EFFECTS OF INCLINED CUTOFFS AND SOIL FOUNDATION CHARACTERISTICS ON SEEPAGE BENEATH HYDRAULIC STRUCTURES

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ABSTRACT

If a dam is founded on a pervious foundation, the differential head formed by the dam acts on the foundation and generates under seepage. The seeping flow generates erosive forces which tend to pull soil particles with the flow. This causes the formation of irregular passages like pipes which move beneath the dam. This process is known as piping phenomenon. Piping occurs if the exit hydraulic gradient at the downstream point approaches the critical hydraulic gradient. In order to prevent piping, it is necessary to reduce the velocity of the seeping water to a safe value. This can be accomplished by lengthening the seepage path. One of the methods of such a lengthening is to introduce sheet piles or cutoff walls within the dam foundation. Using different solution methods, such as the conformal analysis, empirical formulas, electrical analog models, experimental works using physical as well as numerical models and the flow net technique many researchers had examined the confined seepage through pervious soils beneath hydraulic structures and studied their safety against piping phenomenon. Estimation of the exit gradient, uplift pressure, and flow rate were provided for various problems including cases of flat bottom dams with one or more embedded vertical sheet piles. However, limited literature is available concerning the use of inclined walls or sheet piles. Moreover, most of these solutions were based on the assumption of a homogeneous and isotropic foundation soil having an infinite depth. Such an assumption is usually not met depending on various in situ conditions that possess anisotropy, heterogeneity, limited soil depth, ... etc. It appears that no solution is available in the ready accessible literature to determine the effect of using a flat dam base with either toe or heel sheet pile(s) on seepage flow through heterogeneous and anisotropic soils of limited depths. This problem is solved using the finite element method. A model was prepared to compute the piezometric head distribution for different flow conditions and soil characteristics. The calculated exit gradient values, flow rates, and uplift pressure were shown to be affected by changing the slope angle of the sheet pile and varying the soil and flow conditions.

Keywords: Dam; inclined cut-off; finite element; piping; seepage
INTRODUCTION

Seeping water through a permeable soil under dams exerts uplift pressures and may carry soil particles with it which leads to piping. This may result in settlement in the dam base and has the potential to cause failure of dams. Therefore, dams founded on permeable soil have to be designed against uplift and piping, also the seepage losses has to be minimized. If the exit gradient at the downstream side, $I_e$, approaches the critical hydraulic gradient, $I_{cr}$, then piping originates in the soil. Terzaghi and Peck (1967) defined $I_{cr}$ as $I_{cr} = \frac{\gamma_{soil}}{\gamma_w}$. Cutoffs, like sheet piles or concrete curtains, can be provided to reduce both uplift pressure and exit gradient. If the cutoff is inclined toward downstream side, both exit gradient and uplift pressure decrease, this is concluded by Abbas (1994) and Mohamed and Agiralioglu (2005).

LITERATURE REVIEW

Solutions for various problems of seepage under dams with embedded vertical sheet piles and a flat bottom dam with one or more vertical sheet piles were given using different approaches by Khosla (1936), Harr (1962), Leliavsky (1965), Terzaghi and Peck (1967), Rushton and Redshaw (1978), Griffiths and Fenton in (1993) and others. However, limited literature is available for seepage through pervious medium beneath dams with inclined sheet piles. Verigin (1940) analyzed seepage flow around an inclined sheet pile embedded in a porous medium of infinite depth. Polubarinova Kochina (1962) gave the hydrodynamical flownet for an inclined cutoff in a porous medium of infinite depth. Abbas (1994) used conformal transformation and gave a solution for seepage flow beneath a flat bottom dam with an inclined sheet pile at its toe on a homogeneous and isotropic soil of infinite depth. Mohamed and Agiralioglu (2005) used a two dimensional finite difference model to analyze steady state seepage flow beneath a flat bottom dam with an inclined sheet pile at its toe on a homogeneous and anisotropic soil. However, it appears that no solution was available in the readily accessible literature to determine the effect of using an inclined cutoff at the downstream end of a dam on seepage flow under unsteady flow on non-homogeneous and anisotropic soil. Such solution has been obtained in this present study using numerical techniques.

