ESTIMATION OF HYDRAULIC JUMP LOCATION USING NUMERICAL SIMULATION

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ABSTRACT

The Boussinesq equations are numerically solved to simulate the formation of hydraulic jump in a rectangular channel having a small bed slope. The MacCormack scheme is used for their solution by applying specified initial and boundary conditions until a steady state flow is reached. The location of the hydraulic jump is determined as a part of these computations. The artificial viscosity technique should be used in the computations to dampen the superior oscillations near the steep gradient of the simulated hydraulic jump. Twelve laboratory experiments were carried out for verification of the numerical model. Upstream Froude number for these experiments ranged from 2.17 to 3.83. The simulated hydraulic jump profiles using the MacCormack scheme showed good agreement with the experimental data. Application of the model was extended beyond the limits of the data used in the verification process. A power empirical equation was developed to determine the location of hydraulic jump using regression analysis based on 130 simulated data.

INTRODUCTION

Hydraulic jump is the transition phenomenon from supercritical flow to subcritical flow, where water surface rises abruptly, accompanied by considerable turbulence and energy dissipation. The applications of hydraulic jump in open channel flow are vital as energy dissipation device over hydraulic structures, mixing of chemicals used for water purification, and aeration of flows.

Several laboratory and field investigations have been carried out to determine the variables included in the hydraulic jump phenomena such as its length, location, energy dissipation, and efficiency. Chow [4] computed the water surface profiles for both the supercritical flow starting from the upstream boundary and the subcritical flow starting from the downstream boundary to determine the hydraulic jump location in a channel. The jump location is established at the position where the specific forces on both sides of the jump are equal. McCorquodale and Khalifa [10] used a strip
integral method to predict the jump length, velocity distribution, water surface profile and pressure at bed. Katopodes [8] used the finite element method to solve the St. Venant equations numerically until a steady state was reached. The location of hydraulic jump was automatically computed as a part of the solution.

The St. Venant equations are based on the assumption of uniform velocity distribution and hydrostatic pressure distribution [1,3,9]. These equations should be replaced by the equations of the Boussinesq type when the assumption of a hydrostatic pressure distribution is incorrect. In a rapidly varied flow having steep water surface gradient, McCowan and Basco [5,6] supposed that the vertical velocity distribution increases from zero at the channel bed to its maximum value at the free surface to derive Boussinesq equations. The Boussinesq equations can be reduced to the St. Venant equations, by neglecting the Boussinesq terms, which account for the non-hydrostatic pressure distribution.

The use of an explicit finite difference scheme is often favorable when the governing equations are complicated and the solution has a big space time change rate or even discontinuities [6,12]. The Boussinesq equations, for which closed-form solutions are not available, can be solved numerically. Order-2 and higher order schemes often generate spurious oscillation and Gibbs errors in the vicinity of discontinuities, so an artificial viscosity should be added to prevent strong spurious oscillations [6,11,12,14].

In this paper, the MacCormack finite difference explicit scheme [2,3,5,6,12,14,15] is used to solve the Boussinesq equations. The governing equations are first presented, then details of the MacCormack finite difference explicit scheme are explained. Stability conditions and the inclusion of initial and boundary conditions for the MacCormack scheme are discussed. Using the time derivative term as an iterative parameter with the specified boundary conditions, the unsteady flow computations are continued in this scheme until they converged to a steady state flow to determine the location of the hydraulic jump.

The computed results were verified by comparing them with the experimental laboratory data. When a high level of verification can be achieved, then it may be possible to extent the application of the model beyond the limits of the data used in the verification process. A power empirical equation was developed to determine the location of hydraulic jump Using regression analysis based on 130 simulated data obtained from the mathematical model.

