BACKWATER RISE DUE TO FLOW CONSTRICTION BY BRIDGE PIERS

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ABSTRACT

The flow constriction due to piers of highway bridges results in producing backwater rise upstream such bridges, especially at floodplain periods. This backwater rise depends mainly on both flow properties and geometrical characteristics of piers. In the present research paper, the backwater rise due to bridge piers was experimentally investigated for extreme ratios of piers thickness to channel width (t_{PS}/B) under a wide range of both subcritical and supercritical flow conditions between bridge piers. The present study illustrates that the backwater rise upstream bridge piers depends mainly on both flow type between piers and constriction ratio, while it is secondary depends on geometrical shape of pier endnoses. Also, the experimental results illustrates that the backwater rise at supercritical flow conditions between piers agreed satisfactory with the corresponding computed values from the most widely relationships published by Yarnell [10] than that for subcritical flow conditions between piers. The results of the backwater rise due to constriction of flow by bridge piers presented in this research paper are employed in development of two formulas, which could be used in computing backwater rise at both subcritical and supercritical flows between bridge piers.

INTRODUCTION

In most cases, both the highway and the railway bridges cross the streams resting on wide piers only (i.e. without abutments). Also, the flow at bridges site may be obstructed by piers only due to reflowing of channel cross-section resulted from decreasing of flow discharge as that happened in Nile River and its branches after the construction of Aswan High Dam (abutments are outside the waterway). The constriction of flow at bridge site due to piers results in occurrence of backwater rise upstream bridge position, which could be considered as an important problem, especially at highway and railway bridges passing over rivers on relatively wide piers. The importance of backwater rise phenomenon derives from the economic significance of river levels, in which the cost of the protective works depends on the maximum predicted flood level. Hunt et al. [5] stated that the bridge region could be divided into three zones: the contraction reach upstream the constricted section by bridge piers; the constricted reach between bridge piers; and the expansion reach just downstream bridge piers. Hunt et al. [5] concentrated their study on backwater rise at the transition reaches (contraction and expansion reaches). As stated by Henderson [4] and [9], the study of backwater rise due to flow constriction due to bridges piers was started by Yarnell [10]. Also, the backwater rise upstream bridges was studied by Beffa [1], Charbeneau et al. [2], Kaatz [6], Seckin [7], and Soliman et al. [8].

The application of energy equation for horizontal bed between sections 2 and 3 as illustrated in Fig. 1 leads to:

$$(Y_2 + \frac{V_2^2}{2g})r = Y_3 + \frac{V_3^2}{2g} \tag{1}$$

In which

r	the residual ratio of energy between sections 2 and 3 ≈ 0.9-1.0 [8];
V_2	flow velocity at section 2;
V_3	flow velocity at section 3;
Y_2	flow depth at section 2 (minimum flow depth); and
Y_3	downstream flow depth.

Also, for a rectangular open channel flow section, the application of continuity equation between sections 2 and 3 leads to:

$$Y_2 \, b \, V_2 = Y_3 \, B \, V_3 \tag{2}$$

The opening ratio (O_r) at the constricted section is:

$$O_r = b/B = 1 - t_{PS}/B \tag{3}$$

In which

b	flow width at constriction section;
B	flow width of cross section without constriction;
n	number of piers at bridge site;
O_r	opening ratio at constricted section;
t_P	thickness of the pier at bridge site; and
t_{PS}	total thickness of piers at bridge site = n x t_P.

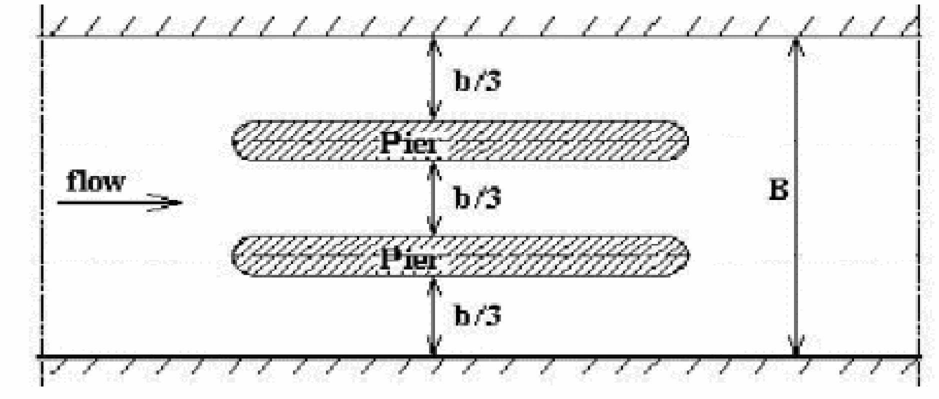
According to Soliman et al. [8], Eqs. 1, 2, and 3 could be concluded in the following relationship:

$$\boldsymbol{O}_r^2 = \frac{\boldsymbol{r}^3 \boldsymbol{F}_{r3}^2 (2 + \boldsymbol{F}_{r2}^2)^3}{\boldsymbol{F}_{r2}^2 (2 + \boldsymbol{F}_{r3}^2)^3} \tag{4}$$

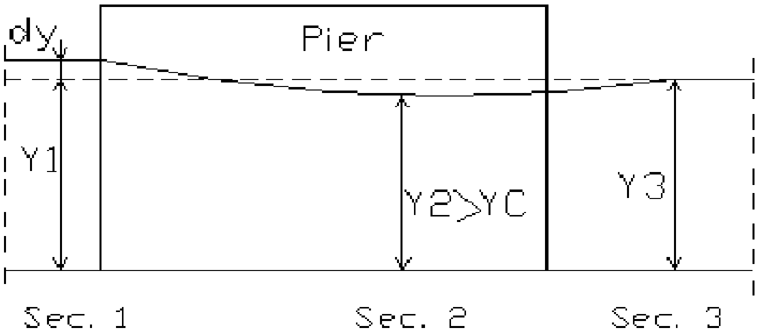
In which F_{r2} and F_{r3} are Froude numbers at cross-sections 2 and 3 as illustrated in Fig. 1.

Due to the difficulty of measuring process between bridge piers, the flow type between bridge's piers could be determined from flow properties at either upstream or downstream bridge piers. As stated by Henderson [4], the limiting value of opening ratio for distinguishing between the unchoked flow (subcritical flow) and the choked flow (supercritical flow) was dealt by Yarnell [10] according to the assumptions; $E_1 = E_2$ and $M_2 = M_3$. Assuming critical flow at section 2, the assumption $E_1 = E_2$ [8] leads to:

$$\boldsymbol{O}_r^2 = \frac{27\boldsymbol{F}_{r1}^2}{(2+\boldsymbol{F}_{r1}^2)^3} \tag{5}$$



Plan



Elevation

Fig. 1 Flow between bridge piers

Also, the second assumption $M_2 = M_3$ [8] leads to:

$$\boldsymbol{O}_r = \frac{(2+1/\boldsymbol{O}_r)^3\boldsymbol{F}_{r3}^4}{(1+2\boldsymbol{F}_{r3}^2)^3} \tag{6}$$

in which

E_1	total energy at section 1;
E_2	total energy at section 2;
Fr_1	Froude number at section 1;
Fr_3	Froude number at section 3;
M_2	momentum at section 2; and
M_3	momentum at section 3.

