AN EXPERIMENTAL MODEL FOR FLOW THROUGH POROUS MEDIA USING WATER FILTER

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ABSTRACT

An experimental study was conducted to examine several selected water filter's media such as crushed silica, crushed anthracite coal, glass beads crushed porcilinaite and crushed garnet. A large spread of particle size (0.5 mm – 2 mm), porosity (35% - 60%) under different temperatures (20^oC - 80^oC) were tested in order to validate the experimental modeling of both Darcy's and Forchheimer's law parameters.

Typical constants of head loss in porous media as a function of velocity using Forchheimer's model has been correlated by using hydraulic conductivity coefficients *a**, *b** which are experimentally evaluated at based conditions ($T_w = 25^oC$), ($n = 50\%$), ($d = 1$ mm). Empirical equations for different selected porous media and filter's characteristics were approximated to use Forchheimer's model.

Keywords: porous media, Forchheimer regime, water filter, pressure loss head

1- INTRODUCTION

The simple linear relationship between flow and head loss by Darcy (1856) was demonstrated by Forchheimer (1901) for condition under which the relationship between flow and head loss in porous media doesn't follow. Forchheimer (1901) proposed the nonlinear behavior of underground hydraulic gradients under certain conditions as follow:

$$\frac{\Delta H}{\Delta L} = av + bv^2 \tag{1}$$

where v: superficial (approach) velocity = $\frac{Q}{A}$ $\left(LT^{-1}\right)$; a: hydraulic conductivity coefficient related to linear head loss (TL^{-1}); b: hydraulic conductivity coefficient related to nonlinear head loss ($T^2 L^{-2}$).

Blake (1923) developed empirical relationship using a plot of friction factor (vs). Reynolds number for flow through tower packing at relatively high Reynolds numbers as in the following form:

$$\frac{\Delta H}{\Delta L} = C_b \ \mu^{0.2} \ \gamma^{0.8} \left[\frac{(1-n)^{1.2}}{n^3}\right] S_v^{\ 1.2} V^{1.8} \tag{2}$$

Burke and Plummer (1928) used dimensional analysis to develop streamlined equation that fit quite well a variety of experimental data collected from the literature:

$$\frac{\Delta H}{\Delta L} = K_2 \left(\frac{1}{g}\right)\left[\frac{(1-n)}{n^3}\right] S_v V^2 \tag{3}$$

where, K_2 is constant; γ: specific weight ($\mathrm{ML^{-2}\,T^{-2}}$); $\mathrm{C_b}$: Blake constant. Kozeny (1927a,b) and White (1929) applied similar principles to characterize the nonlinear resistance of porous media to the flow of gases at high Reynolds number. Nutting (1930) defined the specific resistance and Wyckoff (1933) popularized Nuttings specific resistance parameter, resulting in its wide use whenever the modeling of the flow of fluids underground was under taken. Fancher (1933), Fair and Hatch (1933), and Wallis (1938, 1939) developed a powerful model for predicting Darcy resistance from the characteristics of the porous media. Forchheimer's model (Eq. 1) will also show that this is the best model which characterizes the flow through the porous media over the range of conditions of importance in water filtration. (Rose and Risk, 1949) have argued that an equation of the following form often fits the data as well or better:

$$\frac{\Delta H}{\Delta L} = m v^{z} \tag{4}$$

where, *m* and *z* are coefficients. To create an equation for losses through porous media over a wide range of flow conditions, Ergun and Orning (1949) added to Kozeny (1927) term for linear losses to the new term for kinetic losses. They are argued that the transition form the dominance of viscous to kinetic effects for most packed systems is smooth indicating that there should be a continuous function relating pressure drop to flow rate. The following is the empirical equation that resulted:

$$\frac{\Delta H}{\Delta L} = C_1 \left[\frac{\mu}{\rho_c^a}\right]\left[\frac{(1-n)^2}{n^3}\right] S_v^2 V + C_2 \left[\frac{1}{g}\right]\left[\frac{(1-n)}{n^3}\right] S_v V^2 \tag{5}$$

