THE EFFECT OF WATER JET ON THE HYDRAULIC JUMP

F. Ayça VAROL [1], Esin ÇEVİK [2], and Yalçın YÜKSEL [3]

Yıldız Technical University, Department of Civil Engineering
Hydraulic and Coastal-Harbour Laboratory, İstanbul, Turkey
[1] Research Assistant, E-mail: favarol@yildiz.edu.tr
[2] Prof. Dr., E-mail: cevik@yildiz.edu.tr
[3] Prof. Dr., E-mail: yuksel@yildiz.edu.tr

ABSTRACT

In this study, effect of water jet on the hydraulic jump in horizontal channel was investigated. The aim of this study was to determine effect of the water jet with different flow rates to the hydraulic jump characteristics.

The experiments were carried out in different upstream Froude numbers in the range of 3.43 - 4.83 and five different water jet discharges. Free jump and jumps with jets were analyzed by image processing technique with a high speed SVHS camera. Flow structures, roller lengths, water surface profiles and energy losses during free hydraulic jump and hydraulic jumps with water jet were studied and compared experimentally.

Keywords: Hydraulic jump, energy losses, effect of water jet, roller length.

1. INTRODUCTION

In hydraulic structures, hydraulic jump is used to consume excess energy commonly. Nevertheless scouring may occur at the channel bed depending on the turbulent flow pattern at the hydraulic jump section. Thus, the energy loss dissipation during the jump and the location of the jump are important design considerations. The hydraulic jump is a rapid transitional event from supercritical to subcritical flow. During hydraulic jump, the energy loss occur with large-scale turbulence structure. Hydraulic jumps have been extensively used for energy dissipation below hydraulic structures.

The hydraulic jump was first investigated experimentally by Bidone [1]. This led Belanger [2] to distinguish between subcritical (mild) and supercritical (steep) slopes, since he had observed that in steep channels hydraulic jump is frequently produced by a barrier in orginally uniform flow.

Lots of researchers studied about hydraulic jump. Rajaratnam [3], [4] Rajaratnam and Subramanya [5], Sarma and Newnham [6], Long et al. [7], Ead and Rajaratnam [8], Chaurasia [9], Ohtsu et al. [10] investigated structure of hydraulic jump in their researches. Wielogorski and Wilson [11], Swamee and Rathie [12] studied the equation between sequent depths. Safranez [13], Peterka [14] worked on determining the roller length and jump length. Leutheusser and Kartha [15] studied the mean velocity distribution. Bakhmeteff and Matke [16], Rajaratnam [17] and Rajaratnam and Subramanya [3] analysed the surface profile. Leutheusser and Kartha [18], Resch and Leutheusser [19], Liu et al. [20] looked for turbulence characteristics. Garg and Sharma [21] worked on energy dissipation. Rajaratnam [17] and Hager [22] researched the air conditioning length. Narayanan [23], McCorquodale and Khalifa [24], Madsen and Svendsen [25] worked on the modelling of the hydraulic jump.

Rause et al. [26] made the first attempt to measure the turbulence characteristics in free hydraulic jumps in an air model using a hot-wire anemometer. Resch and Leutheusser [19] used a hot-film anemometer to make some limited measurements on the turbulence in free hydraulic jumps with Froude numbers, Fr, of 2.85 and 6 in a water channel and presented the experimental results including mean velocity, turbulence intensities and Reynolds shear stresses without general analysis. Liu et al. [20] studied about turbulence characteristics of hydraulic jumps with Froude numbers of 2.0, 2.5 and 3.2. A Micro Acoustic Doppler velocimeter was used to obtain measurements of the velocities, turbulence intensities, Reynolds stresses and power spectra.

Yüksel et al. [27], [28] and Bostan [29] used the similarities between the front of a broken wave and a hydraulic jump. This have led to the idea of expressing the energy loss in the surf zone through the dissipation in a bore. The plunging breaker was simulated by a hydraulic jump with a jet impinging at an angle at its toe. Such a hydraulic jump is studied numerically and experimentally to determine its internal flow characteristics and energy dissipation. Therefore, they were carried out experiments in a horizontal channel with a free jump and jumps with jets introduced at various angles. The jet discharge was kept constant during their experiments.

This study is different from the other studies; the effect of water jet with five different flow rates on the hydraulic jump in horizontal channel was investigated.

2. HYDRAULIC JUMP

For supercritical flow in a horizantal rectangular channel, the energy of flow is dissipated through frictional resistance along the channel, resulting in a decrease in velocity an increase in depth in direction of flow.

In open channels, hydraulic jump is characterised by a sharp rise in free surface elevation, strong turbulence, splashing and air entrapment in the roller (Chanson, [31]).

The district of large scale turbulence which occurs during hydraulic jump is called roller length. Lots of researchers explained particularly turbulence roller and mechanism of circulation in this roller region.

Hydraulic jump in horizantal channel is shown in Figure 1. y_1 and y_2 are depth of the upstream and downstream of the jump respectively. In addition to these prejump and postjump depths are collectively called sequent depths. L_r is roller length which is from toe of the jump to the end of the roller, ending at the surface stagnation point. L_j is jump length which is from toe of the jump to section, being uniform velocity distribution, ie water surface to the horizontal position is the distance (Yüksel et al., [27]).

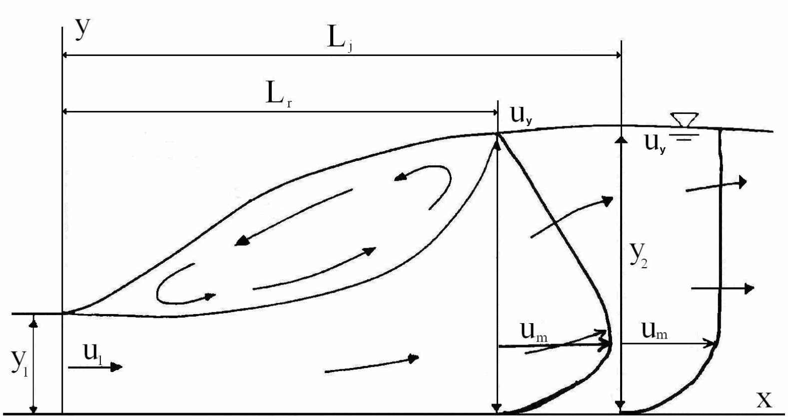

Figure 1 Hydraulic jump (Bostan, [29])

The length of surface roller is smaller than the length of the jump. Velocity distribution in jump which is bisected as inner and outer layer by Narayanan [23] is shown in Figure 2.