The independent variables in Eq. 6 could be derived from section 3 which are initially known. The flow between bridge piers becomes critical when the value of Froude number at section 2 equals unity ($Fr_2=1$). The substitution of the value of $Fr_2=1$ in Eq. 4 leads to a value of downstream Froude number (Fr_{3c}) corresponding to critical flow conditions between piers at section 2. The value of Froude number (Fr_{3c}) depends on the value of opening ratio (O_r) at bridge site as illustrated in the following equation [8]:

$$\boldsymbol{O}_r^{\,2} = \frac{27\boldsymbol{r}^3\boldsymbol{F}_{r3c}^2}{(2+\boldsymbol{F}_{r3c}^2)^3} \tag{7}$$

If the value of Froude number downstream under normal flow conditions (F_{r3}) is less than the value of F_{r3c} computed from Eq. 7, then the flow between piers is subcritical as illustrated in Fig. 2 case-A and vice verse when the value of Froude number in downstream under normal flow conditions (F_{r3}) is greater than the value of F_{r3c}, the flow between piers is supercritical as illustrated in Fig. 2 case-B [8].

1- Case (A) Unchoked flow between bridge piers (subcritical flow)

The following empirical equation was developed by Yarnell [10] for predicting the value of backwater rise in subcritical flow (case-A) [9].

$$\frac{dy}{Y_3} = CF_{r_3}^2[(C+5F_{r_3}^2-0.6)(X+15X^4)] \tag{8}$$

In which

C	coefficient depends on shape of pier endnoses; C = 0.9 for semicircular endnoses, C = 1.05 for triangular endnoses pier, and C = 1.25 for rectangular pier [8];
dy	backwater rise upstream bridge site;
Fr_3	Froude number at downstream section of bridge piers;
X	ratio of pier thickness to the distance between piers axis; and

Y_3 normal flow depth at downstream section of the bridge piers.

It should be mentioned that formula (8) was derived for pier length/pier width ratio (L_P/t_P) = 4. Henderson [4] stated that the value of backwater rise (dy) may be increased by 5% and 10% for the ratio of L_P/t_P= 7 and 13, respectively.

2- Case (B) Choked flow between bridge piers (supercritical flow)

If the value of Froude number (F_{r3}) at normal flow conditions downstream bridge site increases more than the critical value of Froude number (F_{r3c}) computed from Eq. 7 depending on the opening ratio, the flow between piers is supercritical flow. A relationship between dh/Y_3 and Fr_3/Fr_{3c} at supercritical flow conditions was developed by Yarnell [10] as illustrated in Fig. 3 [8]. Chow [3] stated that the relationship illustrated in Fig. 3 was derived for pier length/pier thickness (L_P/t_P) = 4.0.

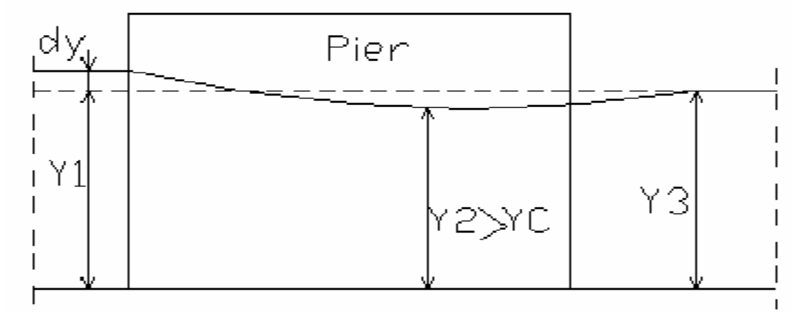
The backwater rise due to piers constriction at bridge site is controlled by different parameters as follows;

$$dy/Y_3 = f[Fr_{3c}, Fr_3, b/B, \text{pier shape}] \tag{9}$$

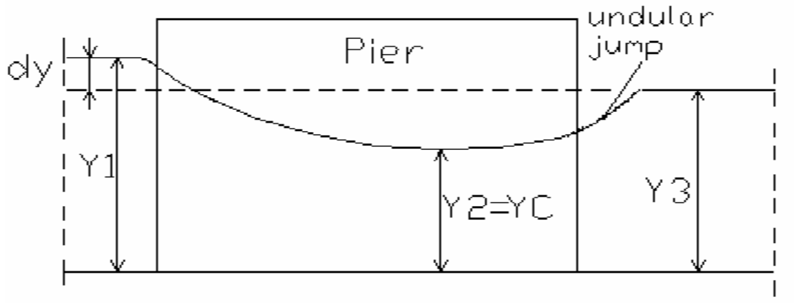
in which

b total opening width between bridge piers;
B initial flow width before bridge construction;
dy backwater rise upstream bridge site;
F_{r3} Froude number downstream bridge piers; and
Fr_{3c} value of downstream Froude number corresponding to critical flow condition between bridge piers (Fr_2=1), which depends on opening ratio, Eq. 7.

Since most of former studies were carried out on the transition reaches at both upstream and downstream of bridge site, the present study is focused on the constricted reach between the bridge piers. In this research paper, the backwater rise resulted from the existence of bridge piers is experimentally investigated under different geometrical characteristics of piers for a wide range of flow properties between bridge piers (subcritical and supercritical flows). Also, the effect of piers endnoses on backwater rise is experimentally investigated.