Ergun (1952) then substituted the diameter of a sphere $\mathrm{d_s}$ having the specific surface $\mathrm{S_v}$ in Equation (5) producing the following result:

$$\frac{\Delta H}{\Delta L} = 150\left[\frac{\mu}{\rho_g}\right]\left[\frac{(1-n)^2}{n^3}\right]\left[\frac{1}{d_s^2}\right]V + 1.75\left[\frac{1}{g}\right]\left[\frac{(1-n)}{n^3}\right]\left[\frac{1}{d_s}\right]V^2 \tag{6}$$

The constants C_1=150 and C_2=75 were found from various sized spheres and also from sand and pulverized coke. Many experimenters have attempted to use Reynolds concept to determine the upper limit of the validity of Darcy's law, for example studies given by Franzini (1951), Hubbert (1956), and Scheidegger (1957).

Irmay (1958), Ward (1964), Sunada (1965), Wright (1968), Fair (1968), Beavers (1969), Ahmed (1969), Bear (1972) and MacDonald (1979) worked to show that Forchheimers nonlinear equation could be derived from the first principles beginning with the Navier-Stokes equation. Recently, modeling progresses of flows within unconsolidated, granular media rely mostly on experimental works using homogeneous, spherical, artificial media (Comiti and Renaud, 1989). In civil engineering, these research works are applied to the study of internal flows within earth and rock structures (Martin, 1990; Shih, 1991; Hansen 1992; Burchart et al., 1991; Hamilton 1997; Wahyudi, 1998; Bingjun et al., 1998) and to the problems of similarities of flow parameters in centrifuged geotechnical small-scale models, within which very large hydraulic gradients are often found (Babendreier, 1991; Burchart, 1991; Khalifa et al., 2000).

Trussell et al. (1999) developed the constants (a and b) for three filter media (glass beads, crushed silica sand and crushed anthracite coal). Summary of design parameters for selected filter medium is tabulated in Table (1).

Table (1): Summary of design parameters for selected filter medium

Medium	coefficient		Typical porosity	Reference
	a	*b*		
Crushed anthracite	210-245	3.5-5.3	47-52	Trussell et al. (1999)
Crushed sand	110-115	2-2.5	40-43	Chang et al. (1999)
Glass beds	130-150	1.3-1.8	38-40	Rumpf and Gupte (1971)

2- EXPERIMENTAL SET-UP AND METHOD

The experimental set-up, the schematic diagram and the photograph of the system are presented in Figure (1) and Plate (1), respectively, showing both a hydraulic device and a measurement device.