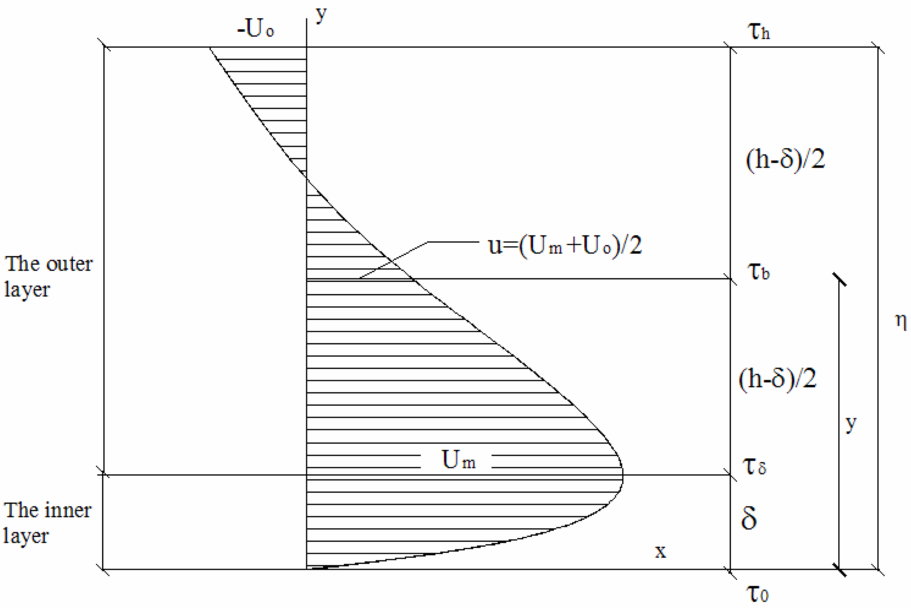

Figure 2 Mean velocity distribution in jump (Narayanan, [23])

It is assumed conservation of mass and momentum in jump. The two equations of motion for a hydraulic jump are the equations of continuity [Eq (1)] and the momentum [Eq (2)].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial \tau}{\partial y}+g\sin\alpha \tag{2}$$

where u: velocity paralel to the bed, v:velocity normal to the bed, p:mean static pressure, α: the bed slope, τ: the Reynolds turbulent shear stress, ρ: density of water and g: the acceleration due to gravity.

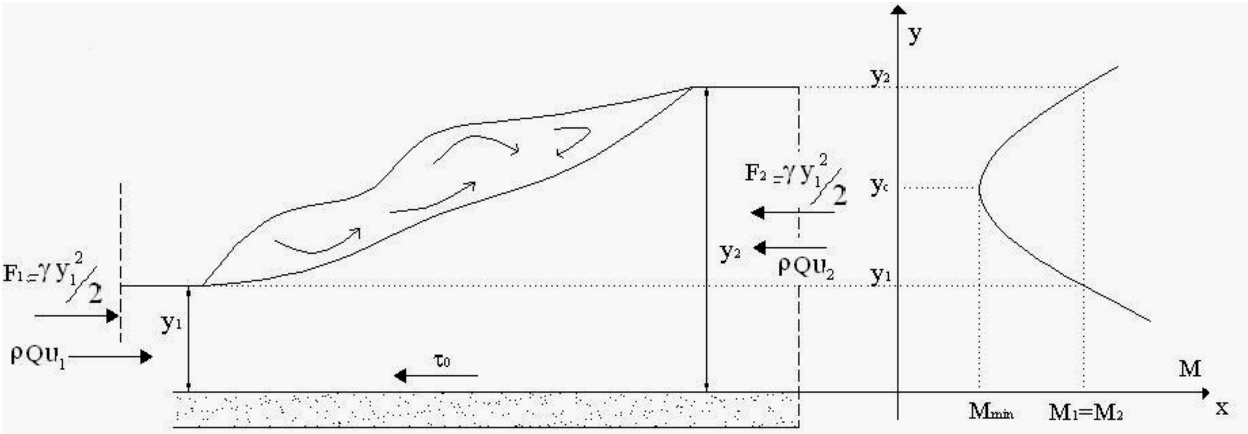

Figure 3 Conservation of momentum in hydraulic jump

If the momentum conservation is written as below

$$\rho Q(u_2 - u_1)= F_1 - F_2 - \tau_0 \tag{3}$$

where Q is discharge of the system, u is velocity, F is force, 1 and 2 are indices showing the upstream and downstream conditions, respectively. If the velocity distribution is assumed to be uniform and the pressure distribution hydrostatic, and if the boundary shear stress is neglected, then the sequent flow depth is given by equation (4).

$$\frac{y_2}{y_1}=\frac{1}{2}\left(-1+\sqrt{1+8Fr_1^2}\right) \tag{4}$$

This equation is known as Belanger Equation, where y_1 is upstream depth of the jump, y_2 is downstream depth of the jump and $Fr_1 = u_1/\sqrt{gy}_1$ is upstream Froude number. The ratio of downstream and upstream Froude Numbers can be written as:

$$\frac{Fr_2}{Fr_1} = \frac{2^{3/2}}{\left(\sqrt{1+8Fr_1^2}-1\right)^{3/2}} \tag{5}$$

in which $Fr_2 = u_2 / \sqrt{gy_2}$ is downstream Froude number.

As seen in Figure 4, energy loss as ΔE is eventuated during hydraulic jump. This loss of energy in the jump is equal to the difference of specific energies before and after the jump.

$$\Delta E = \left(y_1 + \frac{u_1^2}{2g}\right) - \left(y_2 + \frac{u_2^2}{2g}\right) \tag{6}$$

As a result of the basis of mass and momentum conservation and mathematical operation, this equation (6) is took into equation (7), energy loss in hydraulic jump, which is seen below.

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} \tag{7}$$

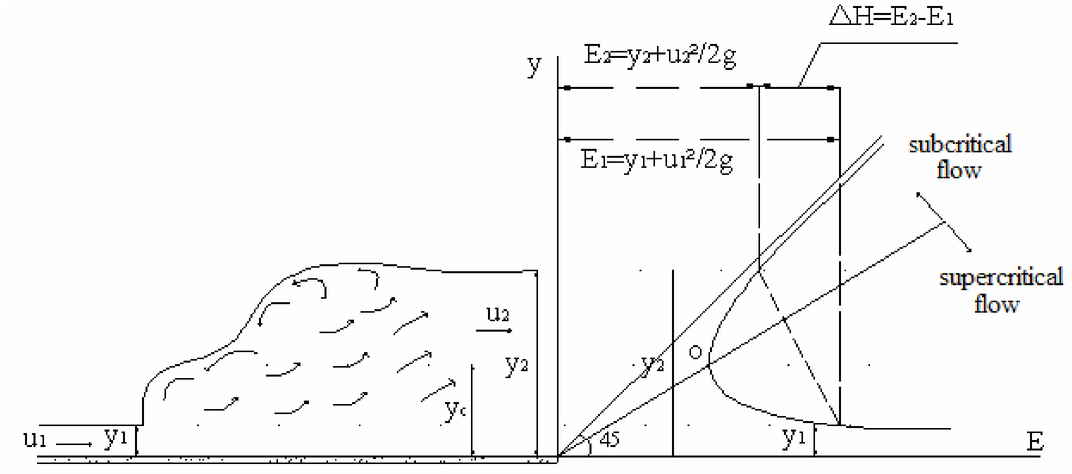

Figure 4 The projection of hydraulic jump and curve of specific energy-flow depth

The location of the jump is an important design consideration on hydraulic structures especially energy breakers. Length of the jump cannot be determined easily by theory, but it has been investigated experimentally by many hydraulicians. Table 1 summarizes these researchers and formulas.