OBJECTIVE

In this present study, the objective is to develop a two dimensional finite element model to analyze seepage flow beneath a dam with an inclined sheet pile under steady or unsteady flow conditions on homogeneous or non-homogeneous soil and isotropic or anisotropic seepage flow. Figure 1 shows a schematic representation of this study.
FORMULATION OF THE PROBLEM

Governing Equation

The groundwater confined transient flow equation for two dimensional case can be expressed by the following linear partial differential equation

\[
\frac{\partial}{\partial t} (K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial z} (K_z \frac{\partial h}{\partial z}) = s \frac{\partial h}{\partial t}
\]

in which: \(h\) is the piezometric head, \(K_x\) & \(K_z\) are the permeabilities in the horizontal and vertical directions, respectively, \(s\) is the storage coefficient, \(t\) is the time.

Solution of Equation (1) represents time and space distributions of piezometric head in non-homogeneous, anisotropic medium with confined transient flow.

Mathematical Formulation

The finite element method is used here to solve Equation (1). Using Galerkin’s method, we seek an approximate solution over each finite element of the domain \(\Omega^e\) and the boundary \(\Gamma^e\). The polynomial approximation of the solution is of the form:

\[
h^e(x, z, t) = \sum_{j=1}^{n} h_j^e(t) \psi_j^e(x, z)
\]

where \(h_j^e\) are the values of the solution at the nodes and \(\psi_j^e\) are the approximation functions over the finite element with \(n\) as the number of nodes. The weak form of Equation (1) is derived, it can be expressed as:
\[
\int_{\Omega^e} \left[ \frac{\partial w}{\partial x} (K_x \frac{\partial h}{\partial x}) + \frac{\partial w}{\partial z} (K_z \frac{\partial h}{\partial z}) + w_s \frac{\partial h}{\partial t} \right] dx dz - \int_{\Gamma^e} w q_n ds = 0
\]  
(3)

Where \( q_n \) is the discharge and \( w \) is known as the weight function, it can be expressed as
\[
w = \psi^e_i (x, z)
\]
(4)

\( K_x \) and \( K_z \) are assumed to be constant within one finite element (Pinder, 1964).

Substitute Equation (2) for one finite element and Equation (4) in Equation (3) yields:
\[
\int_{\Omega^e} \left[ \frac{\partial \psi^e_i}{\partial x} (K_x \sum_{j=1}^{n} h_j \frac{\partial \psi^e_j}{\partial x}) + \frac{\partial \psi^e_i}{\partial z} (K_z \sum_{j=1}^{n} h_j \frac{\partial \psi^e_j}{\partial z}) + s \psi^e_i (\sum_{j=1}^{n} \frac{\partial h}{\partial t} \psi^e_j) \right] dx dz - \int_{\Gamma^e} \psi^e_i q_n ds = 0
\]
(5)

The previous expression can be expressed in a matrix form as follows:
\[
[C^e \{h^e\} + G^e \left\{ \frac{\partial h^e}{\partial t} \right\} = \{Q^e\}
\]
(6)

Where \([C^e]\) is the conductance matrix, \([G^e]\) is the storage matrix and \(\{Q^e\}\) is the discharge vector.

\[
C^e_{ij} = \int_{\Omega^e} \left[ K_x \frac{\partial \psi^e_i}{\partial x} \frac{\partial \psi^e_j}{\partial x} + K_z \frac{\partial \psi^e_i}{\partial z} \frac{\partial \psi^e_j}{\partial z} \right] dx dz
\]
(7)

\[
G^e_{ij} = \int_{\Omega^e} s \psi^e_i \psi^e_j dx dz
\]
(8)

\[
Q^e_i = \int_{\Gamma^e} \psi^e_i q_n ds
\]
(9)