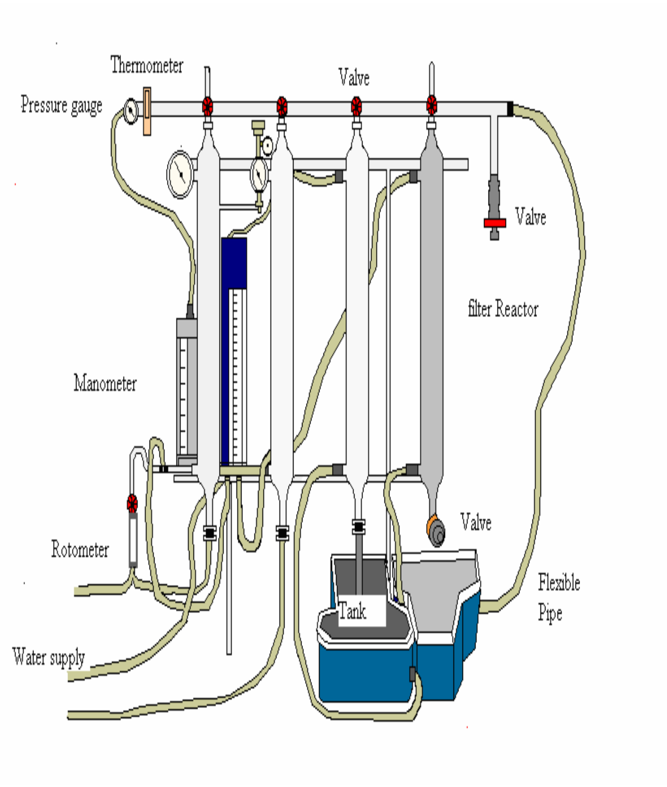

Fig. (1): Experimental system **Plate (1): photograph of system**

The hydraulic device consists of a pump inverting the flow of water from a 400-liter tank through the material placed in the testing cell. When flowing out of the cell, water is forced back into the tank. The temperature is monitored at a quasi-constant value: 24-26°C. Temperature sensors are placed in the tank, at the entrance and at the exit of the test cell. Then the values of the water density, ρ, and of the water viscosity, μ, are quasi constant during the tests (ρ = 998.2 kg /m^3 and μ = 1.008 Pa s).

Some valves are used to select the pressure tapings and the differential pressure gauge suiting the best measurement range. The measurement bench consists of three flow meters and four differential pressure gauges regulated by a valve set. The different measurements ranges of these devices are complementary and can be used to cover all the measurements accurately. The sand was packed as homogeneously as possible using the pluviation method with a constant 1.2 m height of fall. Satisfactory repeatability of the bulk densities was achieved throughout the experiments. The bed porosity was calculated knowing the cell volume and the mass of sand.

3- EMPIRICAL MODELS IN POROUS MEDIA AT FORCHHEIMER'S REGIME

There are four regimes of flow in porous media. Darcy regime is the first which is limited to Reynolds number below 1. In this regime flow is creeping flow i.e. the laminar flow with no significant inertial contribution. Forchheimer regime is the second regime in which initially the flow is steady laminar but as it progresses the initial effects becomes very vital. Initially the head loss is proportional to v with small v^2 dependence but as it progresses the head loss v^2 becomes related with small dependence on v. Furchheimer regime corresponds to a Reynolds number of approximately 100.

The third regime represents more or less inertial flow to full turbulence which Reynolds number is between 600 and 800 depending on the media characteristics and flow conditions. The final regime is fully turbulence in which the velocity is randomly fluctuating. In water filter, Reynolds number in excess of 0.5 and less than 50 has to be kept to put them solidly in the Darcy and Forchheimer flow regimes.

The actual head loss through porous media such as under ground aquifer and water filter was often greater than that derived from Darcy's law, particularly when high flow velocity. Forchheimer's model for hydraulic gradient proposed nonlinear equation as (Eq. 1).

Hydraulic conductivity depends upon a number of factors, which are summarized as (particle size distribution, particle shape and texture, mineralogical composition, voids ratio, degree of saturation, filter fabric, nature of fluid, type of flow and temperature). The hydraulic conductivity coefficient is influenced by particle characteristics. The smaller the particles, the smaller the voids between them, and therefore the resistance to flow of water increases (i.e. the hydraulic conductivity coefficient decreases) with decreasing particle size. Elongated or irregular particles create flow paths, which are more tortuous than those around nearly spherical particles. Particles with a rough surface texture provide more frictional resistance to flow than do smooth-textured particles. Both effects tend to reduce the rate of flow of water through the filter, i.e. to reduce its hydraulic conductivity coefficient.

Applying Poiseuille equation in Eq. (5), *a* and *b* can be calculated from the following Equations:

$$a = C_1\{\frac{(1-n)^2}{n^3}\}(\frac{1}{d})^2 \tag{7}$$

$$b = C_2\{\frac{(1-n)}{n^3}\}(\frac{1}{d}) \tag{8}$$

Set of experiments for different water filters materials were carried in the hydraulic laboratory to evaluate the proposed hydraulic conductivity *a** and *b**at based conditions $T_w = 25^{o}C$, n = 50%, d = 1mm which is tabulated in Table (2).

