EVALUATION OF CAPACITY RELIABILITY-BASED AND UNCERTAINTY-BASED OPTIMIZATION OF WATER DISTRIBUTION SYSTEMS

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ABSTRACT

Two approaches for single-objective reliability-based optimization of water distribution networks are presented. The two approaches link a genetic algorithm (GA) as the optimization tool, the Newton method as the hydraulic simulation solver with the chance constraint or with the Monte Carlo simulation to estimate network capacity reliability. For both approaches, the source of uncertainty analyzed is the future nodal external demands which are assumed to be random normally distributed variables with given mean and standard deviations. The proposed approaches are illustrated by application to the two-loop network.

For the first approach, optimal network design constrained by reliability for water distribution system analysis is formulated as an optimization problem under uncertainty. The optimal design problem is formulated as a chance constraint minimization problem restricted with a pre-specified level of uncertainty. The reliability of the system is then evaluated for the least-cost design of the network using the Monte Carlo simulation.

The second approach uses the Monte Carlo simulation to estimate network capacity reliability. This requires a big number of realizations to estimate network capacity reliability; therefore, the execution time is highly increased.

An evaluation and comparison between the two approaches are demonstrated and the advantages and disadvantages are illustrated.

Keywords: Genetic Algorithms, Optimization, Monte Carlo Simulation, Chance Constraint, Reliability, Water Distribution Systems.

INTRODUCTION

The complexity of water distribution systems makes it difficult to obtain least-cost design systems considering other constraints such as reliability. The most important consideration in designing and operating a water distribution system is to satisfy consumer demands under a range of quantity and quality considerations during the entire lifetime for the expected loading conditions. The accommodation of a water distribution system to abnormal conditions such as breaks and mechanical failure of pipes, valves, etc. is essential and their possibility of occurrence should be examined carefully to estimate the reliability of the system.

There is currently no universally accepted definition or measure of the reliability of water distribution systems. In general, reliability is defined as the probability that a system performs its mission within specified limits for a given period of time in a specified environment [1]. The issue of water distribution system reliability concerns the ability of the system to supply the demands at the nodes within the system at pressure head levels greater than or equal to a pre-specified level [2, 3, 4].

Numerous optimization techniques are used in water distribution systems. These include the deterministic optimization techniques such as linear programming (for separable objective functions and linear constraints), and non-linear programming (when the objective function and the constraints are not all in the linear form), and the stochastic optimization techniques such as genetic algorithms and simulated annealing. Genetic Algorithms (GAs), in general, have an advantage over classical optimization techniques in that they may be used to find optimal or near optimal solutions to nonlinear, discrete, and discontinuous problems. Genetic Algorithms (GAs) have been applied extensively to optimize water distribution systems for hydraulic criteria. The main advantages of GAs are that they use a population of evolving solutions and identify several solutions from which the decision maker can select, rather than a single optimum. The main disadvantage lies in the high computational intensity.

Lansey et al. [3] were among the first to present a chance constraint model for the least-cost design of water distribution systems. The uncertainty in the required demand, pressure heads, and pipe roughness coefficient were explicitly accounted for in the model. The generalized reduced gradient (GRG2) technique was used to solve the nonlinear programming single-objective chance constrained minimization model. The methodology assumed nodal heads to be random, normally distributed variables with given mean and standard deviation. Since head values are functions of many parameters, some of which could be uncertain, they should be treated as a response function rather than independent stochastic variables. Also, the generalized reduced gradient method (GRG2) is a local search method which could be easily trapped in the local minimum (Savic and Walters, [5]).

Goulter et al. [6] incorporated reliability concept into optimal design models for pipe network systems. The measure of the system reliability was used as a criterion to improve the system distribution. The chance constraints were the probability of pipe failure for each link and the probability of demand exceeding design values at each node in the network.

Bao et al. [7] presented a Monte Carlo simulation model that estimated the nodal and system hydraulic reliabilities of water distribution systems that accounted for uncertainties. The model consisted of three major components; random number generation, hydraulic network simulation, and computation of reliability. The model could be applied in the analysis of existing water distribution systems or in the design of new or expanding systems.

Xu and Goulter [8] proposed a two-stage methodology for reliability assessment with uncertainties in nodal demands, pipe capacity, and reservoir levels. To quantify uncertainties in the hydraulic model, the model used the first-order second moment (FOSM) reliability method, which assumed a closely linear relationship between uncertain and response variables which is often not the case for water distribution systems.