Linear triangular finite element (three nodes element) is used here. The integrals in the above equations are evaluated using the natural coordinates \(L_1, L_2\) and \(L_3\). In summary, they can be expressed as:

\[
C^e_{ij} = \frac{1}{4A_e} (K_x \beta^e_i \beta^e_j + K_z \gamma^e_i \gamma^e_j)
\]
(10)

\[
G^e_{ij} = \frac{A_e S}{6}
\]
(11)
\[ G_{ij}^e = \frac{A_e s}{12} \]  

Where \( A_e \) is the area of the finite element

\[ \beta_i^e = z_i - z_k \quad (i \neq j \neq k) \]  

\[ \gamma_i^e = -(x_j - x_k) \]  

\( Q^e_i \) can be set equals to zero for all nodes because of the following:
- \( Q^e_i = 0 \) for no flow boundaries.
- \( Q^e_i = 0 \) for interior nodes.
- \( Q^e_i \) is irrelevant for nodes on specified head boundaries.

The finite element mesh shown in Figure 2 is used. The total number of elements and their geometry are not constant. They depend on the geometry of the study area (dam base, sheet pile length, angle of inclination, etc.). However, mesh generation is done automatically by the prepared computer program. After assembly of the finite elements, Equation (6) becomes:

\[ \begin{bmatrix} C \end{bmatrix} \{ h \} + \begin{bmatrix} G \end{bmatrix} \left\{ \frac{\partial h}{\partial t} \right\} = \{ Q \} \]  

Fig. 2 Finite element mesh
From Taylor series, the derivative \( \frac{\partial h}{\partial t} \) can be approximated as follows:

\[
\left( \frac{\partial h}{\partial t} \right) = \frac{h^{t+\Delta t} - h^t}{\Delta t}
\]

(15)

Therefore, the vector \( \left\{ \frac{\partial h}{\partial t} \right\} \) can be expressed as:

\[
\left( \frac{\partial h}{\partial t} \right) = \frac{1}{\Delta t} \left( \{h\}^{t+\Delta t} - \{h\}^t \right)
\]

(16)

In order to obtain stable solution for Equation (14), an implicit numerical technique is used; which is the backward difference method. The solution of Equation (14) after \( \Delta t \) time becomes:

\[
\{h\}^{t+\Delta t} = \left[ [C] + \frac{1}{\Delta t} [G] \right]^{-1} \left[ \{Q\} + \frac{1}{\Delta t} [G] \{h\}^t \right]
\]

(17)

Using the matrices solution, Equation (17) is solved at each time step.

**Initial and Boundary Conditions**

Consider the case as shown in Figure (3). The boundary conditions are as follows:

\[
\frac{\partial h}{\partial z} (x,0,t) = 0 \quad 0 \leq x \leq TL \quad \text{on bc}
\]

\[
h(x, DP + FD, t) = h_{ds} \quad BS + USL \leq x \leq TL \quad \text{on de}
\]

\[
\frac{\partial h}{\partial x} (USL + BS, z, t) = 0 \quad DP \leq z \leq DP + FD \quad \text{on ef}
\]

\[
\frac{\partial h}{\partial z} (x, DP, t) = 0 \quad USL + XS \leq x \leq USL + BS \quad \text{on fg}
\]

\[
\frac{\partial h}{\partial z^i} (x^i, \frac{SK}{2}, t) = 0 \quad 0 \leq x^i \leq SL \quad \text{on gh}
\]

\[
\frac{\partial h}{\partial z^i} (x^i, -\frac{SK}{2}, t) = 0 \quad 0 \leq x^i \leq SL \quad \text{on hi}
\]
Fig. 3 Schematic sketch for the proposed model showing the boundary conditions
\[
\frac{\partial h}{\partial z} (x, DP, t) = 0 \quad \text{on} \quad ij
\]
\[
USL \leq x \leq USL + XS
\]

\[
\frac{\partial h}{\partial x} (USL, Z, t) = 0 \quad \text{on} \quad jk
\]
\[
DP \leq Z \leq DP + FD
\]

For the boundary \( ak \), if the upstream water level is constant at level \( h_{u1} \), then
\[
h(x, DP + FD, t) = h_{u1} \quad 0 \leq x \leq USL \quad \text{on} \quad ak
\]

For the boundaries \( ab \) and \( cd \), they are neither constant head nor impervious boundaries. Therefore, the effects of choosing their type on the model results have to be investigated.