GOVERNING EQUATIONS

McCowan and Basco [5,6] assumed that the fluid is incompressible, the channel is prismatic, rectangular in cross section, and has small bottom slope to derive the Boussinesq equations. Also, they assumed that the flow velocity in the lateral direction is zero and the vertical velocity distribution increases from zero at the
channel bed to its maximum value at the free surface. The Boussinesq equations for one-dimensional flow can be rewritten as:

\[
\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = 0
\]

(1)

\[
\frac{\partial uh}{\partial t} + \frac{\partial}{\partial x} \left( u^2 h + \frac{1}{2} gh^2 - E \right) - gh(S_o - S_f) = 0
\]

(2)

in which

\[
E = \frac{h^3}{3} \left[ \frac{\partial^2 u}{\partial x \partial t} + u \frac{\partial^2 u}{\partial x^2} - \left( \frac{\partial u}{\partial x} \right)^2 \right]
\]

(3)

where \( x \) = distance along the channel bed; \( t \) = time; \( h \) = water depth; \( u \) = flow velocity in the \( x \) direction; \( g \) = gravitational acceleration; \( S_o \) = channel bed slope; \( S_f \) = friction slope; and \( E \) = Boussinesq term.

The friction slope can be expressed using the Manning's equation as:

\[
S_f = \frac{n^2 |u|}{R^{4/3}}
\]

(4)

in which \( n \) = Manning's roughness coefficient; and \( R \) = hydraulic radius.

The Boussinesq equations (1) and (2) may be rewritten in a vector form as:

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} - S = 0
\]

(5)

in which

\[
U = \begin{bmatrix} h \\ uh \end{bmatrix}; \quad F = \begin{bmatrix} u^2 h + \frac{1}{2} gh^2 - E \\ uh \end{bmatrix}; \quad \text{and} \quad S = \begin{bmatrix} 0 \\ - gh(S_o - S_f) \end{bmatrix}
\]

(6)

**NUMERICAL SOLUTION METHOD**

Explicit difference schemes at a finite number of grid points in the rectangular spatial-time grid can be used to obtain a numerical solution of the Boussinesq equations. The unknown variables in these explicit schemes are computed at a rectangular grid point on an advanced time line using the known values and conditions at grid points on the present time line. The MacCormack explicit scheme was applied for the solution of the Boussinesq equations using specified initial and boundary conditions. The location of hydraulic jump was determined by continuing the solution until a steady state was achieved.

The MacCormack scheme is a predictor corrector scheme, in which the solution is obtained using forward difference approximation for the space derivative in the predictor step and backward difference approximation of the predicted values in the corrector step. The computational grid for the MacCormack scheme is shown in Figure 1.
The substitution of equations (7) and (8) into equation (5) and simplification of the resulted equations yield the following:

**Predictor**

\[ U_i^* = U_i^k - \Delta t \frac{\Delta t}{\Delta x} \left( F_{i+1}^k - F_i^k \right) + \Delta t S_i^k \]  \hspace{1cm} (7)

**Corrector**

\[ U_i^{**} = U_i^* - \Delta t \frac{\Delta t}{\Delta x} \left( F_i^* - F_{i-1}^* \right) + \Delta t S_i^* \]  \hspace{1cm} (8)

\[ U_i^{k+1} = \frac{1}{2} \left( U_i^* + U_i^{**} \right) \]  \hspace{1cm} (9)

**Figure 1. Computational Grid for the MacCormack Scheme**

The substitution of equations (7) and (8) into equation (5) and simplification of the resulted equations yield the following:

**Predictor**

\[ h_i^* = h_i^k - \Delta t \frac{\Delta t}{\Delta x} \left( u_i^{k+1} h_i^{k+1} - u_i^{k} h_i^k \right) \]  \hspace{1cm} (10)

\[ u_i^* h_i^* = u_i^{k+1} h_i^{k+1} - \Delta t \frac{\Delta t}{\Delta x} \left( \left[ \left( u_i^{k+1} \right)^2 h_i^{k+1} + \frac{1}{2} g \left( h_i^{k+1} \right)^2 - E_i^{k+1} \right] - \left[ \left( u_i^{k} \right)^2 h_i^k + \frac{1}{2} g \left( h_i^k \right)^2 - E_i^k \right] \right) + \]  \hspace{1cm} (11)

\[ \Delta t g h_i^{k+1} \left( S_0 - S_f \right)^k \]

\[ E_i^k = \frac{\left( h_i^k \right)^3}{3} \left[ u_i^k \left( \frac{u_i^{k+1} - 2 u_i^k + u_i^{k-1}}{\Delta x^2} \right) - \left( \frac{u_i^{k+1} - u_i^k}{\Delta x} \right)^2 \right] \]  \hspace{1cm} (12)

\[ E_i^{k+1} = \frac{\left( h_i^{k+1} \right)^3}{3} \left[ u_i^{k+1} \left( \frac{u_i^{k+2} - 2 u_i^{k+1} + u_i^k}{\Delta x^2} \right) - \left( \frac{u_i^{k+2} - u_i^{k+1}}{\Delta x} \right)^2 \right] \]  \hspace{1cm} (13)
Corrector

\[ h_i^{**} = h_i^* - \frac{\Delta t}{\Delta x} \left( u_i^* h_i^* - u_{i-1}^* h_{i-1}^* \right) \]  
(14)

\[ u_i^{**} h_i^{**} = u_i^* h_i^* - \frac{\Delta t}{\Delta x} \left\{ \left[ \left( u_i^* \right)^2 h_i^* \right] + \frac{1}{2} g \left( h_i^* \right)^2 - E_i^* \right\} - \left\{ \left( u_{i-1}^* \right)^2 h_{i-1}^* + \frac{1}{2} g \left( h_{i-1}^* \right)^2 - E_{i-1}^* \right\} + \Delta t g h_i^* \left( S_0 - S_f \right)_i \]  
(15)
in which

\[ E_i^* = \left( \frac{h_i^*}{3} \right)^3 \left[ u_i^* \left( u_{i+1}^* - 2 u_i^* + u_{i-1}^* \right) - \left( \frac{u_i^*}{\Delta x} \right)^2 \right] \]  
(16)

\[ E_{i-1}^* = \left( \frac{h_{i-1}^*}{3} \right)^3 \left[ u_{i-1}^* \left( u_{i-1}^* - 2 u_{i-2}^* + u_{i-2}^* \right) - \left( \frac{u_{i-1}^*}{\Delta x} \right)^2 \right] \]  
(17)

The advantages of the MacCormack scheme include simplicity and high accuracy; however, it has a drawback that in the computation of shock waves superior oscillations near the steep gradient of the computed solution may appear, so an artificial viscosity method should be used [2,3,6,11,12].

**ARTIFICIAL VISCOITY**

Artificial viscosity method is a shock-capturing method. It is very simple, and has smoothing or filtering effect in restricting the development of parasite oscillations. The shock-capturing method plays an important role only in the rapidly varying area of the solution, while it has very small effect on the rest part of the solution.

The artificial viscosity should change a discontinuity curve (shock wave) into a narrow layer where the solution has a steep but smooth transition. Also, it should only have a slight impact on the accuracy of the smooth part of the solution. Artificial viscosity form used by Jameson et al. [3,6,12] is introduced as follows. A parameter \( \lambda_i \) is first computed from the initially computed flow depths as:

\[ \lambda_i = \frac{|h_{i+1}^{k+1} - 2 h_i^{k+1} + h_{i-1}^{k+1}|}{|h_i^{k+1} + 2 h_i^{k+1} + h_{i-1}^{k+1}|} \]  
(18)

\[ \lambda_{i+1/2} = \kappa \frac{\Delta x}{\Delta t} \max \left( \lambda_{i+1}, \lambda_i \right) \]  
(19)
in which \( \kappa \) is a dissipation coefficient used to regulate the amount of dissipation.