Table 1 Some empiricial formulas for roller length

Researcher	Res. Date	Formula (L_j)	
Safranez	1929	$5.2y_2$	
Bakhmeeff-Matzke	1936	$5(y_2-y_1)$	
Smetana	1934	$6(y_2-y_1)$	
Page	1935	$(5\approx6)y_2$	
Bradley ve Peterka	1957	$6y_2$	
Peterka	1978	$5\text{-}6y_1$ $6\text{-}6.1y_1$ $6\text{-}5.5y_1$	$Fr_1=2.5\text{-}4.5$ $Fr_1=4.5\text{-}14$ $Fr_1=14\text{-}20$

2.1 The Effect of Water Jet to Hydraulic Jump

The one dimensional momentum equation for the hydraulic jump of unit width with the jet impinging at its toe may be written as (Yüksel et al., [28]):

$$\rho(q_1+q_2)u_2-\rho q_1u_1-\rho q_ju_j\cos\theta=\frac{1}{2}\rho g{y_1}^2-\frac{1}{2}\rho g{y_2}^2 \tag{8}$$

in which u_1 and y_1 are respectively the upstream velocity and depth of the jump, u_2 and y_2 are respectively the velocity and the downstream depth of the jump, u_j and y_j are respectively the velocity and thickness of the impinging jet, θ is the angle of the jet makes with the horizontal and ρ is the density of the liquid. It follows that $q_1=u_1y_1$, $q_2=u_2y_2$ and $q_j=u_jy_j$. Dividing the momentum equation through by $(1/2)\rho g{y_1}^2$ and rearranging the terms gives (Yüksel et al., [28]):

$$2{Fr_1}^2\left[\left(1+\frac{q_j}{q_1}\right)^2\frac{y_1}{y_2}-1-\frac{{q_j}^2}{{q_1}^2}\frac{y_1}{y_j}\cos\theta\right]=1-\frac{{y_2}^2}{{y_1}^2} \tag{9}$$

in which $Fr_1=u_1/\sqrt{gy_1}$ is upstream Froude number of the flow. For $q_j=0$ equation (9) reduce to classical equation for a free jump (Yüksel et al., [28]).

3. EXPERIMENTS

Experiments were conducted in one-dimensinal channel in Hydraulics and Coastal-Harbour Engineering Laboratory, Yıldız Technical University. They were performed in a 150 mm depth, 75 mm wide glass-walled flume 5.0 m long. The slope of channel can be changed by request. Channel system is provided water transfer by pumps in itself as shown in Figure 5.

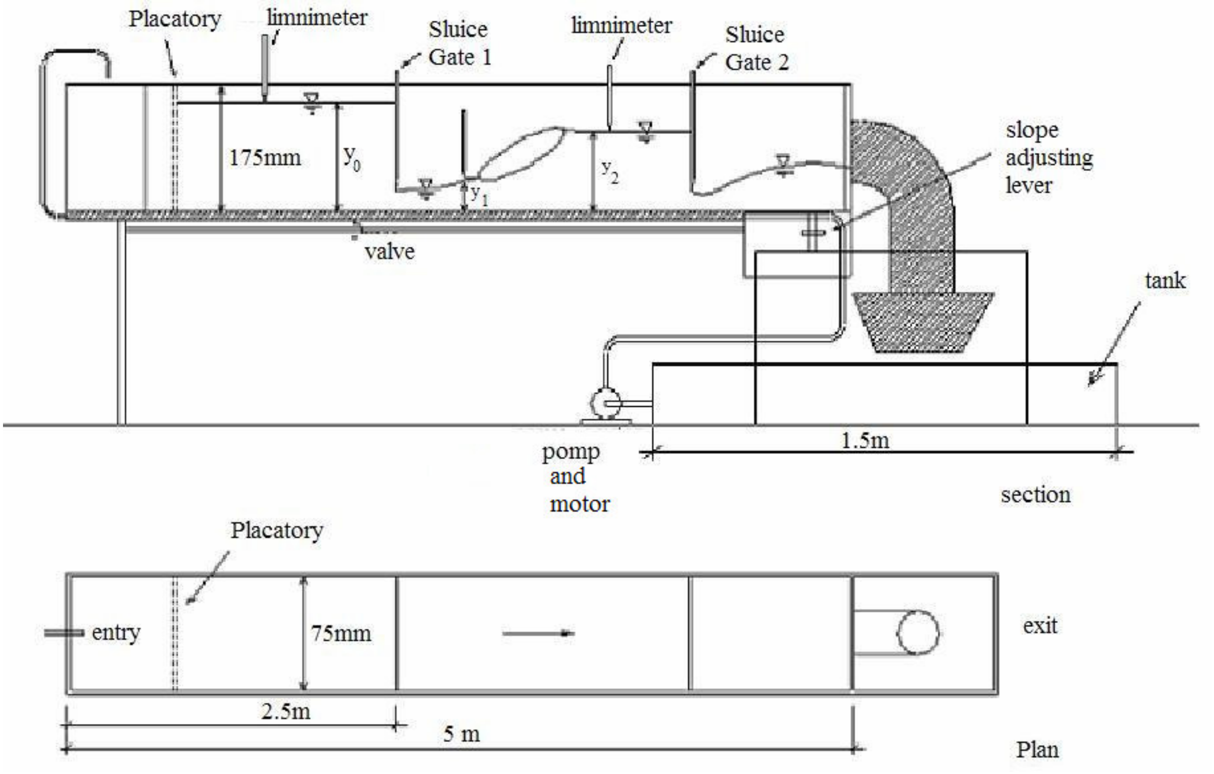

Figure 5 Experimental set-up

Two different sluice gates have been used in channel. Originally a hydraulic jump was formed downstream of sluice gate 1 as shown in Figure 5. After the application of the water jet, the jump was approached to the first sluice gate in all tests. Therefore, by adjusting the downstream tail gate 2, free hydralic jumps were formed as possible as close to the second gate. The measurement of discharges in channel were made with the help of gravity tank. In experiments, the slope of flume bed in all experiments was kept horizontal. Different jumps were obtained for various upstream Froude numbers. The experimental conditions are presented in Table 2.