If they are considered as constant head boundaries, they will have the form:
\[
h(0, Z, t) = h_{u1} \quad 0 \leq Z \leq DP + FD \quad \text{on} \quad ab
\]
\[
h(TL, Z, t) = h_{ds} \quad 0 \leq Z \leq DP + FD \quad \text{on} \quad cd
\]

Alternatively, if they are considered as impervious boundaries, they will be expressed as:
\[
\frac{\partial h}{\partial x} (0, Z, t) = 0 \quad 0 \leq Z \leq DP + FD \quad \text{on} \quad ab
\]
\[
\frac{\partial h}{\partial x} (TL, Z, t) = 0 \quad 0 \leq Z \leq DP + FD \quad \text{on} \quad cd
\]

Two cases are considered for the initial condition:

**Case I: Constant water level at the upstream side**

The initial condition is as follows:
\[
h(x, z, 0) = h_{ds}
\]

**Case II: Variable water level at the upstream side**

As shown in Figure 3, consider that \( h_{u1}, h_{u2} \) as the initial and final water levels at the upstream side and \( h_{ub} \) is a water level between \( h_{u1} \) and \( h_{u2} \). It is required to analyze the seepage flow at level \( h_{ub} \). The first step is to apply the same initial condition as in case I with boundary condition on \( ak \) as follows:
\[
h(x, DP + FD, t) = h_{u1} \quad 0 \leq x \leq USL
\]
In the second step, when steady state is reached, the head values of piezometric head obtained in the first step are considered as initial condition and the following boundary condition will be applied on \( ak \):

\[
h(x, DP + FD, t) = h_{u2} \quad 0 \leq x \leq USL
\]

Here in case II \( h_{ub} \) is a specific water level occurs during the time \( TFE \) (Time required to fill or empty the reservoir). Assuming constant rate to fill or empty the reservoir, then \( h_{ub} \) can be expressed as:

\[
h_{ub} = (h_{u2} - h_{u1}) \times SPT / TFE + h_{u1} \quad \text{in case of filling the reservoir, and}
\]

\[
h_{ub} = (h_{u1} - h_{u2}) \times (TFE - SPT) / TFE + h_{u2} \quad \text{in case of emptying the reservoir}
\]

Where \( SPT \) is the time at which \( h_{ub} \) occurs.

**Foundation Soil**

Three cases of the foundation soil are considered in this study,
- Homogeneous and isotropic.
- Homogeneous and anisotropic.
- Block wise homogeneous.

Selection of \( K_x, K_z \) and \( S \) depends on the type of the foundation soil.

**MODEL APPLICATION AND RESULTS**

The computer program was written in COMPAQ VISUAL FORTRAN 6.1. The output included the uplift head, the exit gradient and the seepage quantity behind the dam. Any required geometry of the case can be analyzed as follows:

- Angle of inclination of the sheet pile \( 0^\circ \leq \theta \leq 180^\circ \).
- Sheet pile location \( 0 \leq XS \leq BS \) (\( XS = 0 \) at heel, \( XS = BS \) at toe).
- Dam base is depressed (\( FD \neq 0 \)) or on ground (\( FD = 0 \)).
- The case with or without sheet pile (\( SL \neq 0 \) or \( SL = 0 \)).
- Other dimensions BS, DP, USL and DSL can be inputted in the model as required.