Then, the computed velocities and water depths are modified as:

\[ f_i^{k+1} = f_i^{k+1} + \lambda_{i+1/2} \left( f_i^{k+1} - f_i^{k+1} \right) - \lambda_{i+1/2} \left( f_i^{k+1} - f_{i-1}^{k+1} \right) \]  
(20)
INITIAL AND BOUNDARY CONDITIONS

Initial and boundary conditions are the essential requirements for initiating the simulation of hydraulic jump. The initial condition describes the velocity and the depth of flow at all computational nodes along the x-direction of the channel at the initial time \( t = 0 \). The flow in the entire channel at initial time is assumed to be steady supercritical. The flow depth and velocity at all the computational nodes at the initial time are determined by the numerical integration of the following differential equation describing the gradually varied flow [3,6,12]:

\[
\frac{dh}{dx} = \frac{S_0 - S_f}{1 - \frac{u^2}{gh}}
\]

The initial flow depth \( h \), and velocity \( u \) describing the upstream boundary conditions at the entrance of the channel are kept unchanged during the computations. A constant flow depth is specified at the downstream boundary, while the flow velocity is computed at advanced time level \( k+1 \) using the characteristic form of equations (1) and (2) [3,6] as:

\[
u_{i+1}^{k+1} = u_i^k - \left( \frac{g}{c} \right)^k_i \left( h_{i+1}^{k+1} - h_i^k \right) + u_i^k \Delta t g \left( S_0 - S_f \right)_i
\]

in which \( c = \sqrt{gh} \) is the celerity of a gravity wave in a rectangular channel.

STABILITY CRITERION

For explicit methods, the value of \( \Delta t \) must be less than some maximum value allowable for stability. The MacCormack scheme is stable if it satisfied the following Courant-Friedrichs-Lewy (CFL) criterion:

\[
\Delta t = C_n \frac{\Delta x}{\max(|u| + \sqrt{gh})}
\]

in which \( C_n \) = Courant number which must be less than or equal to one [2,3,6,11,12,14]. Equation (23) shows that the explicit time step mustn’t be greater than the time required for a wave to propagate from one grid point to the next one.

DESCRIPTION OF LABORATORY EXPERIMENTS

The laboratory experiments were carried out to get the required data for the verification of the MacCormack scheme. A rectangular recirculation perspex flume 7.5 cm width, 15 cm height, and 4.8 m length, located at the fluid mechanics laboratory, Higher Institute of Engineering, Hon, Libya, was used to get the required data, Figure 2. The water is supplied from a constant head tank through a sharp edged sluice gate. The flume is equipped with a tailgate to control the tail water depth and a
controlling screw to control the slope of the flume. At the end of the flume, the water discharges into the sump through a weighting tank.

The weighting tank was used to measure the discharge and a point gage operating along the flume was used to measure the water depths. A trial and error procedure was used to determine the Manning’s coefficient for the flume. During the initial steady supercritical flow, the Manning’s coefficient $n = 0.01$ was determined by matching the computed water surface profile with the measured water depths in the flume.

Twelve laboratory experiments were carried out for verification of the MacCormack scheme. The details of the laboratory experiments are listed in Table 1.

![Figure. 2. Layout of Experimental Installation](image)

### Table 1. Details of Experiments

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>Discharge (m$^3$/sec)</th>
<th>Water depth at vena contracta (m)</th>
<th>$F_r$ at vena contracta</th>
<th>Bed slope</th>
<th>Tail water depth (m)</th>
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<tbody>
<tr>
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<td>0.003075345</td>
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<td>2.80</td>
<td>0.002836167</td>
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<td>0.0218</td>
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</tr>
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</table>
MODEL APPLICATION

Numerical model were constructed to solve the Boussinesq equations using the second-order MacCormack scheme. The Courant stability criterion and the space grid size restricted the time step size. The Courant number was used equal to 0.65. Following the increase of the downstream water depth at time $t = 0$, the unsteady flow computations were continued until the steady state was reached using the time derivative term as an iterative parameter. The jump traveled from the downstream to the upstream and then moved little back and forth until it was stabilized in one location as shown in Figure (3).