Case A – Unchoked flow



Case B – Choked flow

Fig. 2 Types of flow between bridge piers

EXPERIMENTAL WORK

The experimental data used in investigation of backwater rise phenomenon due to bridge piers were resulted from the experimental work conducted in a re-circulating Plexiglas flume in the Hydraulic laboratory of Al-Tahady University, Libya, through 180 run. Timber piers models painted with a non-permeable material were used. The experiments were carried out within downstream Froude number ranged between 0.19 and 0.93. Six pier's models of different pier thicknesses to channel width ratio (t_{PS}/B) equal to 0.1, 0.19, 0.29, 0.38, 0.48, and 0.58 were used to cover a wide range of channel blockage by bridge piers. The aforementioned ratios of pier thickness to channel width ratio (t_{PS}/B) results in six values of opening ratio at bridge site (0.9, 0.81, 0.71, 0.62, 0.52, and 0.42). For each pier model, five values of discharge were used. Also, for each model the downstream depth was controlled to obtain both subcritical and supercritical flow cases between bridge piers. The effect of geometrical shape of pier was studied through three geometrical shapes of piers (rectangular, triangular endnoses, and semicircular endnoses). Both a flowmeter fitted behind the pump of the flume, and a calibrated triangular V-notch at the end of the flume were used in measuring the flow discharge through the flume. Also, the flow depths were measured by using a point gauge supplied with verniers allowing measurements accuracy of ±0.1 mm, and the flow velocity was measured by using pitot-tube. The following parameters were measured for each run; upstream flow depth (Y_1), approach velocity (V_1), flow depth through vents (Y_2), flow velocity through vents (V_2), downstream flow depth (Y_3); and downstream flow velocity (V_3). Some samples of experiments are illustrated through photos 1 and 2.

Photo 1 Backwater rise due to subcritical flow condition between piers.

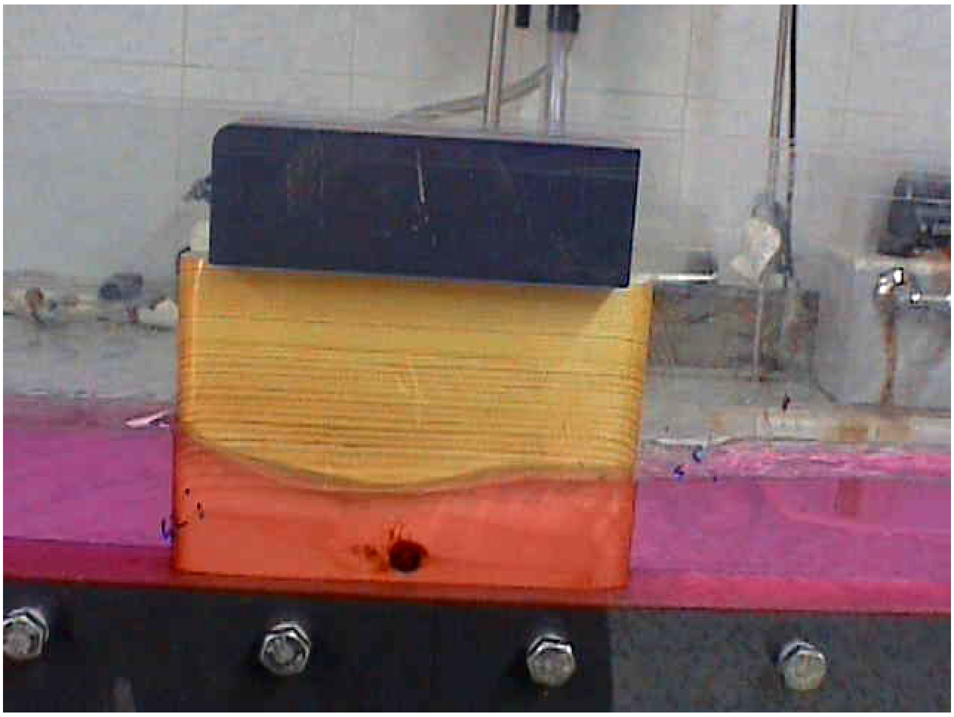


Photo 2 Backwater rise due to supercritical flow condition between piers.

ANALYSIS AND DISCUSSION OF THE RESULTS

The downstream Froude number (F_{r3c}) corresponding to critical flow conditions between piers for each value of opening ratio (O_r) is computed from Eq. 7 depending on each run data. The downstream flow parameters (Y_3, V_3) are used in computing the value of Froude number (Fr_3). If the value of F_{r3} is less than the value of F_{r3c}, the flow between piers is subcritical; otherwise the flow between piers is supercritical. Both the subcritical and the supercritical flow conditions between bridge piers are included in the present research paper.

A- Subcritical flow between bridge piers

Depending on downstream flow parameter, the value of Froude number (Fr_3) is computed, while the value of Fr_{3c} is computed from Eq. 7 depending on opening ratio at bridge site. If the value of (Fr_3) is less than the corresponding value of Fr_{3c}, the flow is subcritical. The dimensionless backwater rise (dy/Y_3) due to subcritical flow between bridge piers is illustrated through Figs. 4 to 10. Figures 4 and 5 represent rectangular piers, Figs. 6 and 7 represent triangular endnoses piers, Figs. 8 and 9 represent semicircular endnoses piers, and Fig. 10 shows a comparison between the dimensionless backwater rise resulted from the three types of piers. From figures, the backwater rise due to piers constriction at subcritical flow between piers is inversely proportional to the opening ratio for different flow conditions (directly proportional to t_{PS}/B). Figure 4 shows that the backwater rise is directly proportional to F_{r3}/F_{r3c} at the different values of opening ratio. This could be referred to the increase in flow discharge resulted from the increase of F_{r3}, in which the value of Fr_{3c} is constant for each value of opening ratio as illustrated in Eq. 7. The increase of flow rate between piers at constant blockage ratio resulted in increasing of the backwater rise upstream the bridge site, which is confirmed from Eq. 8, in which the backwater rise depends mainly on the value of F_{r3}. Fig. 5 shows a comparison between the measured and the calculated values of the dimensionless backwater rise at rectangular piers. From Fig. 5, it could be concluded that the difference between the experimental values of dimensionless backwater rise upstream rectangular piers and the corresponding predicted ones according Yarnell [10] at subcritical flow between piers is in acceptable range (8% -14%).

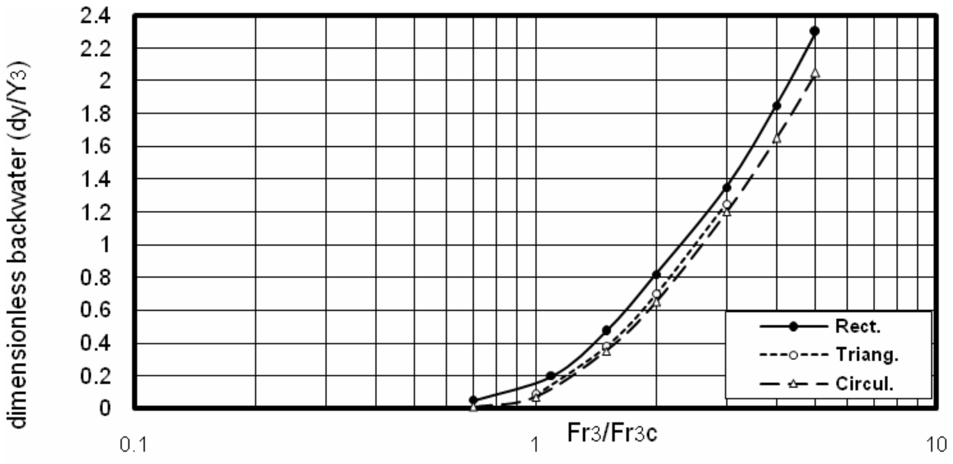


Fig. 3 Dimensionless backwater rise at supercritical flow between piers due to Yarnell [10], after Soliman et al. [8].

