutite intercalating thin clay bed. The transmissivity is \( 10^{-5} \) m\(^2\)/s or under in wadi Ahmar and in the uplift area, but it is \( 10^{-3} \) m\(^2\)/s or over in the Muqzzahat area owing to the presence of karstic zones. The Alabraq formation is composed of calca-calcilutite intercalating thin clay bed in two or three horizons. The transmissivity, in general, is approximately \( 10^{-5} \) m\(^2\)/s, but it is approximately \( 10^{-2} \) m\(^2\)/s in the Muqzzahat area. With regard to the Alfaidiyah formation, it consists of calca-calcilutite intercalating marly limestone in the downstream basin of wadi Albab; although karstic pervious zones are not found, the transmissivity is approximately \( 10^{-3} \) m\(^2\)/s.
6. MODEL DEVELOPMENT STEPS

Before entering the modelling phase, it is necessary to prepare the required input data such as: Boundary conditions of the basin: Two types of artificial boundaries are commonly encountered. The first type is the specified piezometric head (Dirichlet boundary condition). The condition on such a boundary is \( h = \) constant, the second type is the specified flux (Neumann boundary condition). Flux estimates are much less accurate; however, this type of boundaries is usually employed in the exceptional case when there is no flow across the boundary. The condition on such impermeable boundaries is \( \partial h / \partial n = 0 \) with \( n \) defined as the direction normal to the boundary, Yachiyo [6]. The boundaries of the model have been defined by taking advantage of the attitude of the available piezometric surface map. Making use of the flowlines and iso-piezometric contours, the lateral boundaries of the study area are set as shown in Fig. (1), Yachiyo [6]. The bottom boundary was taken to be the layer through which no leakage occurs. From the delineation of the ground water system in the study area, this layer corresponds approximately to the base of the Eocene rock. It should also be noted that in order to obtain a unique distribution of heads in a steady-state problem, there must be at least one node on a first type boundary. This requirement is a mathematical one, Hildebrand [7]. In unsteady-state problems, initial conditions are sufficient to ensure uniqueness. Division of the basin area: the entire basin has been divided into polygons, paying special attention for regions of particular interest. Keeping this in view, 39 nodes have been chosen in the area in a manner such that the water levels at these nodes are known for some period.

Most of the water wells are located at nodes in this model. Thus the true aquifer head and the distribution of water extraction were simulated almost accurately in space and time. Polygons have been constructed according to the Thiessen method, Fig. (1). Geometrical and Hydrogeological data: the geometric factors include aquifer top and bottom elevations, horizontal extent of the aquifer including the location and definition of the boundaries of the model. The hydrogeological parameters include the piezometric elevation of each node, the permeability and/or the transmissivity coefficient, and the storativity. The bottom elevation was measured from the geological cross-sections based on the interpretation of the lithological log information of the wells available in the region. Since the layers of the sedimentary rocks are connected to each other by vertical cracks and leakage, the aquifer of the study area is assumed to be an unconfined. Therefore, the top elevation of each node repeats the same elevation of the ground surface. The area of each polygon, \( A \), the length of the perpendicular bisector associated with each pair of nodes, \( w \), and the distance between the nodes, \( D \), have been measured. This can be used to find the conductance factor for sub-surface flow between nodes for known values of transmissivity.
The evaluation of piezometric head has been obtained as an average of the elevations measured through the years 1979-1986 and it was assumed equal to the value on the piezometric head map, Fig. 2, corresponding to the node location. The permeability has been evaluated from the available transmissivity data, Yachiyo [6], Fig. (3), divided by the thickness of the saturated section. As the main aquifer can approximately be considered as an unconfined aquifer, the storage coefficient corresponds to its specific yield. Although it is difficult to evaluate the correct storage coefficient from the obtained data, it can be considered as a percentage (approximately 50%) from the mean value of the observed porosity in the study area, Yachiyo [6].
Figure 2 Piezometric surface map

Figure 3 Generalized transmissivity map
Unsteady simulation time step: large time steps lead to cheap but inaccurate solutions, whereas small time steps give better accuracy and greater computational costs. The ideal situation is to use the largest possible time step that does not seem to significantly change the solution. The time step used in this study is taken to be one month.

7. AQUIFER RECHARGE

The parameters used in the computation of the aquifer net recharge are the precipitation, evapo-transpiration, runoff, and extraction by pumping wells. The recharge of the aquifer takes place according to the following equation:

\[ I = P - ET - R \]  

(3)

where \( I \) is the effective infiltration, \( P \) is the precipitation, \( ET \) is the evapo-transpiration, and \( R \) is the surface runoff.