![Figure 3. Water Surface Profile at Different Times, Experiment No. (1)](image)

The numerical schemes were calibrated using the experimental results. The simulated jump profiles using dissipation coefficient $\kappa = 0.03$ and $\Delta x = 0.1 \text{ m}$ gave the most accurate results. The obtained results using $\Delta x = 0.1 \text{ m}$ seems to be more accurate than that obtained using $\Delta x = 0.05 \text{ m}$, it could be due to the effect of the used artificial viscosity technique and the truncation error [12].

The comparison between the simulated results, obtained using $\Delta x = 0.1 \text{ m}$, Courant number = 0.65 and $\kappa = 0.03$, and the measured data shows a good agreement as illustrated from Figure (4) to Figure (8).

Figures (4) and (5) illustrate the effect of tail water depth on the location of hydraulic jump. The hydraulic jump moves towards the upstream boundary when the tail water depth increases. Figure (6) shows that, for the same discharge, the hydraulic jump moves towards the downstream boundary as upstream water depth decreases due to the increase of Froude number. Figure (7) indicates also that the hydraulic jump moves towards the downstream boundary as Froude number increases (discharge increases) for the same upstream water depth. The hydraulic jump moves also towards the downstream boundary when the flume slope increases as shown in Figure (8).
The specific force was computed at the beginning of the jump and the approximate location at the end of the jump. It may be concluded that the computed results satisfy the equality of specific force at the beginning and the end of hydraulic jump because their values differed from each other by 0.9% to 4.9%.

Figure 4. Comparison between Measured and Simulated Jump Profiles Using Different Tail Water Depths, Experiments No. (2), (3) and (4).

Figure 5. Comparison between Measured and Simulated Jump Profiles Using Different Tail Water Depths, Experiments No. (5) and (6).
Figure 6. Comparison between Measured and Simulated Jump Profiles Using Different Upstream Water Depths, Experiments No. (4) and (7).

Figure 7. Comparison between Measured and Simulated Jump Profiles Using Upstream Fraud Numbers, Experiments No. (8) and (9).

Figure 8. Comparison between Measured and Simulated Jump Profiles Using Different Bed Slopes, Experiments No. (10), (11) and (12).
GENERAL EQUATION USING REGRESSION ANALYSIS

It is assumed that the dependent variable \( L \), which is the distance from the beginning of the flume to the location of hydraulic jump as shown in Figure (9) is a function of the following independent variables: density of flow \( \rho \), the upstream water depth \( h_u \) at vena contracta, the tail water depth \( h_t \) at the end of the channel, the velocity at vena contracta \( u_v \), the acceleration of gravity \( g \), bed slope \( S_0 \).

The general function relationship between the above variables can be written as:

\[
f(L, \rho, h_u, h_t, u_v, g, S_0) = 0.0
\]  

(24)

Using the dimensional analysis, the \( \pi \) terms obtained are, \( \pi_1 = L/h_u \), \( \pi_2 = h_t/h_u \), \( \pi_3 = u_v^2 / gh_u = F_r^2 \), and \( \pi_4 = S_0 \). These \( \pi \) terms may be arranged in the following non-dimensional form:

\[
f\left( \frac{L}{h_u}, \frac{h_t}{h_u}, F_r, S_0 \right) = 0.0
\]  

(25)

The general form of equations relating a dependant Pi-term with a number of independent Pi-terms using the regression analysis [13] in this work is in the form of the product of powers of relevant Pi terms, i.e.,

\[
\pi_1 = C \pi_2^{a_2} \pi_3^{a_3} \pi_4^{a_4} \cdots \pi_m^{a_m}
\]  

(26)

Equation (26) can be transformed to a linear expression by taking logarithms of both sides of the equation, as follows:

\[
\log \pi_1 = \log C + a_2 \log \pi_2 + a_3 \log \pi_3 + a_4 \log \pi_4 + \cdots + a_m \log \pi_m
\]  

(27)

Finally, the equation can be rewritten in matrices form as follows:
Where, $N$ is the number of observations. Then, the values of the parameters $C, a_2, a_3, \ldots, a_m$ are obtained and can be replaced back into equation (26).