In Table 2, where $q_j^* = \frac{q_j}{u_1 y_1}$ represents the dimensionless unit jet discharge and $u_j^* = \frac{u_j}{u_1}$ the dimensionless jet velocity. q_j is the unit jet discharge (m^3/sm) and u_j; the jet velocity (m/s).

Table 2 Experimental conditions

		Discharge of system	**Discharge of jet (m^3/s)**	**qj (m^3/sm)**	**q_j^***	**y_1 (cm)**	**y_2 (cm)**	**u_1 (m/s)**	**V_j (m/s)**	**u_j^***
1	**FR=3.43**	**Q_S=** 66,80x10^{-5}	Free jump			0.85	4.3	0.99		
			Q_{J1}=4,67x10^{-5}	6,970x10^{-4}	0.073	0.75	4.65	1.271	0.697	0.548
			Q_{J2}=5,385x10^{-5}	8,037x10^{-4}	0.084	0.9	4.7	1.069	0.804	0.752
			Q_{J3}=6,389x10^{-5}	9,535x10^{-4}	0.098	0.7	4.85	1.394	0.954	0.684
			Q_{J4}=7,825x10^{-5}	11,679x10^{-4}	0.117	0.8	5	1.244	1.168	0.939
			Q_{J5}=9,71x10^{-5}	14,493x10^{-4}	0.142	0.4	5.15	2.55	1.5	0.588
2	**FR=3.71**	**Q_S=** 74,42x10^{-5}	Free jump			0.9	4.6	1.103		
			Q_{J1}=4,67x10^{-5}	6,970x10^{-4}	0.066	0.8	4.95	1.318	0.697	0.529
			Q_{J2}=5,385x10^{-5}	8,037x10^{-4}	0.076	0.9	5.1	1.182	0.804	0.68
			Q_{J3}=6,389x10^{-5}	9,536x10^{-4}	0.089	0.9	5.15	1.197	0.954	0.797
			Q_{J4}=7,83x10^{-5}	11,687x10^{-4}	0.107	0.7	5.25	1.567	1.169	0.746
			Q_{J5}=9,718x10^{-5}	14,504x10^{-4}	0.129	0.6	5.4	1.8697	1.45	0.776
3	**FR=4.12**	**Q_S=** 62,87x10^{-5}	Free jump			0.75	4.2	1.118		
			Q_{J1}=4,67x10^{-5}	6,970x10^{-4}	0.077	0.7	4.75	1.287	0.697	0.542
			Q_{J2}=5,39x10^{-5}	8,045x10^{-4}	0.088	0.75	4.8	1.214	0.804	0.662
			Q_{J3}=6,38x10^{-5}	9,522x10^{-4}	0.103	0.55	4.9	1.679	0.952	0.567
			Q_{J4}=7,83x10^{-5}	11,687x10^{-4}	0.124	0.65	5.1	1.45	1.169	0.806
			Q_{J5}=9,72x10^{-5}	14,508x10^{-4}	-	Submerged	-	Submerged	1.45	-
4	**FR=4.37**	**Q_S=** 73,49x10^{-5}	Free jump			0.8	4.65	1.225		
			Q_{J1}=4,669x10^{-5}	6,969x10^{-4}	0.067	0.85	5.05	1.226	0.697	0.569
			Q_{J2}=5,39x10^{-5}	8,045x10^{-4}	0.077	0.85	5.15	1.237	0.804	0.65
			Q_{J3}=6,386x10^{-5}	9,531x10^{-4}	0.089	0.85	5.25	1.253	0.953	0.761
			Q_{J4}=7,813x10^{-5}	11,661x10^{-4}	0.108	0.6	5.35	1.807	1.166	0.645
			Q_{J5}=9,709x10^{-5}	14,491x10^{-4}	-	Submerged	-	Submerged	1.45	-
5	**FR=4.59**	**Q_S=** 77,12x10^{-5}	Free jump			0.8	4.7	1.285		
			Q_{J1}=4,67x10^{-5}	6,970x10^{-4}	0.064	0.7	5.15	1.558	0.697	0.447
			Q_{J2}=5,39x10^{-5}	8,045x10^{-4}	0.073	0.8	5.25	1.375	0.804	0.585
			Q_{J3}=6,385x10^{-5}	9,529x10^{-4}	0.086	0.9	5.3	1.237	0.953	0.77
			Q_{J4}=7,82x10^{-5}	11,671x10^{-4}	0.103	0.65	5.4	1.742	1.167	0.67
			Q_{J5}=9,72x10^{-5}	14,508x10^{-4}	0.123	0.8	5.6	1.477	1.45	0.982
6	**FR=4.83**	**Q_S=** 66,37x10^{-5}	Free jump			0.7	4.45	1.264		
			Q_{J1}=4,67x10^{-5}	6,970x10^{-4}	0.074	0.6	4.95	1.579	0.697	0.441
			Q_{J2}=5,39x10^{-5}	8,045x10^{-4}	0.084	0.45	5.05	2.126	0.804	0.378
			Q_{J3}=6,38x10^{-5}	9,522x10^{-4}	0.098	0.45	5.1	2.156	0.952	0.442
			Q_{J4}=7,825x10^{-5}	11,679x10^{-4}	0.118	0.7	5.25	1.413	1.168	0.827
			Q_{J5}=9,718x10^{-5}	14,505x10^{-4}	0.143	0.85	5.45	1.193	1.45	1.215

The water jet apparatus was made of stainless chrome-nickel alloy. The rectangular cross-section and water jet's dimensions are given in Figure 6.

The water of jet was supplied from a tank. Continuity of the jet flux is provided with the help of 5.80 m high, constant-head tank. Hydraulic jump with water jet is shown in Figure 7.