Table (2): Hydraulic conductivity coefficients at based condition (T_w=25°C), (n=50%), (d=1mm) for different selected porous media

Medium	***Sg* (specific gravity)**	***a** (s/m)**	***b** (s^2/m^2)**
Crushed sand	2.6	31.2	1513
Crushed coal	1.4	56.3	1902
Glass beads	2.4	44.8	1082
Crushed garnet	3.8	26.2	1618
Crushed porcilinaite	1.12	36.4	1807

Forchheimer's model constants *a* and *b* (Eq. 1) can be corrected and expressed in the forms:

$$\frac{a}{a*} = 2.64\left(\frac{n^{1.4}}{d^2}\right)(1.028)^{(25-T)} \tag{9}$$

$$\frac{b}{b*} = 2.514\left(\frac{n^{1.333}}{d}\right) \tag{10}$$

From experiments on the selected materials used in water filters, the effect of various materials, grain sizes, temperatures and porosities was shown in Fig. (2), Fig. (3), Fig. (4) and Fig. (5). The properties of fluid, which are relevant to hydraulic conductivity coefficient, are density and dynamic viscosity. For water the density ρ_w varies little over the range of temperatures normally experienced (10°C - 80°C), but viscosity μ_w decreases by factor of about 4 over this range. For a laboratory test the standard temperature is 25°C, while the temperature for atypical field hydraulic conductivity test in U.S.A. may be 20°C, and in Britain is 10°C.

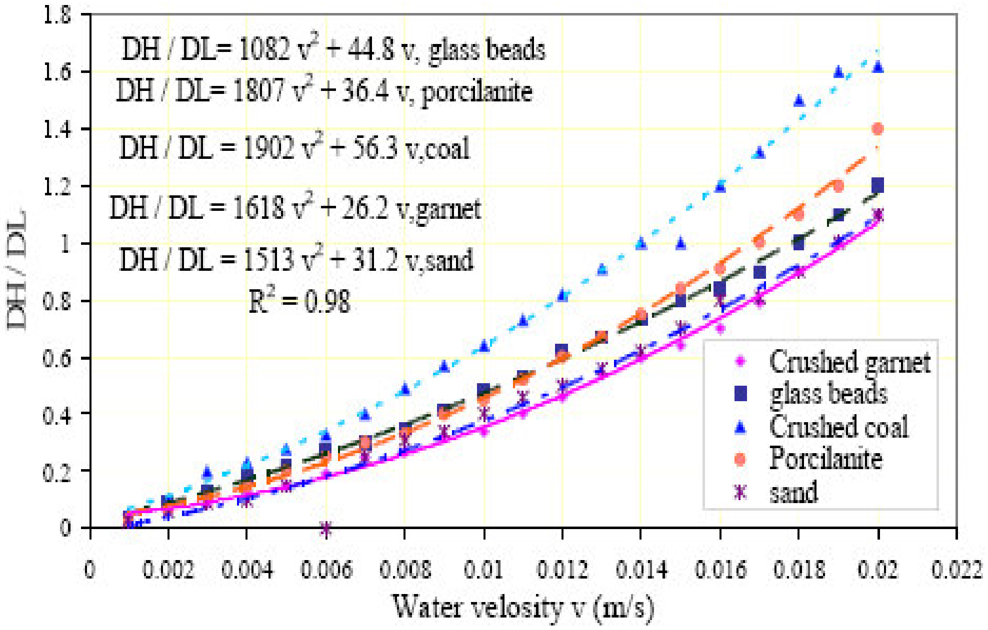

Fig. (2): Relationship between pressure loss head per unit filter depth and water velocity for different bed materials (d = 1 mm, n = 0.5 and water temperature = 25°C)