The model can analyze steady or unsteady seepage flow. For the foundation soil, any of the three cases presented can be selected. The exit gradient can be obtained as follows:

\[
I_e = \frac{h_i - h_{di}}{d_i}
\]

Where \( h_i \) are the values of the piezometric head at nodes vertically below the exit nodes, and \( d_i \) are the vertical distances as shown in Figure 4.
Different cases are selected and analyzed using the model. The following represents some of these cases under steady state seepage flow.
As shown in Fig. 5, when the sheet pile is at the toe, high values for exit gradient are developed if the sheet pile is inclined towards the upstream side (\( \theta \) is less than 90°), also both of the uplift head and the seepage behind the dam are greater than those of vertical sheet pile (Fig. 7 and Fig. 8). On the other hand, the exit gradient decreases as \( \theta \) increases towards the downstream (\( \theta \geq 90° \) Fig. 6) for a distance \( Xt / SL \approx 0.75 - 0.8 \) beyond the toe, then the exit gradient starts increasing slightly with increasing \( \theta \). It is also clear that the maximum exit gradient decreases for \( \theta = 120° \) and starts increasing for \( \theta = 135° \). From Fig. 7, the uplift head decreases as \( \theta \) increases. From Fig. 8, the minimum seepage quantity behind the dam is at \( \theta \) approximately equals to 120°.

![Fig. 6 Variation of exit gradient for different values of \( \theta \) (\( \theta \geq 90° \))](image)

![Fig. 7 Variation of uplift head along the dam base for different values of \( \theta \)](image)
Fig. 8 Variation of seepage behind the dam for different values of $\theta$

For Figures (5, 6, 7 and 8) $SL = XS = BS, h_{ds} = FD = 0, Kx = Kz$

From Fig. 9, when the sheet pile is at the heel, high values for exit gradient are developed if the sheet pile inclined towards the upstream side ($\theta$ is less than 90°), also the uplift head is greater than that of vertical sheet pile (Fig. 10). The exit gradient decreases as $\theta$ increases towards the downstream ($\theta \geq 90^\circ$) until $\theta$ approximately equals to 120°, then the exit gradient starts increasing with increasing $\theta$. From Fig. 11, the minimum seepage quantity behind the dam is at $\theta$ approximately equals to 60°.

Fig. 9 Variation of exit gradient for different values of $\theta$
For Figures (9, 10 and 11) SL = BS, \( h_{ds} = FD = XS = 0 \), \( Kx = Kz \)

From Fig. 12, when the sheet pile was at the mid distance of the dam base, high values for exit gradient were developed if the sheet pile inclined towards the upstream side (\( \theta \) is less than 90°), also both of the uplift head and the seepage behind the dam were greater than those of vertical sheet pile (Fig. 13 and Fig. 14). The exit gradient near the toe decreases as \( \theta \) increases towards the downstream (\( \theta \geq 90^\circ \)) until \( \theta \) approximately equals to 120°, then the maximum exit gradient starts increasing with increasing \( \theta \). From Fig. 14, the minimum seepage quantity behind the dam is at \( \theta \) approximately equals to 90°.
Fig. 12 Variation of exit gradient for different values of $\theta$

Fig. 13 Variation of uplift head along the dam base for different values of $\theta$

Fig. 14 Variation of seepage behind the dam for different values of $\theta$

For Figures (12, 13 and 14) $SL = BS$, $h_{ds} = FD = 0$, $XS = BS/2$, $Kx = Kz$
The effect of anisotropy is clearly indicated in Figures 15, 16, and 17. Exit gradient, uplift head and seepage increase as the anisotropy ratio \( \frac{K_x}{K_z} \) increases.

**Fig. 15** Effect of anisotropy ratio \( \frac{K_x}{K_z} \) on exit gradient

**Fig. 16** Effect of anisotropy ratio \( \frac{K_x}{K_z} \) on uplift head
Fig. 17 Effect of anisotropy ratio ($K_x / K_z$) on seepage behind dam

For Figures (15, 16 and 17) $SL = BS = XS$, $h_{ds} = FD = 0$, $\theta = 120^\circ$

For a heterogeneous foundation soil, the variation of exit gradient does not follow a general trend. According to the specifications of the foundation soil, each case has to be analyzed and described individually. For the case shown in Figure 18, four soil layers are entered to the model with specifications as in Table 1.