8. STEADY-STATE CALIBRATION

For the rainfall, an average annual isohytal map covering the study area is prepared using the existing rainfall data for the hydrological years 1979 and 1986, as shown in Fig. (4), Yachiyo [8]. The rainfall values were assumed as an average value of the contour lines within each polygon. However, for the Evapo-transpiration: There is not much difference in value between evaporation and evapo-transpiration, since the study area is not richly vegetated. The estimation of evapo-transpiration was calculated from the readings of class A evaporation pans already established in the wadi Qattarah reservoir area. The evaporation and evapo-transpiration are calculated using \( EO = \alpha EP \) and \( ET = \alpha EO \), respectively, where \( EO \) is the evaporation form free water surface mm/day, \( \alpha \) is the pan coefficient (assumed to be 0.7), Linsley and Franzini [9], \( EP \) is the evaporation from pan mm/day, \( ET \) is the evapo-transpiration mm/day, and \( F \) is a reduction factor (varies from 0.6 to 0.8 with an average of 0.7). An iso-evapotranspiration map was prepared, Fig. (5), and the evapotranspiration value for each node was computed in a similar way as for rainfall. From the existing runoff data at the hydrometric stations already established, Yachiyo [8], the characteristics of the runoff of the study area are as follows: 1) the runoff coefficient varies with every flood due to differences of rainfall intensity, duration, soil moisture conditions, vegetation, etc; 2) annual runoff coefficient has a small value. Mean annual runoff coefficients for wadi Ahmar and wadi Albab were estimated to be 8% and 4%, respectively. Moreover, for the Artificial discharge (Extraction by pumping): the water consumption for domestic and agriculture use has been distributed among the individual meshes on the basis of the position of the producing wells. Two assumptions have been made in the estimation of the extraction amount: namely the operation days are 335 day/year and the operation hours are 8 hours/day. The annual extraction amount by pumping can be expressed by \( P = 3600*8*335(y_i / 1000) \),
where $P$ is the annual extraction amount by pumping in $m^3$/year and $y_i$ is the yield of the $i^{th}$ production well in operation in lt/sec. The final values of the annual net recharge are obtained (tables are not given for limiting the space).

Figure 4 Average isoheytal map
9. UNSTEADY STATE VERIFICATION

The average annual recharge values should be replaced with time dependent values. For this, actual monthly data (1979-1986) which were obtained through observation at the installed 14 rainfall stations in the study area was used. The monthly rainfall value at each node was calculated using interpolation and extrapolation between the stations. Estimation of monthly evapo-transpiration at each node in the study area was calculated as a percentage of the actual rainfall value. These represent the average evapo-transpiration values for each month of the rainy season (from Oct. to April). The estimation of the monthly runoff at each node was obtained using the same percentages as in the evapo-transpiration calculation. Monthly water abstraction was assumed as follows: (1) 30% of the total abstraction is pumped out through the months Oct., Nov., Dec., Jan., Feb., and March, (2) 70% of the total abstraction is pumped out through the months April, May, June, July, Aug., and Sept. These abstractions are equally divided and assumed constant throughout these two groups of months. For each of the steady and unsteady state cases, the evaluation of the infiltration is then defined for each node as the product of the infiltration height, obtained from Eq. (3), times the area of the node divided by time during which the rainfall has occurred. The net recharge was calculated at each node after subtraction of the artificial discharge, $q_{\text{out}}$, and a percentage reduction, $\% \text{red}$, from the effective infiltration. The percentage reduction is assumed as a percent from the net recharge (15% to 25%) in order to account for the presence of perched aquifers and chalky limestone whose permeability is small (tables are not given to limit the space).
10. MODEL DESCRIPTION