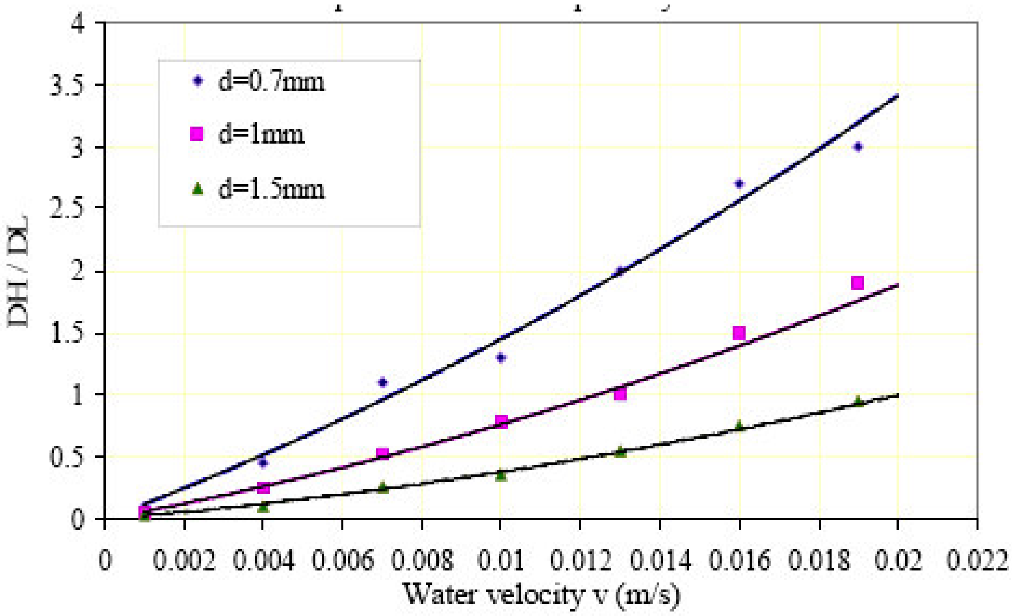

Fig. (3): Effect of grain size of sand on pressure head drop per unit depth of sand filter for different velocities at temperature = 9°C and porosity = 0.57