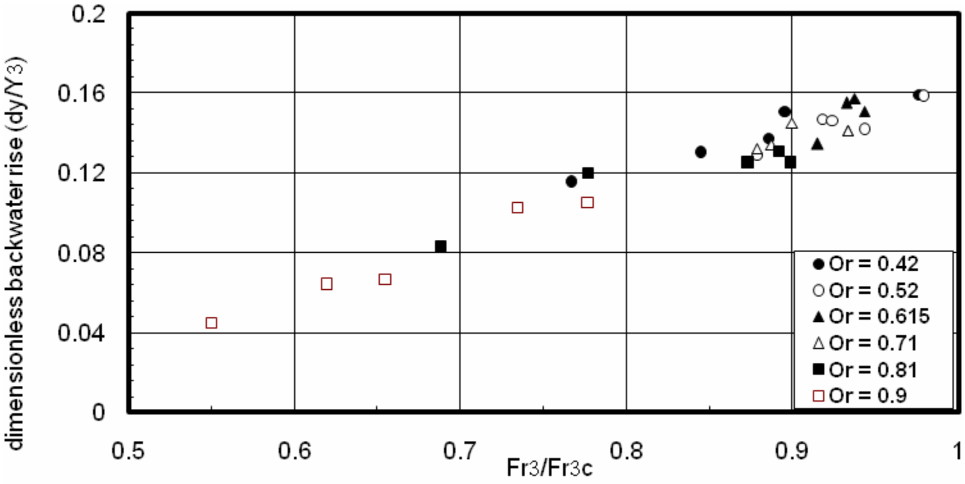


Fig. 4 Relationship between dimensionless backwater rise (dy/Y3) and Fr3/Fr3c at subcritical flow between rectangular piers.

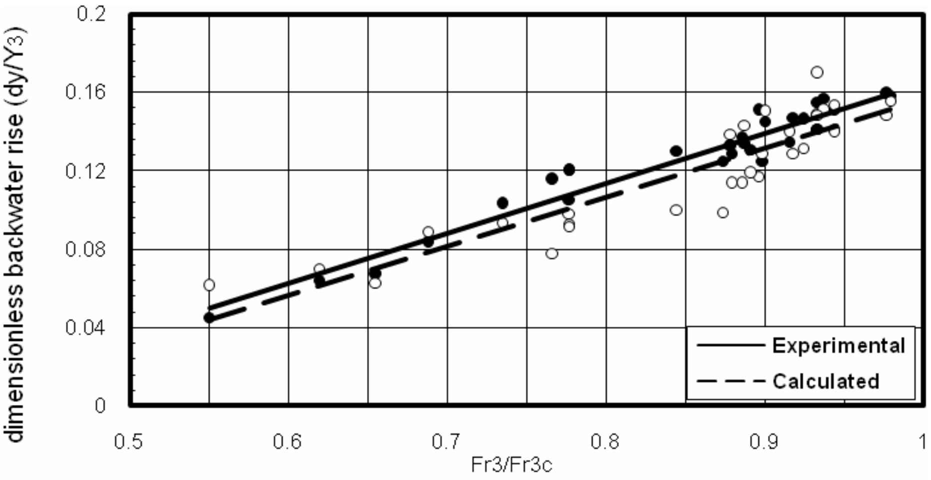


Fig. 5 Relationship between the dimensionless backwater rise (dy/Y3) and (Fr3/Fr3c) at subcritical flow between rectangular piers.

Figures 6 and 7 show the relationship between the dimensionless backwater rise (dy/Y_3) and F_{r3}/F_{r3c} for triangular endnoses piers (angle of slope =60°). From Fig. 6, it could be concluded that the backwater rise upstream triangular endnoses piers follows the same trend as in rectangular piers, but the values of backwater rise in case of triangular endnoses pier are smaller than those for rectangular ones for the same flow conditions. This could be referred to the sloping sides of upstream endnose of pier, which results in redirection the flow velocities and decreases the blockage effect of pier. This is confirmed by the former studies carried out by Charbeneau, et al. [2] and Yarnell [10]. Figure 7 illustrates a comparison between the experimental values of backwater rise and the corresponding computed ones by Yarnell formula for triangular endnoses piers. From this figure, it is found that the difference between the measured values and the corresponding computed ones was about 15%.

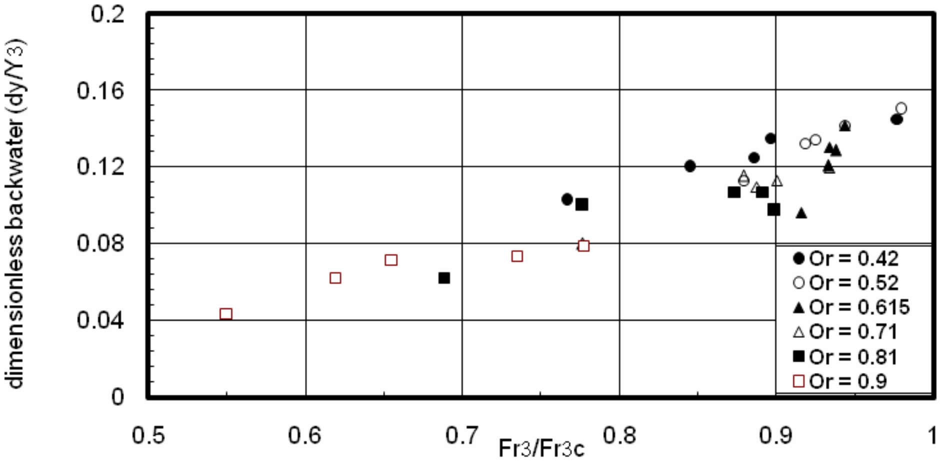


Fig. 6 Relationship between dimensionless backwater rise (dy/Y3) and Fr3/Fr3c at subcritical flow between triangular endnoses piers.