The connecting structure of the model can be represented by using a topological matrix $A$ of size $nc \times nn$ where $nc$ is the number of connections and $nn$ is the number of nodes. Elements of $A$ (i.e., $a_{ij}$) are defined as $-1$ if the flux enters connection $i$ from node $j$, $0$ if the nodes $i$ and $j$ are not linked, and $+1$ if the flux leaves connection $i$ towards node $j$. Given the above mentioned definition of matrix $A$, the flow can be related to the head loss in each connection by Darcy’s law as:

$$ T \, dh = -T \, A \, h = Q $$

where $h$ is the piezometric head at each node, $T$ is the transmissivity of each connection, $dh$ is the head loss along each connection, and $Q$ is the flow in each connection. In addition the continuity equation can be written as:

$$ A^T \, Q = -q $$

where $-q$ is the net flow of water leaving (or entering) the node, and $A^T$ is the transpose of $A$. By substituting, in Equation (5), for $Q$ given by Equation (4) the steady-state problem leads to the following equation:

$$ (A^T \, T \, A) \, h = q $$

in the most general case where one accounts for the effect of time variation of head, with the consequent storage accumulation or depletion, Equation (6) becomes:

$$ (S \, + \, A^T \, T \, A) \, h_t = S \, \frac{h_{t-1}}{dt} + q $$

where $S$ is the diagonal matrix $nn \times nn$ of storativity. The $i^{\text{th}}$ element of matrix $S$ is obtained for the unconfined aquifer by the product of the storativity coefficient times the surface area relevant to the node. $T$ is the diagonal matrix $nc \times nn$ of transmissivity coefficients which is computed in the medium point of the $i$-$j$ connection according to the aquifer thickness (In the case of unconfined aquifer the thickness is a function of the final solution , therefore the adopted scheme is totally implicit), $q$ is the vector $nn \times 1$ of net volumes, leaving or entering the aquifer per time unit, $h_t$ is the solution vector $nn \times 1$ of the piezometric level at the current step, and $h_{t-1}$ is the solution vector $nn \times 1$ at the previous time step.
11. SOLUTION OF THE SYSTEM OF EQUATIONS BY THE CONJUGATE GRADIENT ALGORITHM

The conjugate gradient method, due to Hestness and Stiefel [10] allows for the solution of a system of linear equations with a symmetric positive definite matrix of coefficients. The computational procedure for minimizing a positive definite quadratic function \( F \) consists of minimizing \( F \) successively along lines. The steps involved in the conjugate gradient algorithm are as follows:

- **Initial step**: select a point \( x_1 \) and compute \( P_1 = r_1 = -F'(x_1) = b - A x_1 \).

- **Iterative steps**: having obtained \( x_k, r_k, \) and \( P_k \), compute \( x_{k+1}, r_{k+1}, \) and \( P_{k+1} \) by the formula

  \[
  a_k = c_k / d_k, \quad c_k = P_k^T r_k, \quad d_k = P_k^T A P_k, \quad x_{k+1} = x_k + a_k P_k, \quad r_{k+1} = r_k - a_k A P_k, \quad P_{k+1} = r_{k+1} + b_k P_k, \quad b_k = -P_k^T A r_{k+1} / d_k.
  \]

- **Termination**: terminate at the \( m^{th} \) step if \( r_{m+1} = 0 \). Then \( m \leq n \) and \( x_{m+1} = x_0 \), the minimum point of \( F \).

12. STEADY STATE CALIBRATION

The reasons for performing steady state calibration are that it does not require adjusting storage coefficients along with transmissivities and the average change in water levels in the basin is small. The objective was to obtain the initially assigned water levels of the hydrological years 1979–1986 by changing the transmissivity matrix and the average net recharge. The steady state calibration started with the already computed values of the permeability and net recharge. An initial permeability value is assigned to each node by linear interpolation between the contours. The next step is to adjust these values together with the net recharge by trial and error until the computed values of the hydraulic head were reasonably close to the observed values. After a few steady state runs, it is observed, as expected, that lowering permeability while holding the net recharge the same raises the heads and increases the gradients. Similarly, the head rises if the net recharge is increased and the permeability is held unchanged. This observation was used in the adjustment procedure. The final product of the steady state simulation was developed as water levels for all the nodes. These results are presented in Fig. (6) which shows the observed and the computed water level values. The output parameters obtained in this phase are the permeability of all the connections and the average annual net recharge of all the meshes (values are not given here for the limited space). The simulated water levels matched closely the known elevation for all wells. Also, the transmissivities that were accepted after the last calibration run are in rather good conformance with the reported values. The changes made to the net recharge during calibration show that the average annual net recharge is different and approximately 17% higher than those reported in the table of annual net recharge. The bigger differences between the simulated and the observed recharge values occur at the nodes located adjacent to the flood areas of wadi Albab and wadi Ahmar. This observation indicates that additional recharge from rainwater coming from ground surface through vertical transits is possible and is responsible for building up the water table at these areas.
13. UNSTEADY STATE VERIFICATION