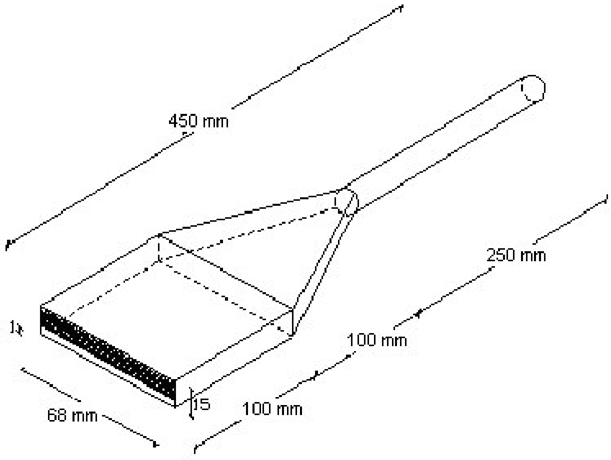

Figure 6 Jet apparatus

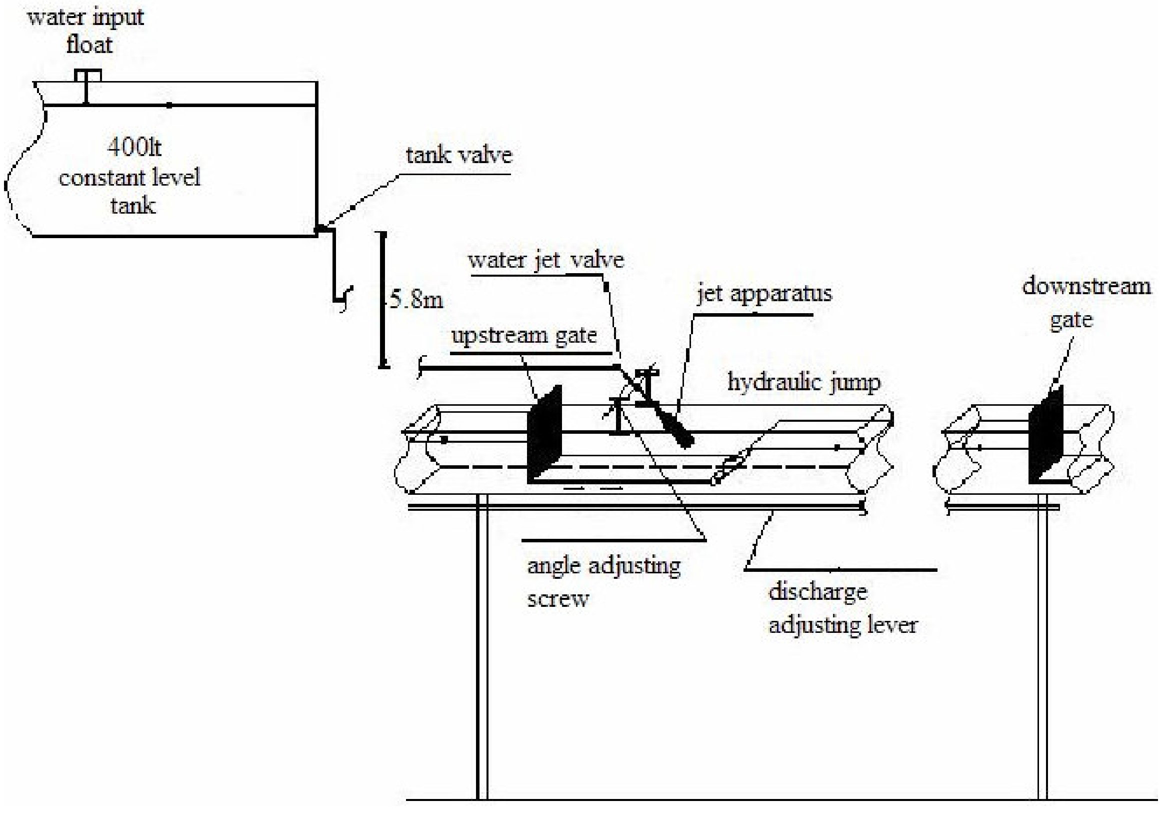

Figure 7 The formation hydraulic jump with water jet

Experiments were carried out under the conditions of free jumps with and without the jet for five different discharges, six different Froude numbers and constant jet angle θ of 60^o to the horizontal channel. To obtain different water jet discharges, flow is controlled by a valve. The jet discharges are respectively 0.0467, 0.0539, 0.0638, 0.0782 and 0.0972 lt/s.

The tank had been worked full to keep constant hydraulic jump conditions. Due to water jet discharge, extension stream had been overflowed from channel tank. Thus upstream Froude numbers was kept constant. The measurements were made primarily for free hydraulic jump, than with five different water discharges. The measurement of water jet discharges were repeated in all experiments.

To measure the flow velocity in the roller image processing technique was used. In this measurement, the flow was seeded with ABS (Acril Nitril Butadien Styren) particles of diameter less than 1.5 mm and specific gravity of 1.04. The ABS particles were recorded and observed by a video camera.

The images were processed at the rate of 25 frames per second. Surface profiles were defined from the coloured printouts and with the help of AUTOCAD programme of the captured video frames.

4. EXPERIMENTAL RESULTS

4.1 Surface Profiles

The depth at assorted positions in the course of the jump defining surface profiles were considered from video frames. The graphics were plotted for different discharges of the water jet for each Froude number.

The hydraulic jumps were observed to move to the upstream with increasing water jet flow rate regardless Froude number. Even the jump has so much approached to the upstream gate; the jet could not affect to jump because submerged jump was formed, so the measurements of these experiments could not be evaluated.

The steepness of the surface profiles during hydraulic jumps with water jet increased when compared with free hydraulic jump, also downstream water depth, y_2, increases with increasing water jet discharge. The surface profiles can be seen from Figures 8 to 13 for each Froude Number.

