AN ANALYTICAL MODEL STUDY FOR FLOW THROUGH POROUS MEDIA

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ABSTRACT

Hydrodynamic modeling of viscous flow in porous media was investigated for five selected filter media: crushed silica, crushed coal, glass beads, crushed porcilinaite and crushed garnet. Typical constant that can be used to estimate head loss for some of the most common design of granular media filters were correlated.

An experimental set-up has been specially conceived to generate Reynolds number between 100 and 800. A capillary-type flow model was constructed to determine structural parameters of media and predict pressure gradients.

Using one dimensionless equation of the capillary-type flow model with set of experiments, friction coefficient can be correlated as a function of pore Reynolds number in transition zone.

Empirical relationships were developed using a plot of friction vs. Reynolds number similar to those that had been successfully used for the pipe flow. Analytical models were constructed to develop an equation for friction factor in transition zone (Inertial flow with increasing random irregular) in porous media as a function of flow type from first hydraulic principles and a semi-empirical equation of head loss in porous media for ($100 \le \mathrm{Re} \le 800$) was evaluated.

Keywords: porous media, water filter, friction coefficient, pressure head loss

1- INTRODUCTION

The steady flow through filter having sand bed of various thickness and under various pressure is directly proportional to the hydraulic gradient, and the Darcy's equation in the following commonly used (except at high velocities when turbulence occurs) is:

$$\frac{\Delta H}{\Delta L} = \frac{1}{K}\frac{Q}{A} \tag{1}$$

where, ΔH : head loss through medium (L), ΔL: depth of filter bed (L), Q: flow rate through medium (L^3/T), A: area of the filter bed in plan (L^2) and K: Darcy's coefficient (L/T).

The hydrodynamics studies were investigated firstly by hydro-geologists, petroleum engineers and civil engineers who were interested in characterizing the flow of various fluids (water, oil and gas) in the under ground environment. Mechanical and Chemical engineers are interested in predicting the head loss in engineered media beds and specifying the characteristics of the materials that make up the porous bed such as (water and air filter, cooling tower, scrubbers, absorber columns, ion exchange columns and evaporative beds).

For laminar flow (low flow velocity in porous media), Poiseuille equation (Poiseuille, 1841) can be applied to estimate the head loss from the velocity of flow in each individual capillary as follow:

$$\frac{\Delta H}{\Delta L} = 32\left(\frac{\mu}{\rho g}\right)\frac{V_c}{d_c^2} \tag{2}$$

$$V_c = \frac{Q}{nA_b} = \frac{V.\zeta}{n} \tag{3}$$

and

$$n = \frac{\forall_v}{\forall} \tag{4}$$

where, L: length of capillary (L), μ: dynamic viscosity (M/L.T), ρ: density (M/L^3), g: standard acceleration of gravity, (9.81 m/s^2), d_c: inside diameter of capillary (L), V_c: velocity of fluid in capillary (L/T), n: bulk porosity of bed of porous media (L^3/L^3), $\forall_v$: volume of the voids (L^3), $\forall$: total volume of the bed (L^3), and ζ: tortuosity (L/L).

2- LITERATURE REVIEW

The simple linear relationship between flow and head loss by Darcy (1856) have been demonstrated by Forchheimer (1901) for condition under which the relationship between flow and head loss in porous media doesn't follow. Also, Kozeny (1927a,b) and White (1929) applied similar principles to characterize the nonlinear resistance of porous media to the flow of gases at high Reynolds number. Nutting (1930) defined the specific resistance and Wyckoff (1933) popularized Nuttings specific resistance parameter, resulting in its wide use whenever the modeling of the flow of fluids underground was undertaken. Fancher (1933), Fair and Hatch (1933), and Wallis (1938, 1939) developed a powerful model for predicting Darcy resistance from the characteristics of the porous media. Many experimenters have attempted to use Reynolds concept to determine the upper limit of the validity of Darcy's law by Ergun (1949), Rose (1949), Franzini (1951). Hubbert (1956), and Scheidegger (1957).

In civil engineering, research works were applied to the study of internal flows within earth and rock structures (Martin, 1990; Shih, 1991; Hansen, 1992; Burchart et al., 1991; Hamilton, 1997; Wahyudi, 1998; Bingjun et al., 1998) and to the problems of similarities of flow parameters in centrifuged geotechnical small-scale models, within which very large hydraulic gradients are often found (Babendreier, 1991; Burchart, 1991; Khalifa et al., 2000). The extrapolation of these models to wide particle-size distributed natural media requires, therefore, specific studies.