The second phase of modelling consisted of simulating the historic record of water levels at some piezometers in the study area for the years 1979-1986. The computer model must be able to reproduce the observed levels at these piezometers to a sufficient accuracy before the model is to be used for future predictions. The changes in this phase should be made in the storage coefficient matrix and in replacing the average annual values of recharge with time dependent values. The recharge values used in this phase are calculated for each month. The model was run after regional transmissivities and other aquifer characteristics were determined from the regional steady state model. Given these characteristics from the steady state model, the storage coefficient could be determined from the transient state model. For the transient state,
the criterion of calibration is to compare the simulated and observed hydrographs at several piezometer locations. Since the grid system for the modelled area was selected to correspond exactly with piezometer locations, the output water level elevations from the model could be compared easily with the actual piezometer hydrographs. Examples from the piezometers used for comparison were PZ3388-I-I-2 in wadi Ahmar basin, and PZ3388-II-C-1 in wadi Al Bab basin. Their piezometric behaviour graphs were plotted using the computer. The transient state verification starts with the storage coefficient value, derived from the mean value of the porosity observed in the study area, which equals 9%. Since the aquifer is considered as unconfined, the storage coefficient corresponds to its specific yield. The storage coefficients around the selected nodes were then adjusted to get a good match between the physically observed and computer simulated hydrographs. The adjustment procedure of the storage coefficient values can be illustrated as follows: from the definition of the storage coefficient, the water volume at any node is equal to $V = A \cdot h \cdot S$ where $V$ is the water volume, $A$ is the surface area, $h$ is the hydraulic head, and $S$ is the storage coefficient.

If a volume of water equals $1 \cdot A \, \text{m}^3$ is taken into or released from storage, the initial water level will rise or drop by an amount equals $1/S$ meter. This indicated that the new head, for the same water volume, will be higher if $S$ is reduced. On the contrary, the head will drop down if $S$ is increased. Many computer runs with different storage coefficient values were made until a good match between the observed and the simulated hydrographs is obtained. This observation is used in the verification. From the average standard values of the specific yield and porosity in limestone aquifers, the storage coefficient value of 9% in the study area was thought to be high. This is proved from the drawn graphs, (only two graphs are presented here to save the space, Fig. 7). From this figure it is clear that the amplitude of oscillation of the hydraulic head from the mean is high for $S = 9\%$ while a better matching is achieved when reducing the storage coefficient to a value ranging from 2 to 3.3.

The final storage coefficient values obtained during the unsteady state verification are tabulated for all the nodes (table are not given to limit the space).
13. FORECAST OF FUTURE AQUIFER BEHAVIOUR

The items relevant to the projections and simulations are the abstractions for domestic use and for agricultural projects foreseen in the development plans. The plans for agricultural development of the study area include two projects: (1) in the plain near the lower reach of wadi Albab, the wadi Albab agricultural project consists of 50000 hectares of tilled land out of which 2000 hectares are planned to be irrigated, and (2) in the plain near the lower reach of wadi Ahmar, the Benghazi south west agricultural project consists of 100000 hectares of tilled land out of which 2000 hectares are going to be irrigated. Although no data is available for these plans, the necessary water volume for both projects can roughly be estimated as 3000 to 8000 m$^3$/ha/year. The net
monthly recharge used in the verification phase is calculated based on the available monthly data for the hydrological years 1979-1986. However, these data do not represent the actual average values, it was necessary to adjust them to give the average before entering the future behaviour prediction phase of the model. The adjustment is done using the following scale factor:

\[ y_{i,j} = x_{i,j} \left( \frac{12 \bar{x}_i}{\sum_i x_i} \right) \quad (8) \]

where \( y_{i,j} \) is the new monthly recharge value of node \( i \) for the month \( j \), \( x_{i,j} \) is the old monthly recharge value of node \( i \) for the month \( j \), \( \sum_i x_i \) is the sum of the old monthly recharge values of node \( i \), and \( \bar{x}_i \) is the average annual recharge value of node \( i \) for the month \( j \). The yearly increase of the water needs for domestic use was estimated according to an increment of 10% from the average of those existing at the years 1979-1986. The distribution of the yearly agricultural consumption into monthly consumption has been made according to the following monthly percentages:

<table>
<thead>
<tr>
<th>Month</th>
<th>Sept</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
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<td>0.07</td>
<td>0.06</td>
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<td>0.08</td>
<td>0.1</td>
<td>0.11</td>
<td>0.14</td>
<td>0.15</td>
</tr>
</tbody>
</table>

14. FUTURE SIMULATIONS

Three simulation runs of 10 years each (2000-2010) have been executed. These runs were based on the existing abstraction and on the addition of the water demand for both newly irrigated farms and increasing domestic uses with time.