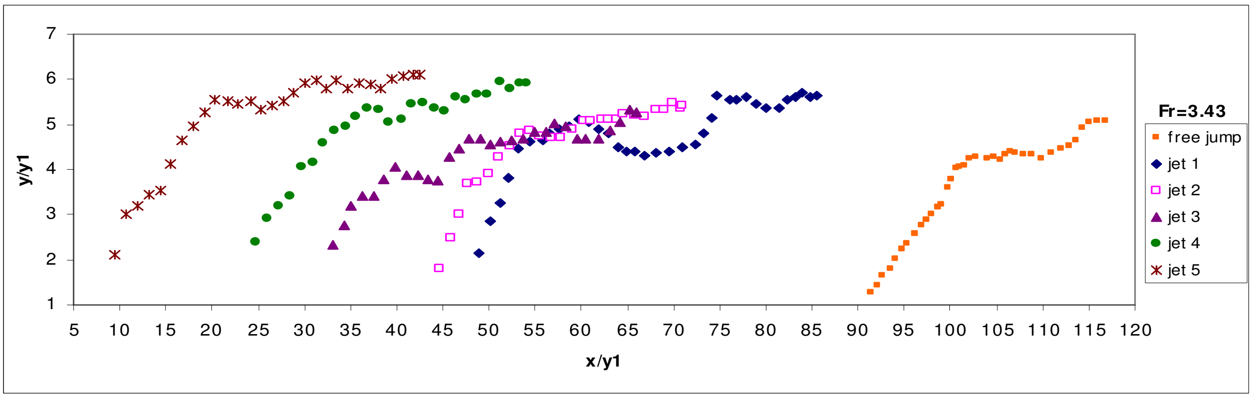

Figure 8 Surface profiles for Fr_1=3.43

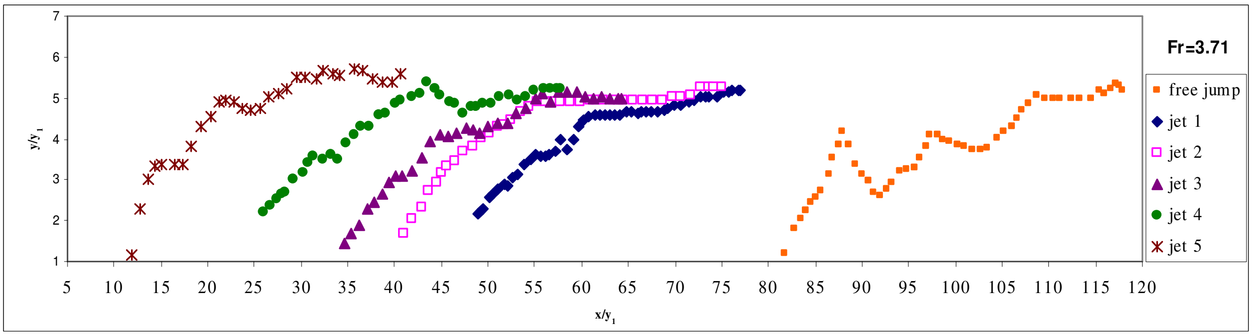

Figure 9 Surface profiles for Fr_1=3.71

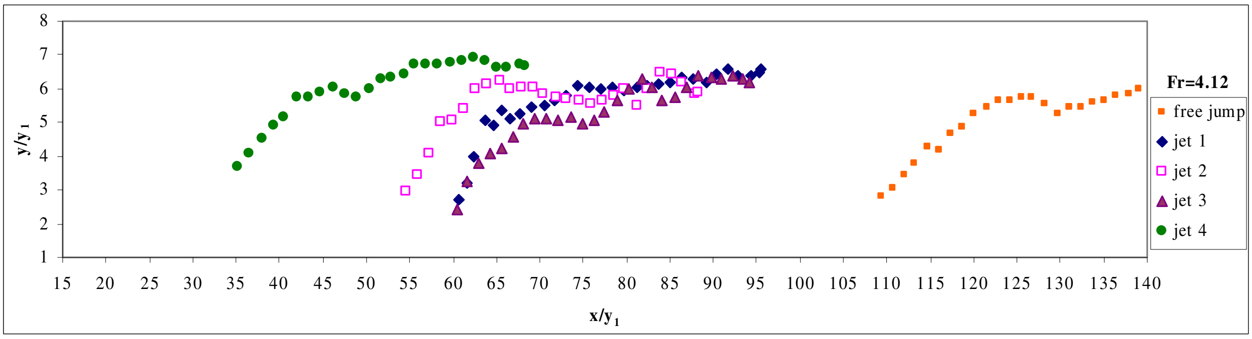

Figure 10 Surface profiles for Fr_1=4.12

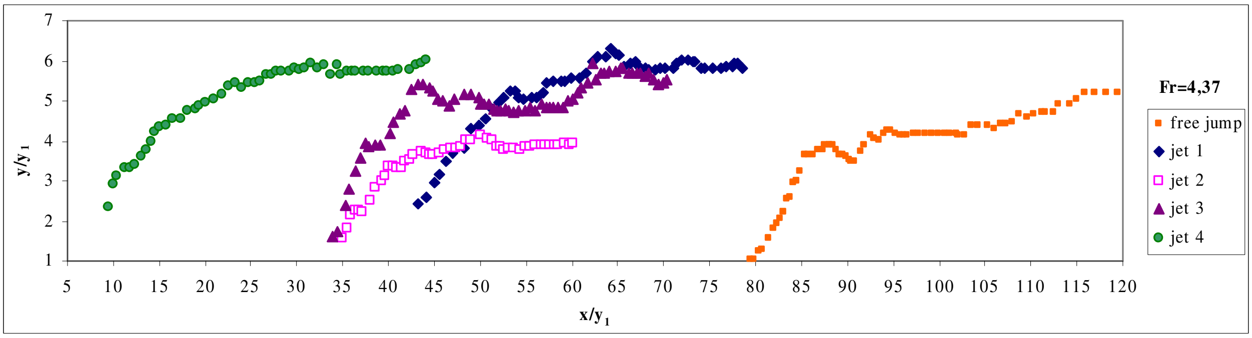

Figure 11 Surface profiles for Fr_1=4.37

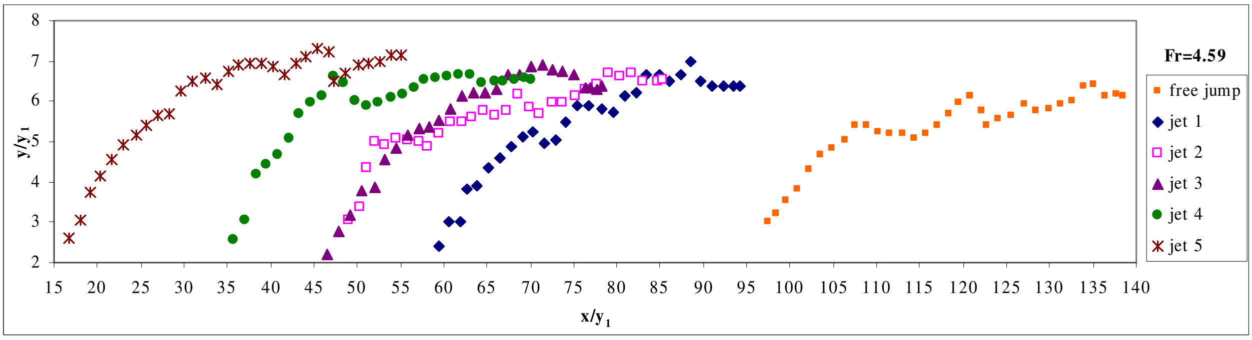

Figure 12 Surface profiles for Fr_1=4.59

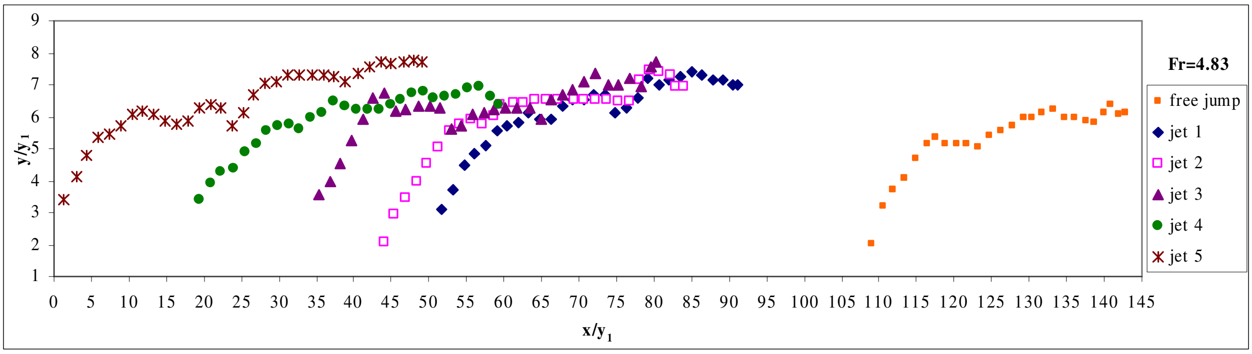

Figure 13 Surface profiles for Fr_1=4.83

4.2 The Roller Lengths

The roller length of the hydraulic jump was determined using the image processing technique. The beginning of jump was assigned from the video frames as well as during the experiment. The end of the roller was defined as the point where the reverse flow velocity became zero. The location of the end of the roller length were confirmed by to put drops of potassium permanganate toward end of the jump. Thus L_r/y_1 is plotted against Fr_1, the upstream Froude number.