Few studies, on the other hand, have been conducted to examine post-Darcy flows within sands (Babendrier 1991); Revil (1999)).

3- EXPERIMENTAL SET-UP AND METHOD

The schematic diagram of the experimental set-up, and the photograph are presented in Plates 1 and 2, respectively, which consists of both a hydraulic device and a measurement device.

The hydraulic device consists of a pump inverting the flow of water from a 400-liter tank through the material placed in the testing cell. When flowing out of the cell, water is forced back into the tank. The temperature is monitored at a quasi-constant value: 24-26°C. Temperature sensors are placed in the tank, at the entrance and at the exit of the test cell. Then the values of the water density, ρ, and of the water viscosity, μ, are quasi constant during the tests (ρ = 998.2 kg/m^3 and μ = 1.008 Pa s).

Some valves are used to select the pressure tappings and the differential pressure gauge suiting the best measurement range. The measurement bench consists of three flow meters and four differential pressure gauges regulated by a valve set.

The different measurement ranges of these devices are complementary and can be used to cover all the measurements accurately. The sand was packed as homogeneously as possible using the pluviation method with a constant 1 m height of fall. Satisfactory repeatability of the bulk densities was achieved throughout the experiments. The bed porosity is calculated knowing the cell volume and the mass of sand.

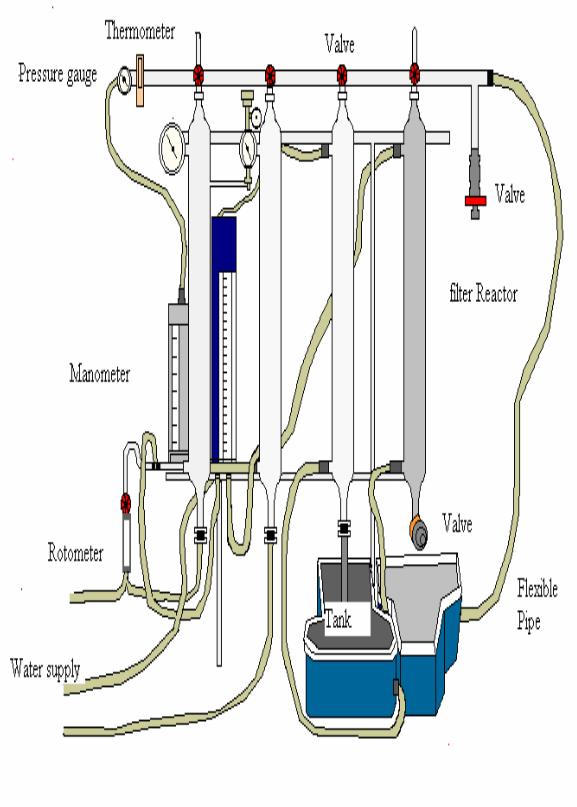

Plate (1): Setup of hydraulic system

Plate (2): photograph of hydraulic system of water filter

4- ANALYTICAL MODEL IN POROUS MEDIA AT HIGH FLOW VELOCITY

Solving practical filter design problems in hydrodynamic phenomena usually requires both theoretical and experimental results. By grouping significant quantities into dimensional parameters it is possible to reduce the number of variables appearing and to make this compact results (equation or chart) applicable to all similar situation.

Investigations consider the bed of porous media as composed of a number of capillary pores, passing in parallel, straight through the depth of the bed. In this case the head loss in high flow velocity (turbulent flow) depend upon capillary water velocity (V_c), dynamic viscosity (μ), density (ρ). Using dimensional analysis (Buckingham Π theorem), general form of the empirical relationship can be written as follows:

$$\frac{\Delta H}{\Delta L} = \Psi(\mathrm{Re}) \frac{1}{d_c} \frac{V_c^2}{2g} \tag{5}$$

$$\mathrm{Re} = \frac{\rho V d}{\mu} \tag{6-a}$$

and

$$\mathrm{Re}_{\mathrm{pore}} = \frac{4\,\zeta\,\rho V}{\mu\,(1-n)\,S} \tag{6-b}$$

where $\Psi(\mathrm{Re})$ is function of Reynolds number (Re) which is one of the most important dimensionless parameters in high flow velocity and its value determines the nature of flow and S is surface area. Friction factor can be evaluated in transition zone $(100 < \mathrm{Re} < 800)$ in porous media as function of media characteristics and flow phenomena as follows:

$$C_f = \left(\frac{4\,\zeta\,\rho V}{\mu\,(1-n)\,S}\right)^n \tag{7}$$

Similar to (Blasius formula) the hydraulic resistance factor (C_f) in porous media can be simulated as function to friction factor and as the form:
The hydraulic resistance factor (C_f) in porous media can be simulated as function of the friction factor as follows:

$$C_f = \frac{\text{constant}}{\mathrm{Re}^{0.25}} \tag{8}$$

where the constant depends upon porosity (n), uniform factor (Φ), and grain shape of media.