**Simulation 1:** in each of the wadi Ahmar and wadi Albab, 2000 hectares (200 farms of 10 ha each) are going to be irrigated were the abstraction is taken as 8000 m³/ha/year. The farms have been equally distributed among the meshes 24, 25, 26, 27, and 28 in wadi Ahmar and 33, 34, 37, 38, and 39 in wadi Albab.

**Simulation 2:** in the second simulation, both the area to be irrigated and the abstractions at wadi Ahmar were kept as that in the first simulation while the abstraction at wadi Albab is reduced to 3000 m³/ha/year.

**Simulation 3:** both the area to be irrigated and the abstractions at wadi Ahmar were also kept constant while those at wadi Albab, reduced to 1000 hectares and 3000 m³/ha/year, respectively.

15. ANALYSIS OF RESULTS OF FUTURE PREDICTIONS

The information given by the simulation runs can be analysed on the basis of the yearly decline of the water level. For the details on the areal impact of the imposed abstractions, the piezometric behaviour graphs during 120 months of simulation at the nodes of some typical meshes (nodes 24 and 37) have been given in Fig. (8) for each of the three simulations. A description of these graphs is given here for some nodes:
Nodes 24, 25, 26, 27, and 28
- Simulation 1: the aquifer is not affected by any appreciable depletion.
- Simulations 2 and 3: these simulations do not affect the meshes considered. The curve remains almost the same as that of the first simulation.

Nodes 33 and 34
- Simulation 1: gradual decline with a total of 10.4 meters for node 33 and 14.15 meters for node 34 is reached at the 10th year.
- Simulations 2 and 3: regular decline of not more than 3.5 meters is reached at the 10th year.
Node 37
- Simulation 1: strong decline and drying up occurs after 36 months.
- Simulation 2: same as simulation 1 with drying up after 108 months. This indicates that in both simulations the abstractions exceed the aquifer capacity.
- Simulation 3: the drawdown is about 15 meters at the end of the 10th year.

Node 38
- Simulations 1 and 2: drying up after 48 and 108 months, respectively.
- Simulation 3: the final drawdown is reduced to about 16 meters and the water level declines with the increase of time.

Node 39
- Simulations 1 and 2: strong decline with drying up after 24 and 60 months, respectively.
- Simulation 3: the final drawdown is reduced to about 26 meters and the water level declines with the increase of time.

16. SUMMARY AND CONCLUSIONS

A mathematical model, based on the integrated finite difference method has been developed to study and simulate the ground water flow system of wadi Albab and wadi Ahmar catchment areas near Benghazi city. The model is also used to analyze the effects of increased pumping due to the increased ground water abstraction required for irrigating the newly developed agricultural projects in the study area. The following points are concluded:

1- Transmissivity values are irregularly distributed in the project area confirming the great heterogeneity of the basin. The lowest values are found at the upstream and downstream areas of wadi Albab and at the upstream area of wadi Ahmar with an average of $1 \times 10^{-4}$ m$^2$/s. The highest values are found at the downstream area of wadi Ahmar, particularly at the nodes adjacent to Muqzzahat area with an average of $2 \times 10^{-2}$ m$^2$/s.

2- The storage coefficient values obtained from the available data (i.e. $S = 9\%$) is found to be high and a better matching is achieved between the simulated and the observed water levels when reducing its values. The storage coefficient range is found to be 2 to 3.3 %.

3- The average capacity of the aquifer in supporting the future domestic consumption and abstraction for the agricultural development plans is confirmed at wadi Ahmar area where gradual piezometric drawdown, which is acceptable, is obtained proving that considerable ground water quantities may be developed at this area. At wadi Albab the drawdown exceeds the exploitability limits.

4- Reducing the area to be irrigated at wadi Albab by 50% and abstracting the minimum water quantities required for the development plans does not show an improved behaviour to the aquifer at this area.

5- Even though the developed model is able to predict the behaviour of the basins for the future abstractions reasonably well, it has some limitations in the sense that the data used for calibration and verification is only for a small period. Using a longer period data will further facilitate in the refining of the model.
REFERENCES