Table 1 Details of the soil layers used in the model

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer thickness, (m)</td>
<td>28</td>
<td>40</td>
<td>65</td>
<td>50</td>
</tr>
<tr>
<td>$K_x$ (m/day)</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$K_z$ (m/day)</td>
<td>1</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Fig. 18 Variation of exit gradient for different values of $\theta$ with heterogeneous soil

For Figure 18, $SL = BS = XS$, $h_{ds} = 0$, $FD = BS/10$, $\theta = 120^\circ$
The following cases represent some results under unsteady state seepage flow:

**Fig. 19** Variation of exit gradient (unsteady flow, constant upstream head)

**Fig. 20** Variation of uplift head along (unsteady flow, constant upstream head)

**Fig. 21** Variation of seepage behind the dam (unsteady flow, constant upstream head)
Fig. 22 Variation of exit gradient (unsteady flow, variable upstream head)

Fig. 23 Variation of uplift head (unsteady flow, variable upstream head)

Fig. 24 Variation of seepage behind the dam (unsteady flow, constant upstream head)

For Figures (19 - 24), SL = BS = XS, $h_{ds} = FD = 0$, $\theta = 120^\circ$, $K_x = K_z$
Reliability of the Model

The validity of the present numerical model was verified by comparing its results with the analytical solution given by Abbas (1994), and the finite difference solution given by Mohamed and Agiralioglu (2005), also the comparison is done with the flow net method. A close agreement is reached as follows:

- The difference in seepage computation behind the dam using the flow net method and the present model is about 7.5%.
- For the exit gradient and the uplift head, Figures 25 and 26 show the comparisons of the present model with previous studies.

For Figures (25 and 26), SL = BS = XS, $h_{ds} = FD = 0$, $\theta = 120^\circ$, $K_x = K_z$
CONCLUSIONS

On the basis of finite element approximations, a two-dimensional numerical model was developed to solve the governing equations of groundwater seepage under hydraulic structures. The goal was to study the effect of inclined cutoffs, permeability ratio, and foundation soil depth on exit gradient, uplift pressure and flow rate. The model calculated the piezometric head at all nodal points in the problem solution domain for steady and unsteady flow conditions. Using these head values, the exit gradient, uplift pressure and flow rate were determined. The model results indicated that the finite element technique gave comparable values for similar cases solved with other solution methods and offer a more general approach that permits the usage of the desired permeability, foundation soil depth, and flow type. From the model results, the following remarks can be concluded:

1- The exit gradient, after a distance $X_t / SL \approx 0.85$ downstream from the dam toe, decreases with increasing the sheet pile inclination and the reduction increases as $\theta$ increases. The result is an increase in the safety factor against piping phenomenon.

2- Using an inclined sheet pile towards the downstream side with $\theta$ less than 120° is beneficial in increasing such a safety factor. On the other hand, the danger from undermining will be shifted further downstream from the toe of the structure.

3- Placing the sheet pile at the dam heel is not recommended under any angle of inclination. The same conclusion was also valid for the case of toe sheeting inclined towards the upstream side. Doing so produces a singularity at the end of the dam with a mathematically infinite velocity. This will produce unstable soil condition in close proximity to the toe.

4- Complete analysis for various cases was considered in order to ascertain the optimum location and the optimum angle of inclination of the sheet pile. However, as preliminary conclusions, if the exit gradient and the seepage behind the dam are considered as the major factors in the design of the dam, the optimum location of the sheet pile is at the toe of the dam with inclination angle equals to 120°. While if the uplift head is considered as the major factor, the optimum location of the sheet pile is at the heel of the dam with inclination angle equals to 120°. These conclusions hold when the soil beneath the dam is homogeneous. However, if it is non-homogeneous, each particular case has its own optimum criteria.

REFERENCES