The modeling approach presented here is cantered on the characterization of macroscopic parameters (pore diameter, tortuosity and porosity of the medium, etc.), on one hand, and on the determination of the range of validity of Darcy linear law based on a unique criterion common to all sands, on the other hand. The capillary model applied here has been developed by Comiti and Renaud (2000) and was first used for artificial, porous media (packing of spheres and plates). For laminar, creeping flow in Darcy's regime, friction factor can be evaluated as:

$$f = \frac{64}{\mathrm{Re}} + 0.194 \quad , \quad (\mathrm{Re} \le 4.3) \tag{9}$$

The only adjustment needed to optimize agreement with experimental measurements is to replace the factor 64 by 150, and 0.194,

$$f = 150\,/\,\mathrm{Re} \quad , \qquad (\mathrm{Re} < 10) \tag{10}$$

Equation (10) is known as the "Blake-Kozeny" equation, and reproduces experimental data for porous media of uniform spheres rather well up to Reynolds numbers of about 10. Sometimes, an empirical coefficient of 180 instead of 150 is recommended, and equation (10) is then usually referred to as the Carman-Kozeny equation.

Above Re of about 10 the flow begins to depart from laminar. For highly turbulent flows, characterized by Re > 1000, experiment shows that the friction factor becomes nearly independent of Re. In this regime of high Reynolds number and turbulent flow the friction factor is well represented by an expression known as the "Burke-Plummer" equation:

$$f = \text{constant} = 1.75 \quad (\text{Re} > 1000) \tag{11}$$

To derive an expression for f that could be applied over intermediate Reynolds numbers, $10 < \text{Re} < 1000$, the common (simplest) approach is to add the laminar and turbulent contributions to f. This is not rigorous, but turns out to work reasonably well:

$$f = 150 / \text{Re} + 1.75 \quad (\text{Ergun equation, } 10 < \text{Re} < 1000) \tag{12}$$

Because of extreme complexity of grains shape of the media and its arrangement (spacing between the grains) most of the advance in understanding the basic relation have been developed around experiments on grains shape or specific surface (S_v) which is defined as:

$$S_v = \frac{S_m}{L_b A_b (1-n)} \tag{13}$$

For circular capillary tube,

$$S_v = \frac{4n}{d_c (1-n)} \tag{14}$$

$$\forall_b = L_b A_b (1-n) \tag{15}$$

where, S_m: surface of the media (L^2) and $\forall_b$: volume of the bed (L^3).

Substituting Eqs. (6,7,8,14) into Eq. (5), we get:

$$\frac{\Delta H}{\Delta L} = C'_f \left(\frac{\mu}{\rho}\right)^{0.25} \left\{\frac{(1-n)^{1.25}}{n^3}\right\} S_v^{\;1.25} \zeta^{1.75} V^{1.75} \tag{16}$$

From the experimental data C'_f is $(0.1 \mp 4.2\%)$ for $(100 \le \text{Re} \le 800)$, (Inertial flow with increasing random irregular flow), and friction factor is evaluated from Fig. (1).

Comparison between experimental and predicted friction coefficient in sand filter for Reynolds number between 100 and 800 is shown in Fig. (2).