The roller length of a jump with a jet are longer than that of the free jump for the same upstream Froude number. Roller length increased with increasing upstream Froude number. A significant effect of water jet discharge on roller length could not be observed as shown in Figure 14.

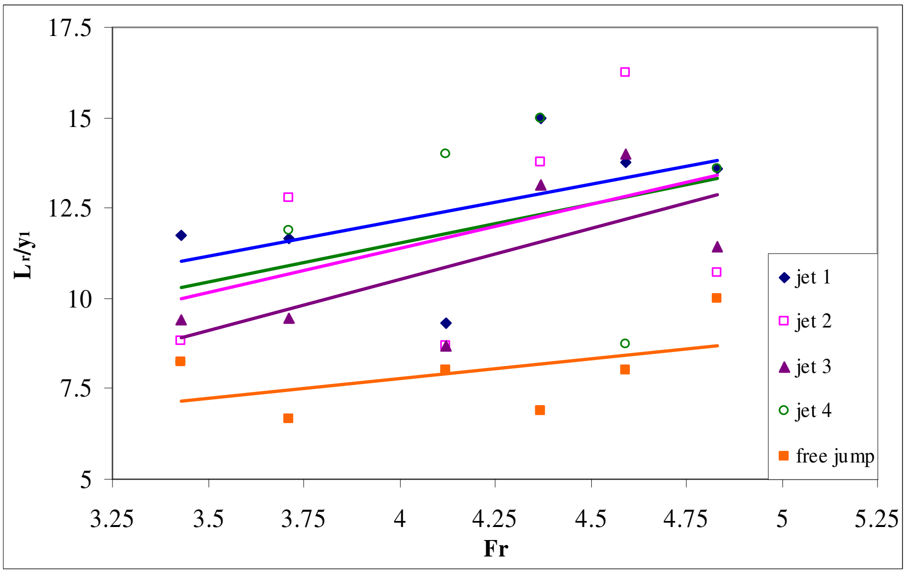

Figure 14 Roller length variation against upstream Froude Number (Fr_1)

4.2 The Energy Losses

The energy losses were calculated theoretically with equation (7). Figure 15 shows energy dissipation, normalized with the upstream depth y_1, plotted against upstream Froude number (Fr_1) for jumps with and without jets. The normalized energy losses with respect to upstream water depth and the energy loss of the free hydraulic jump increased with increasing Froude number both in free hydraulic jump and the hydraulic jump with water jet. Relative energy dissipation was calculated by dividing to energy dissipation in free jump. Relative energy dissipation was plotted against upstream Froude number as shown in Figure 16. The energy losses in hydraulic jump with water jet were obtained higher than that of the free hydraulic jump. The energy loss increased as the jet flow rate increases.

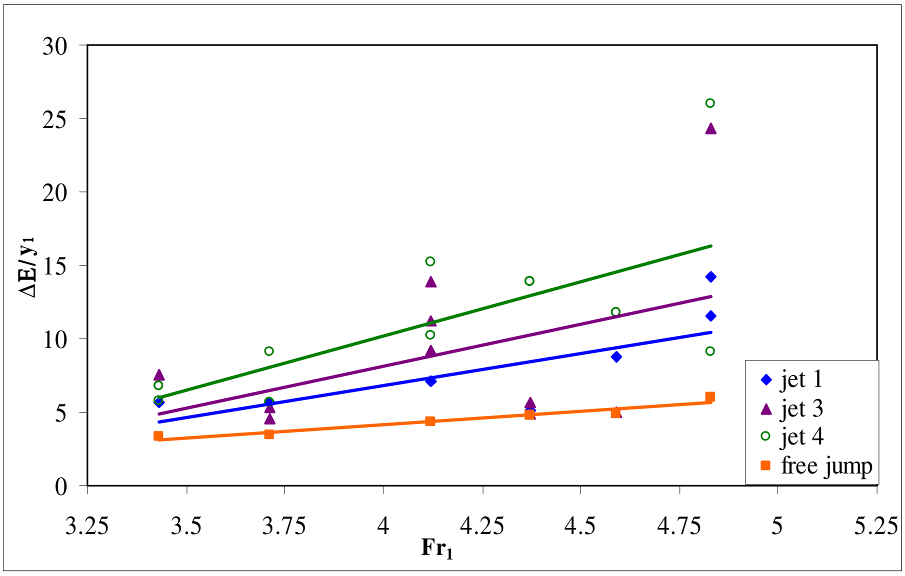

Figure 15 Normalized energy dissipation against upstream Froude Number (Fr_1)

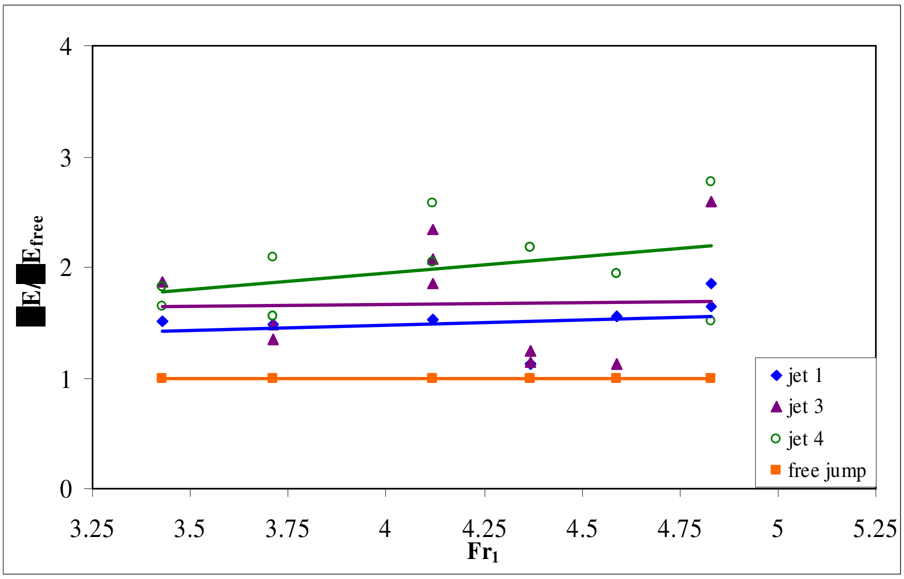

Figure 16 The variation of relative energy dissipation against upstream Froude Number (Fr_1)

5. CONCLUSIONS

The essential results are on follows:

1. According to all graphics plotted to determine the water surface profiles, hydraulic jumps were observed moved to upstream of the flow with increasing water jet flow rate.
2. The steepness of the surface profiles during hydraulic jumps with water jet increased when compared with free hydraulic jump. In other words, downstream water depth, y_2, increases with increasing water jet discharge.
3. The roller length for a jump with water jet increased with respect to free hydraulic jump.
4. Roller length increased with increasing upstream Froude number both in free hydraulic jump and the hydraulic jump with water jet.
5. The normalized energy losses with respect to upstream water depth and the energy loss of the free hydraulic jump increased with increasing upstream Froude number both in free hydraulic jump and the jump with water jet.
6. The energy losses in hydraulic jump with water jet were obtained higher than that of the free hydraulic jump.
7. The energy loss increased as the jet flow rate increases.