If the term ($L/h_u$) is taken as a dependent term, the equation form will be as follows:

$$\left(\frac{L}{h_u}\right) = 365 \left(\frac{F_r}{F_r}\right)^{3.29} \left(\frac{h_u}{h_i}\right)^{1.7} (S_0)^{0.66}$$  \hspace{1cm} (29)$$

Or

$$L = \frac{365 \ F_r^{3.29} h_u^{2.7} S_0^{0.66}}{h_i^{1.7}}$$  \hspace{1cm} (30)$$

Equation (30) was used to predict the location of hydraulic jump. The predicted values were compared with the simulated values to check the validity of the above equation. The comparison between the predicted and observed values shows a good agreement as illustrated in Figure (10).

![Figure 10. Comparison between Predicted and Simulated Values for The Location of Hydraulic Jump](image-url)

**SUMMARY AND CONCLUSION**

Boussinesq equations were solved numerically, using the MacCormack Scheme to simulate the hydraulic jump in a rectangular channel. The unsteady flow computations were continued in this scheme until they converged to a steady state flow. The
location of the hydraulic jump was determined as a part of these computations. Details of the MacCormack Scheme, stability conditions, and the initial and boundary conditions were explained.

Twelve laboratory experiments were carried out to get the required data necessary for verification of the MacCormack scheme. The computed results using the MacCormack schemes were verified by comparing them with the experimental data. The comparison between the simulated results, obtained using $\Delta t = 0.1 \, m$, Courant number $= 0.65$ and $\kappa = 0.03$, and the measured data shows a good agreement as shown from Figure 4 to Figure 8.

Using regression analysis the power empirical equation (30) was developed to predict the location of hydraulic jump. The predicted values were compared with the observed values and a good agreement was achieved as shown in Figure 10.

REFERENCES


**NOTATION**

The following symbols are used in this paper:

\[ C_n = \text{Courant number}; \]
\[ c = \text{wave celerity}; \]
\[ F = \text{vector of fluxes in } x\text{-direction}; \]
\[ F_r = \text{Froude number at vena contracta}; \]
\[ f = \text{general function}; \]
\[ g = \text{acceleration due to gravity}; \]
\[ h = \text{flow depth}; \]
\[ h_t = \text{tail water depth}; \]
\[ h_u = \text{upstream water depth at vena contracta}; \]
\[ i = \text{computational nodes}; \]
\[ k = \text{time level}; \]
\[ L = \text{the distance from the upstream boundary to the location of jump}; \]
\[ n = \text{Manning's roughness coefficient}; \]
\[ Q = \text{discharge}; \]
\[ R = \text{hydraulic radius}; \]
\[ S = \text{vector of source terms (friction and bottom slope)}; \]
\[ S_f = \text{friction slope}; \]
\[ S_0 = \text{bed slope of the channel}; \]
\[ t = \text{time}; \]
\[ U = \text{vector of flow variables } = (h, uh)^T; \]
\[ u = \text{flow velocity in } x\text{-direction}; \]
\[ u_v = \text{the velocity at vena contracta } (u_v); \]
\[ x = \text{distance along channel bottom positive in downstream direction}. \]
\[ \Delta t = \text{time interval}; \]
\[ \Delta x = \text{space interval}; \text{ and } \]
\[ \kappa = \text{dissipative coefficient}. \]