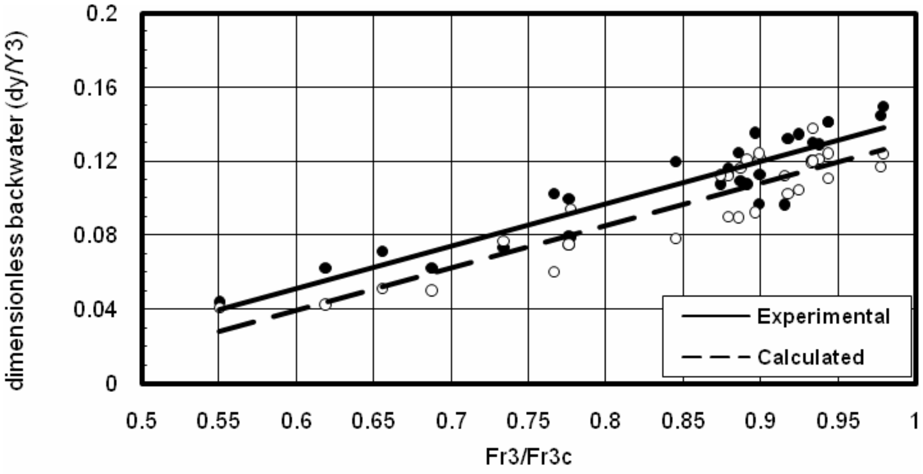


Fig. 7 Comparison between experimental and computed values of dimensionless backwater rise(dy/Y3) at subcritical flow between triangular endnoses piers.

Figures 8 and 9 show the relationship between the dimensionless backwater rise and the term F_{r3}/F_{r3c} for semicircular endnoses piers. From Fig. 8, it is found that the relative backwater rise is directly proportional to the relative value of downstream Froude number. The comparison between measured and computed values of backwater rise is illustrated in Fig. 9, in which the maximum difference between the measured and the computed values is about 19%. This relatively big difference for the three geometrical shapes of piers under subcritical flow could be referred to the fact that Yarnell Eq. 8 was derived for a pier length to pier width equal to 4:1 [4], while the pier length to pier width ratio in this research ranges between 5:1 to 30:1. The effect of geometrical shape of pier endnose on upstream backwater rise at subcritical flow condition between piers is illustrated in Fig. 10, in which the backwater rise for semicircular endnoses pier is smaller than those for both triangular and rectangular endnoses piers, respectively.

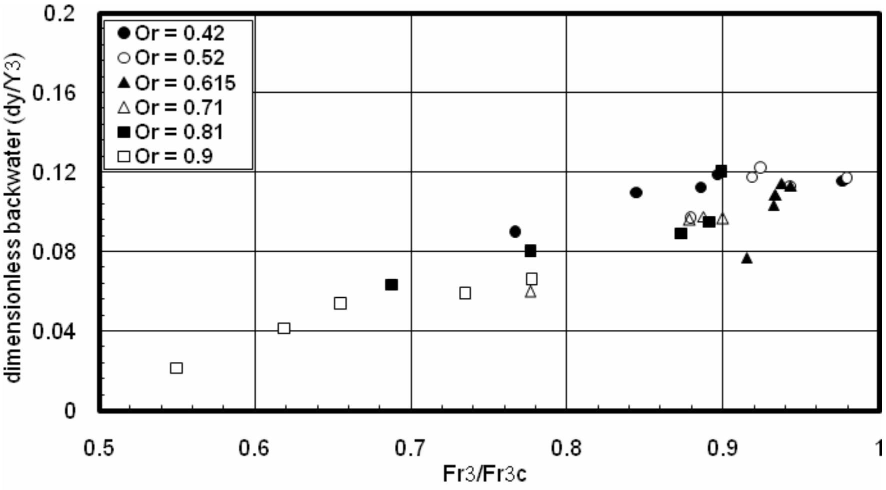


Fig. 8 Relationship between dimensionless backwater rise (dy/Y3) and Fr3/Fr3c for subcritical flow between semicircular endnoses piers for different opening ratio.

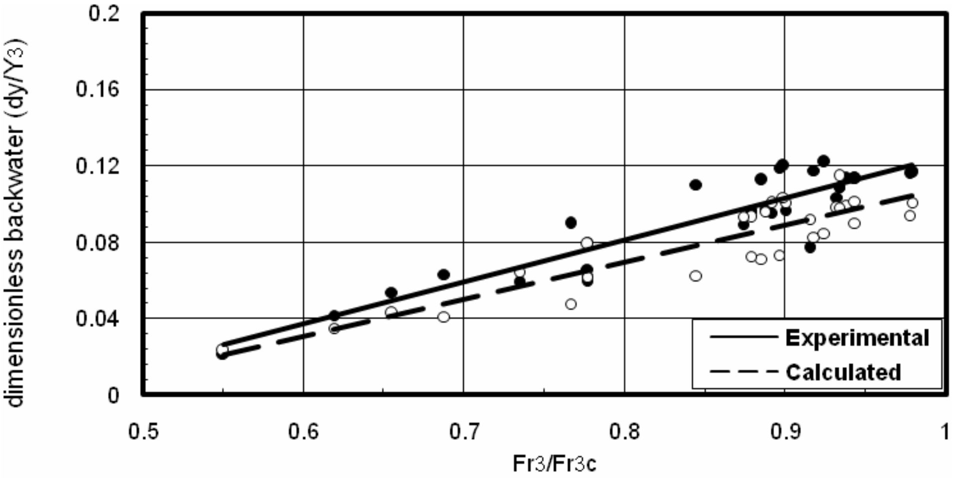


Fig. 9 Comparison between experimental and computed values of dimensionless backwater rise (dy/Y3) at subcritical flow between semicircular endnoses piers.

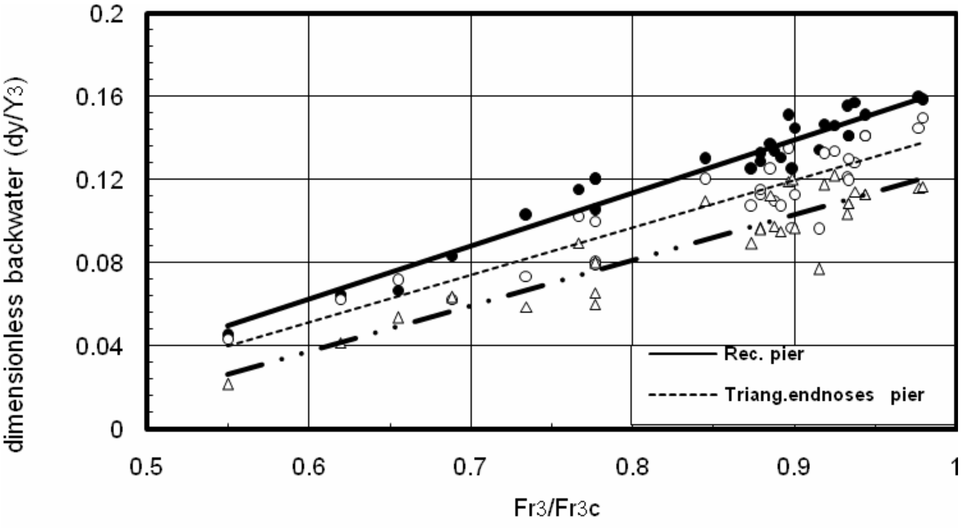


Fig. 10 Effect of geometrical shape of piers on dimensionless backwater rise for subcritical flow between bridge piers.