REFERENCES

1. Bidone, G., (1820), Expériences sur le remous et la propagation des ondes, Memorie della Realle Accademia dele Science di Torino, 30, 195-292.

2. Bélanger, J.B., (1828), Essai sur la Solution Numérique de quelques Problémes Relatifs au Mouvement Permanent des Eaux Courantes (‘Essay on the Numerical Solution of Some Problems relative to Steady Flow of Water’), Carrilian-Goeury, Paris, France.
3. Rajaratnam, N., (1965), “The Hydraulic Jump as a Wall Jet”, Journal of Hydraulic Division, Proc. of the ASCE, 91(5): 107-132, September.
4. Rajaratnam, N., (1967), “The Hydraulic Jumps”, Advances in Hydroscience, Ed. V.T. Chow, Academic Press, New York, USA, 4: 197-280.
5. Rajaratnam, N. and Subramanya K., (1968), “Profile of the Hydraulic Jump”, Journal of Hydraulic Division, ASCE, 94(3): 663-673.
6. Sarma, K.V.N. and Newnham, D.A., (1973), “Surface Profile of Hydraulic Jump for Froude Numbers Less than Four”, Water Power, April, 139-142.
7. Long, D., Steffler, P.M., and Rajaratnam, N., (1990), “LDA study of flow structure in submerged hydraulic jump”, Journal of Hydr. Research, 28:437-460.
8. Ead, S.A. and Rajaratnam, N., (2002), “Hydraulic Jumps on Corrugated Beds”, Journal of Hydraulic Eng., ASCE, 128:656-663.
9. Chaurasia, S.R. (2003), “Direct Equations for Hydraulic Jump Elements in Rectangular Horizontal Channel”, Journal of Irrigation and Drainage Eng., ASCE, 129:291-294.
10. Ohtsu, I., Yasuda, Y., and Gotoh, H., (2003), “Flow Conditions of Undular Hydraulic Jump in Horizontal Rectangular Channels”, J. Hydraul. Eng., 129(12): 948-955.
11. Wielogorski, J.W. and Wilson, E.H., (1970), “Nondimensional Profile Area Coefficients for Hydraulic Jump in Sloping Rectangular Channels”, Water Power, 22(4):144-150.
12. Swamee, P. K., and Rathie, P.N., (2004), “Exact solutions for Sequent Depths Problem”, Journal of Irrigation and Drainage Eng. ASCE, 130:520-522.
13. Safranez, K., (1929) Untersuchung über den Wechselsprung (Investigation on the hydraulic jump), Der Bauingenieur, Vol. 10, Heft 37, pp. 649-651; Heft 38, pp. 668-678.
14. Peterka, A.J., (1978), Hydraulic design of stilling basins and energy dissipators, Engineering, Monograph No.25, USBR.
15. Leutheusser, H.J. and Kartha, V.C., (1972), “Effects of Inflow Condition on Hydraulic Jump”, Journal of Hydraulic Division, Proc. of the ASCE, 98:1367-1385, August.
16. Bakhmeteff, B.A. and Matzke, A.E. (1936), “The Hydraulic Jump in Terms of Dynamic Similarity”, Trans. ASCE, 101:630-680.
17. Rajaratnam, N., (1962), “An Experimental Study of Air Entrainment Characteristics of the Hydraulic Jump”, J. of Instn. Eng. India, 42(7):247-273.
18. Leutheusser, H.J. and Kartha, V.C., (1972), “Effects of Inflow Condition on Hydraulic Jump”, Journal of Hydraulic Division, Proc. of the ASCE, 98:1367-1385, August.
19. Resch, F.J. and Leutheusser, H.G., (1972), “The Hydraulic Jump: Turbulence Measurements in the Two-Phase Flow Region”, J. Houille Blanche 4:279-293 (in French).
20. Liu, M., Rajaratnam, N. and Zhu, D.Z., (2004), “Turbulence Structure of

Hydraulic Jumps of Low Froude Numbers", Journal of Hydraulic Division, ASCE, 130:511-520.

21. Garg, S.P. and Sharma, H.R., (1971), "Efficiency of Hydraulic Jump", Journal of the Hydraulic Divisions, Proc.of the ASCE, 97:409-419, March.
22. Hager, W.H., (1992) Energy Dissipators and Hydraulic Jump, Kluwer Academic Publishers, Dordrecht, Netherlands.
23. Narayanan, R., (1975), "Wall Jet Analogy to Hydraulic Jump", Journal of Hydraulic Division, ASCE, 101(3): 347-359
24. Mccorquodale, J. and Khalifa, A., (1983), "Internal Flow in a Hydraulic Jump", Journal of Hydraulic Division, ASCE, 106(3): 355-367
25. Madsen, P.A. and Svendsen, I.A., (1983), "Turbulent Bores and Hydraulic Jumps", Journal of Fluid Mechanics, 129:1-25
26. Rouse, H., Siao, T.T., and Nagaratnam, S., (1959), "Turbulence Characteristics of the Hydraulic Jump", Trans. ASCE, 124:926-966.
27. Yüksel, Y., Narayanan, R., Kapdaşlı, S., Günal, M., Çevik, E., Bostan, T., and Çelikoğlu Y., (1999), "Surf Bölgesinde Çift Fazlı Hidrodinamik Yapının İncelenmesi", Tübitak, Proje No: Intag 823, Istanbul, Turkey.
28. Yüksel, Y., Günal, M., Bostan, T., Çevik, E., and Çelikoğlu, Y., (2004), "The Influence of Impinging Jets on Hydraulic Jumps", Proc. of the Institution of Civil Eng., Water Management 157, WM2, 63-76.
29. Bostan, T., (2004), Modelling of Wave Breaking with Hydraulic Jump Anology, Yıldız Technical University, PhD Thesis, İstanbul.
30. Chanson H. and Brattberg T., (2000), "Experimental study of air-water shear flow in a hydraulic jump", International Journal of Multiphase Flow 26, pp. 583-607.