$$f = \frac{10.7349}{\text{Re}^{0.25}} \tag{17}$$

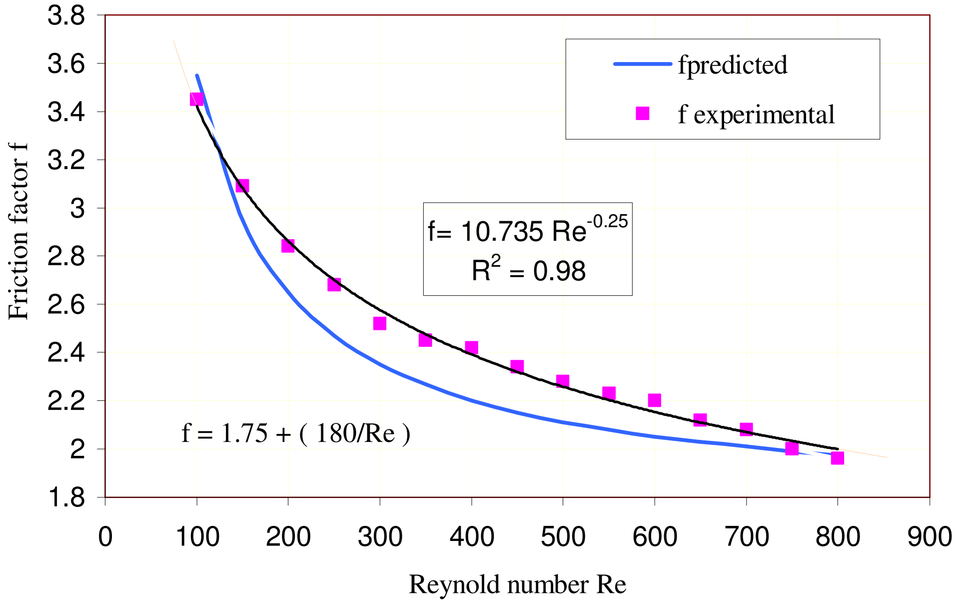

Fig. 1: The relationship between friction factor and Reynolds number in sand filter in transition regime

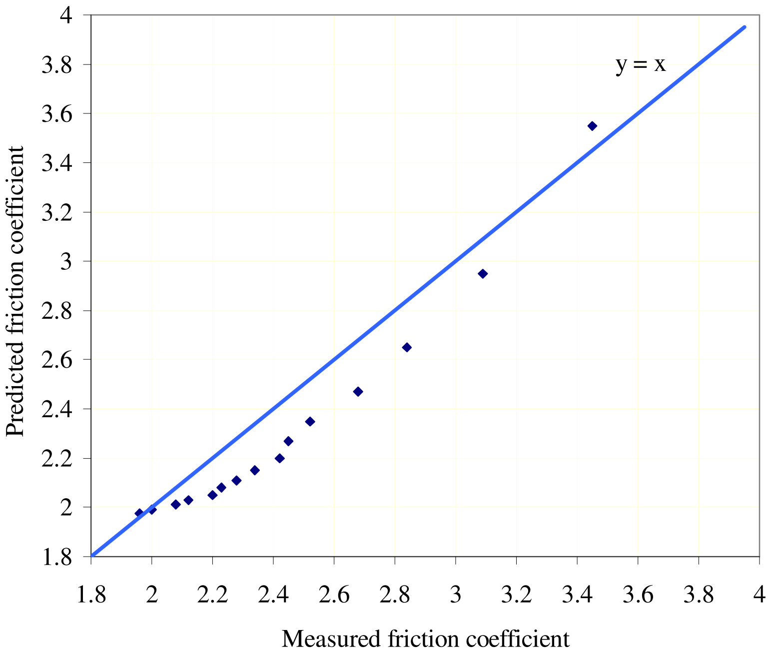

Fig. 2: Comparison between experimental and predicted friction coefficient in sand filter for Reynolds number between 100 and 800

Head loss in water filter can be estimated directly as pipe flow by using the correlation of friction coefficient from equation (17) in the following equations:

$$\frac{\Delta H}{\Delta L} = f\left(\frac{1}{n^2 d}\right)\frac{V^2}{2g} \tag{18}$$

or

$$\frac{\Delta H}{\Delta L} = f\left(\frac{S_v}{4}\right)\left(\frac{1-n}{n^3}\right)\frac{V^2}{2g} \tag{19}$$

5- CONCLUSIONS

1- Hydrodynamic modeling for viscous flow in porous media was investigated to estimate head loss in water filter.

2- Using dimensional analysis (Buckingham Π theorem) and the basic relation around experiments on grains shape or specific surface (S_v), porosity (n), tortuosity (ζ) were developed and empirical equation of head loss in porous media was evaluated as follows:

$$\frac{\Delta H}{\Delta L} = C'_f \left(\frac{\mu}{\rho}\right)^{0.25}\left\{\frac{(1-n)^{1.25}}{n^3}\right\} S_v^{\ 1.25}\zeta^{1.75}V^{1.75},$$

C'_f is $(0.1 \mp 4.2\%)$ for $(100 \leq \mathrm{Re} \leq 800)$

3- Empirical equation of friction factor in transition zone (Inertial flow with increasing random irregular) in porous media for $(100 \leq \mathrm{Re} \leq 800)$ as function to flow type can be evaluated as:

$$f = \frac{10.7349}{\mathrm{Re}^{0.25}}$$

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