The decrease in backwater rise for both semicircular and triangular endsoses piers could be referred to their effect in redirecting flow velocities, which decreased the backwater rise. Based on the experimental data, the following formula is derived using regression analysis to predict backwater rise at subcritical flow conditions between bridge piers with a correlation coefficient equals 0.97 depending on opening ratio, geometrical shape of pier endnoses, and downstream flow properties at bridge site.

$$\frac{dy}{Y_3} = C_1(0.256 - 0.367\,O_r + 0.389\,F_{r3}) \tag{10}$$

in which

C_1 constant depends on pier shape; C_1= 1.0, 0.89, and 0.85 for rectangular, triangular, and semicircular endnoses piers, respectively;
dy backwater rise upstream bridge;
F_{r3} downstream Froude number;
O_r opening ratio between piers; $O_r = 1 - t_{PS}/B$; and
Y_3 downstream flow depth.

Formula (10) could be applied with the following precautions; Fr_3 = 0.2-0.62, O_r = 0.42–0.9, and ratio of pier length to pier width ranges between 5:1 and 30:1.

B- Supercritical flow between bridge piers

If downstream Froude number (Fr_3) is greater than the value of Fr_{3c} corresponding to critical flow conditions between piers, the flow between bridge piers is supercritical flow. The relationships between the dimensionless backwater rise and the term Fr_3/Fr_{3c} at supercritical flow conditions between piers for rectangular, triangular and semicircular endnoses piers are illustrated in Figs. 11 to 13, respectively. The figures show that the backwater rise upstream the aforementioned three types of piers depends mainly on the value of opening ratio ($O_r = 1 - t_{PS}/B$), while for the constant value of opening ratio the backwater rise is directly proportional to the ratio of Fr_3/Fr_{3c} for the three types of piers. This could be referred to the increase of the term Fr_3/Fr_{3c} at constant value of opening ratio means an increase of Fr_3 (Fr_{3c} was constant for each value of opening ratio), which results from the increase of the discharge between piers. The increase of the discharge for constant blockage ratio results in increasing the backwater rise upstream bridge piers.

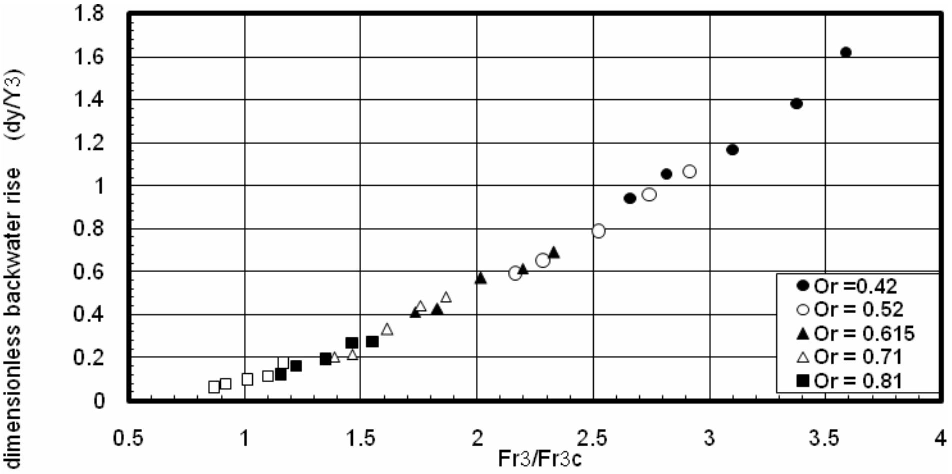


Fig.11 Relationship between dimensionless backwater rise (dy/Y3) and Fr3/Fr3c for supercritical flow between rectangular piers.

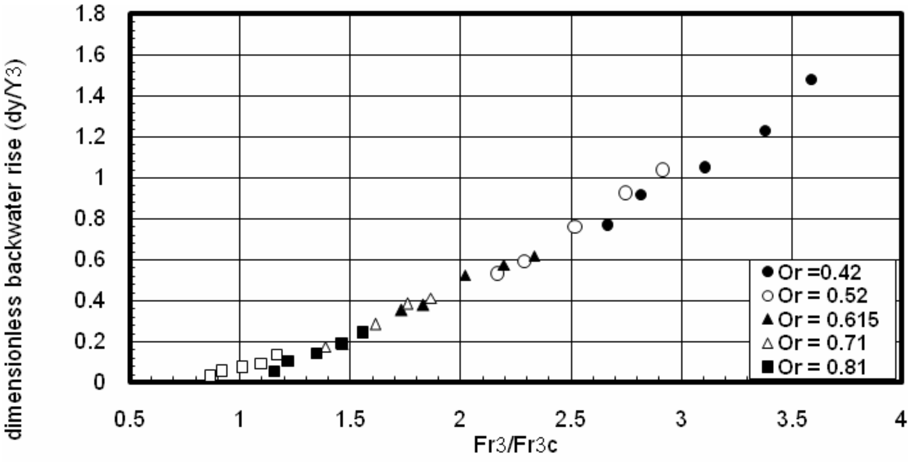


Fig.12 Relationship between dimensionless backwater rise (dy/Y3) and Fr3/Fr3c at supercritical flow between triangular endnoses piers.

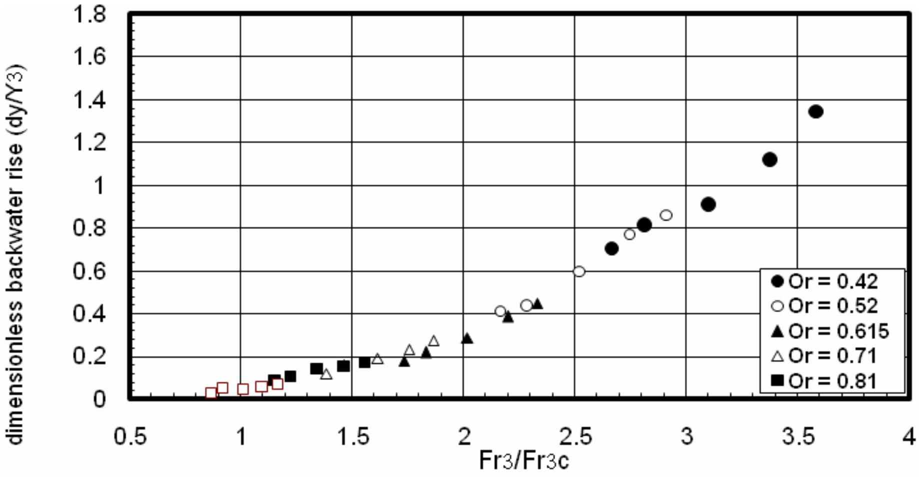


Fig.13 Relationship between dimensionless backwater rise (dy/Y3) and Fr3/Fr3c for supercritical flow between semicircular endnoses piers.