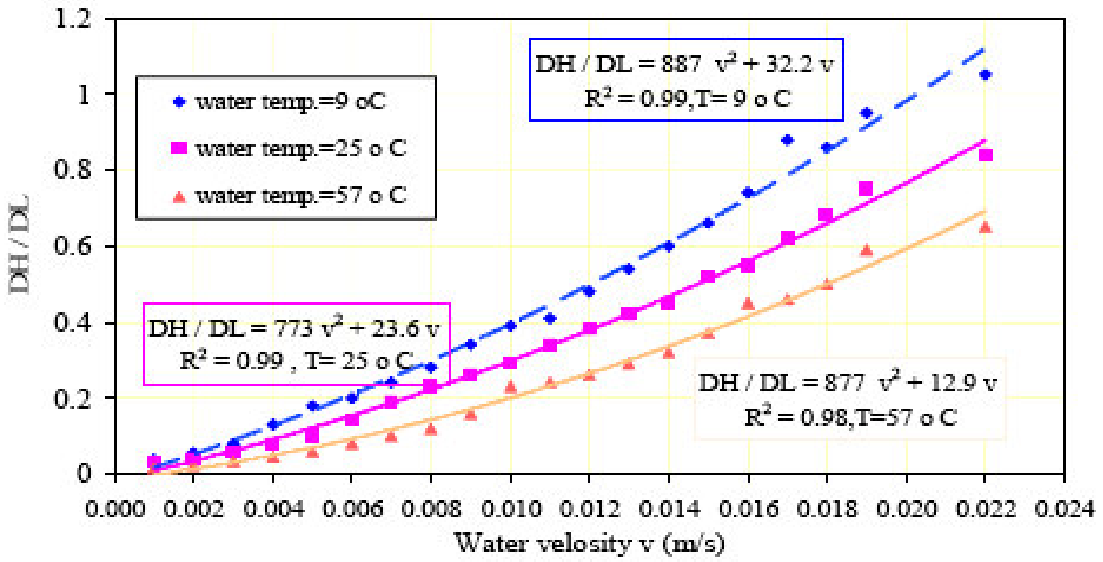

Fig. (4): Relationship between pressure loss head per unit sand filter depth and water velocity for different water temperatures with d = 1.5 mm and n = 0.57

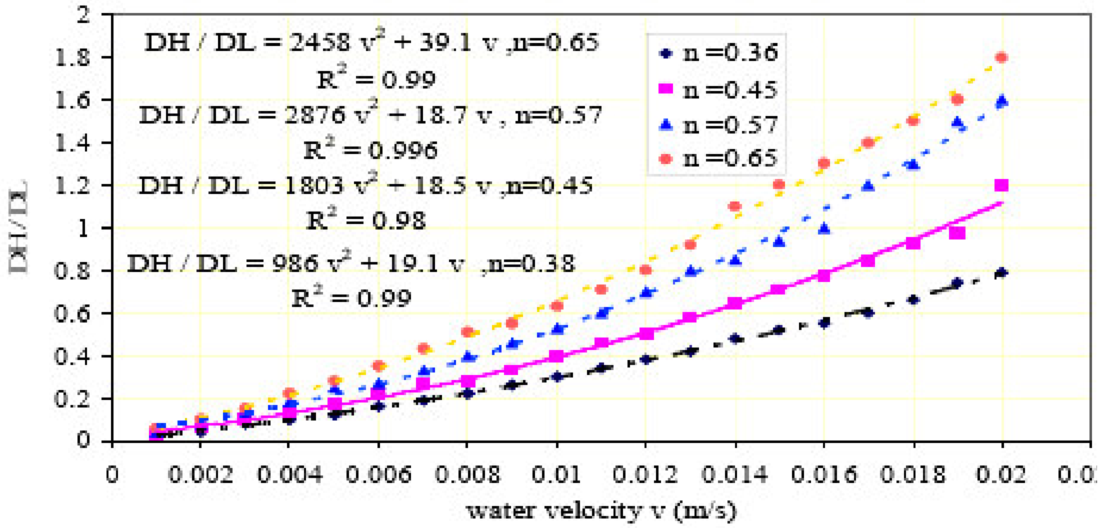

Fig. (5): Relationship between pressure head loss per unit sand filter depth and water velocity for different porosities (d = 1mm, temperature = 25°C)

4- CONCLUSIONS

Hydrodynamic modeling of viscous flow in porous media was investigated for five selected water filters media: crushed silica, crushed coal, glass beads, crushed porcilinaite and crushed garnet. Typical Forchheimer's model constants *a* and *b* that can be used to estimate head loss for some of the most common design of water filters were correlated.

Hydraulic conductivity coefficients at based condition ($T_w = 25^oC$), (n = 50%), (d = 1 mm) *a** and *b** for different selected porous media were evaluated and *a* and *b* correlations for various grain sizes (0.5 mm – 2 mm), porosity (35% - 60%) and temperature (20°C - 80°C) could be found in the empirical forms:

$$\frac{a}{a*} = (\frac{n}{50\%})^{1.4}(\frac{1}{d})^{2}(1.028)^{(25-T)} \quad \text{or} \quad \frac{a}{a*} = 2.64(\frac{n^{1.4}}{d^{2}})(1.028)^{(25-T)}$$

and

$$\frac{b}{b*} = (\frac{n}{50\%})^{1.333}(\frac{1}{d}) \quad \text{or} \quad \frac{b}{b*} = 2.514(\frac{n^{1.333}}{d})$$

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