Figures 14 to 16 illustrate a comparison between both the measured and the computed values of backwater rise for the three types of piers at supercritical flow condition between bridge piers. The figures show that the differences between measured and computed values of dimensionless backwater rise are about 5%, 7% and 3% for rectangular, triangular and semicircular piers, respectively. The analysis of the results of both subcritical and supercritical flows illustrates that the difference between the laboratory data and those predicted from Yarnell equation for supercritical flow is small compared with that occurred for subcritical flow between piers. Also, it could be concluded that the ratio of L_P/t_P has no significant effect on backwater rise at supercritical flow conditions between piers.

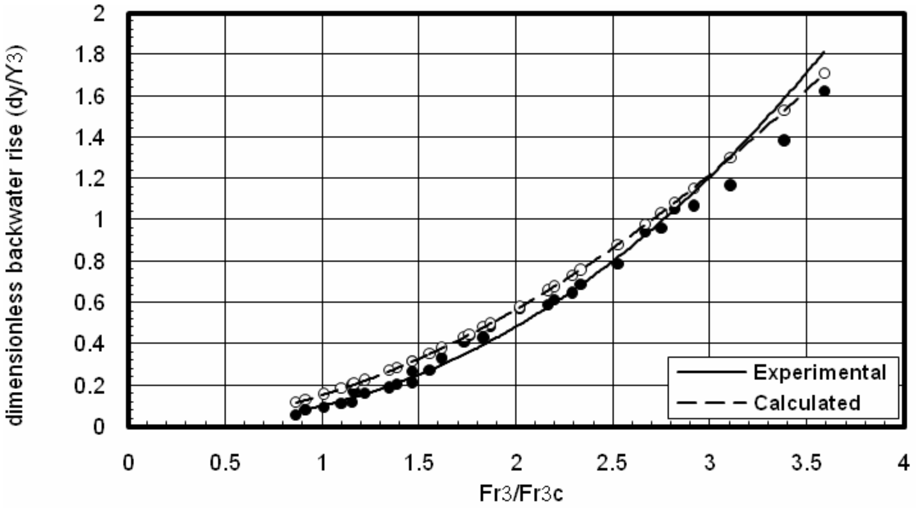


Fig.14 Comparison between the experimental and the calculated values of dimensionless backwater rise for supercritical flow between rectangular piers as a function of Fr3/Fr3c.

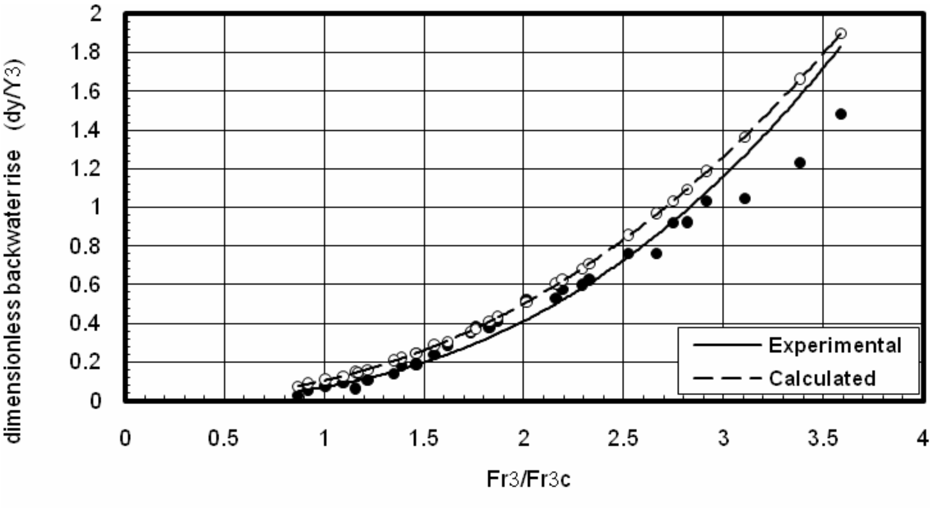


Fig.15 Comparison between experimental and calculated values of dimensionless backwater rise for supercritical flow between triangular endnoses piers as a function of Fr3/Fr3c.

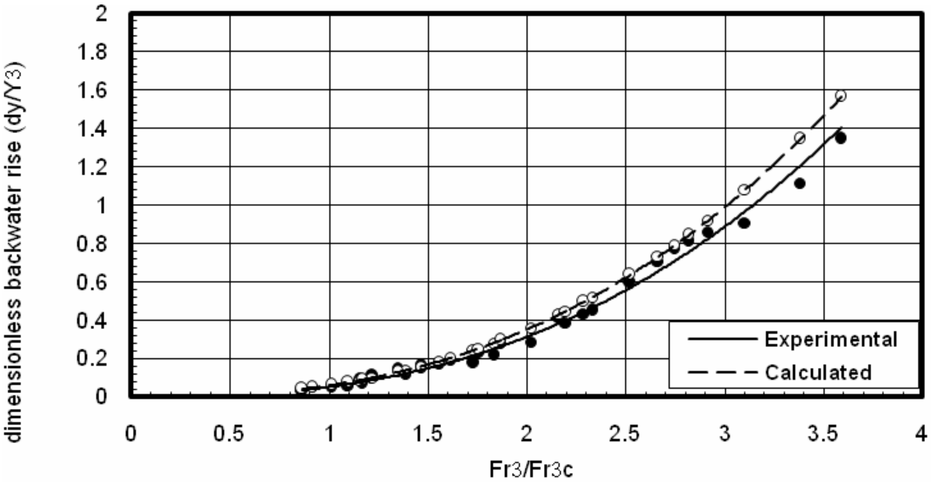


Fig.16 Comparison between experimental and calculated values of dimensionless backwater rise for supercritical flow between semicircular endnoses piers as a function of Fr3/Fr3c.

Figure 17 shows a comparison between the measured values of dimensionless backwater rise for three types of piers at the same supercritical flow conditions. From Fig. 17, it is noticed that the backwater rise upstream semicircular endnoses pier is less than those for both triangular and rectangular endnoses piers, respectively and the backwater rise for triangular endnoses pier is less than that for the rectangular one. This could be explained due to fact that the semicircular endnoses pier reduces the blockage effect and helps the streamlines to be volplaned more easily than that in both triangular and rectangular endnoses pier, which results in decreasing the upstream backwater rise. Based on laboratory data, the following formula for predicting the backwater rise due to supercritical flow between piers is proposed using regression analysis with a correlation coefficient equal to 0.99.

$$\frac{dy}{Y_3} = 0.1013\ C_2\ (\frac{Fr_3}{Fr_{3c}})^{2.586} \tag{11}$$

In which C_2 = 1.0, 0.69, and 0.53 for rectangular, triangular, and semicircular endnoses piers, respectively. The foregoing formula could be used in predicting the backwater rise upstream bridge piers for downstream Froude number ranges from 0.69 to 0.93 and opening ratio varies from 0.42 to 0.9.

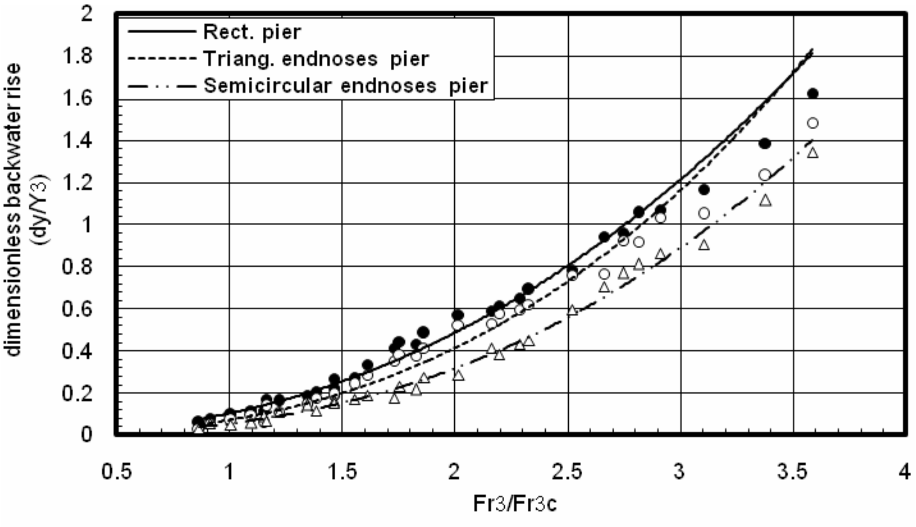


Fig.17 Effect of geometrical shape of pier endnoses on dimensionless backwater rise for supercritical flow between piers.

CONCLUSIONS

From this research, it could be concluded that:

1- The backwater rise upstream bridge piers for both subcritical and supercritical flow between piers is inversely proportional to the value of opening ratio, while it is directly proportional to the value of the downstream Froude number (F_{r3}) for different types of the used piers.

2- The difference between the backwater rise resulted from experimental results and the corresponding computed ones from Yarnell equation for subcritical flow between piers could be referred to the difference in ratio of pier length to pier width in both studies.

3- The following formula was developed for predicting the backwater rise due to flow constriction by bridge piers at subcritical flow condition between bridge piers:

$$\frac{dy}{Y_3} = C_1(0.256 - 0.367\,O_r + 0.389\,F_{r3})$$

in which C_1 is a constant depends on pier shape; C_1 = 1.0, 0.89, and 0.85 for rectangular, triangular, and semicircular endnoses piers, respectively. The foregoing formula could be applied with the following precautions; F_{r3} = 0.2-0.62, O_r = 0.42 – 0.9, and the pier length to pier width ranges from 5:1 to 30:1.

4- The following formula was developed for predicting the backwater rise upstream bridge piers in case of supercritical flow condition between piers at the following conditions; Fr_3 ranges from 0.69 to 0.93, O_r ranges from 0.42 to 0.9, and L_P/t_P ranges from 5:1 to 30:1.

$$\frac{dy}{Y_3} = 0.1013 \quad C_2 \left(\frac{Fr_3}{Fr_{3c}}\right)^{2.586}$$

In which C_2 =1.0, 0.69, and 0.53 for rectangular, triangular, and semicircular endnoses piers, respectively.

5- The difference between the measured data and the corresponding computed results at both subcritical and supercritical flows, between piers indicated that the ratio of pier length to pier thickness was more effective on backwater rise for subcritical flow than that for supercritical flow.

6- The measurements of backwater rise for both subcritical and supercritical flows between bridge piers reflects the importance of geometrical shape of pier, especially at high values of Froude number (supercritical flow).

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NOTATIONS

The following symbols were used in this paper:

B	width of unconstricted flow;
b	constriction opening width between bridge piers;
C	coefficient depends on pier shape;
C_1	constant;
C_2	constant;
dy	backwater rise upstream bridge piers;
E_1	total energy at section 1;
E_2	total energy at section 2;
F_{r1}	upstream Froude number;
F_{r2}	Froude number between piers;
F_{r3}	Froude number downstream of bridge piers;
Fr_{3c}	downstream Froude number corresponds to critical flow condition between bridge piers (Fr_2=1);
L_P	pier length;
M_2	momentum at section 2;
M_3	momentum at section 3;
n	number of piers at bridge site;
O_r	opening ratio = $1 - t_{PS}/B$;
r	the residual ratio of energy between sections 2 and 3;
t_P	piers thickness;
t_{PS}	total thickness of piers at bridge site = n x t_P;
V_2	flow velocity at section 2;
V_3	flow velocity at section 3;
X	ratio of pier thickness to the distance between piers axis;
Y_2	flow depth at section 2 (minimum flow depth); and
Y_3	downstream normal flow water depth.

APPENDIX

APPLIED EXAMPLE

A lined canal of average width 13 m, flow depth 1.2 m, and discharge 31 m^3/s is obstructed by two piers of a highway bridge. If the width of vent is 3 m and thickness of pier is 2 m, calculate the backwater rise for the traditional three types of piers.

Solution

$O_r = 9/13 = 0.69$

$$\boldsymbol{O}_{\boldsymbol{r}}^{2} = \frac{27\,\boldsymbol{r}^{3}\boldsymbol{F}_{\boldsymbol{r}3\boldsymbol{c}}^{2}}{(2 + \boldsymbol{F}_{\boldsymbol{r}3\boldsymbol{c}}^{2})^{3}}$$

By trial and error $Fr_{3c} = 0.54$
Area of canal cross-section = 15.6 m^2
v = 31/15.6 = 1.98 m/sec
$Fr_3 = 1.98/(9.81*1.2)^{0.5} = 0.577$
As Fr_3 is greater than Fr_{3c}, then the flow between bridge piers is supercritical.

$dy/Y_3 = 0.1013\ C_2\ (0.577/0.54)^{2.586}$

dy = 14.3 cm for rectangular endnoses pier;
dy = 9.8 cm for triangular endnoses pier, and
dy =7.5 cm for semicircular endnoses pier.