Xu and Goulter [9] used the first order reliability-method-based algorithm that computed the capacity reliability of water distribution networks. The sensitivity-analysis-based technique was used to derive the first-order derivatives. The (FORM) algorithm required repetitive calculation of the first order derivatives and matrix inversion which was very computationally demanding even in small networks and may lead to a number of numerical problems.

Tolson et al. [10] used GAs to solve the optimal water distribution system design problems along with the first order reliability method (FORM) method to quantify uncertainties. They demonstrated that the Monte Carlo Simulation critical node capacity reliability approximation can significantly underestimate the true Monte Carlo Simulation network capacity reliability. Therefore, they developed a more accurate FORM approximation to network capacity reliability that considers failure events at the two most critical nodes in the network.

Abdel-Gawad [11] presented an approach for water network optimization under a specific level of uncertainty in demand, pressure heads, and pipe roughness coefficient. The approach depends on using the chance constrained model to convert uncertainties in the design parameters to form a deterministic formulation of the problem. The GA method was adopted to solve the nonlinear optimization problem settled in a deterministic form. A hypothetical example was solved and compared with previous solution from the gradient approach [3]. From the results it can be found that the construction cost of the pipe system increases, with an increasing rate, as the reliability requirement increases. Uncertainties in demand nodes or roughness coefficients have a more pronounced effect on final construction cost, than the effect of the required minimum pressure heads.

Babayan et al. [12] presented a methodology for the least cost design of water distribution networks considering uncertainty in node demand. The uncertain demand was assumed to follow both truncated Gaussian (normal) probability density function (PDF) and uniform probability density function. The genetic algorithm was used to solve the equivalent deterministic model for the original stochastic one to find reliable and economic design for the network and the system reliability was then determined using full Mont Carlo simulation with 100,000 sampling points. The model was tested on the New York tunnels and Anytown problems and then compared to available deterministic solutions. The results demonstrated the importance of applying the uncertainty concept in designing water distribution systems.

Babayan et al. [13] developed two new methods to solve an optimization problem under uncertainty. Uncertainty sources used were both future water consumption and pipe roughness. The stochastic formulation was solves after being replaced by a deterministic one using numerical integration method, while the optimization model was solved using a standard genetic algorithm. The sampling method solved the stochastic problem directly by using the newly developed robust chance constraint genetic algorithm. Both methods had their own benefits and drawbacks.

The objective of the present paper is to demonstrate and compare two methodologies which identify least cost solutions that meet the cost reduction targets with a specified degree of reliability. The optimization tool is the Genetic Algorithm (GA) which is linked in the present work with the reliability formulation. The first formulation is the Monte Carlo simulation (MC) and the second one is the chance-constraint (CC) formulation. These formulations were used with a specified degree of reliability when the future demands at different nodes are uncertain. The way Monte Carlo Simulation and Chance Constrained formulation can be used to estimate the different measures of reliability is discussed below.

OPTIMIZATION MODEL FORMULATION

The water distribution network optimization aims to find the optimal pipe diameters in the network for a given layout and demand requirements. The optimal pipe sizes are selected in the final network satisfying the conservations of mass and energy, and the constraints (e.g. hydraulic and design constraints).

1- Deterministic Model

The formulation of the optimization model for water distribution system design can be generally written in the following form, [3]:

Objective function:

$$\text{Min. Cost} = \text{min. } C_T = \sum_{i,j \in M} f\left(D_{i,j}\right) \tag{1}$$

Model Constraints:

$$\sum_{j} q_{i,j} = Q_j \qquad j = 1,\dots,J \text{ (nodes)} \tag{2}$$

$$\sum_{i,j\in n} h_{f_n} = 0 \qquad n = 1,\dots,N \text{ (loops)} \tag{3}$$

$$H_j \geq H_{j,\min} \qquad j = 1,\dots,J \text{ (nodes)} \tag{4}$$

$$D_{\min} \leq D_{i,j} \leq D_{\max} \tag{5}$$

The main objective of the model, Eq. (1), is to minimize the construction cost of the water distribution network as a function of the pipe diameter $D_{i,j}$, for the set of possible links, *M*, connecting nodes *i*, *j* in the system. $q_{i,j}$ is the flow rate in the pipe connecting nodes *i*, *j*. h_f is the head loss in the pipe and expressed by the Hazen-Williams formula:

$$h_f = \frac{K}{C_{i,j}^{\ 1.852}} \frac{L_{i,j}\ q_{i,j}^{\ 1.852}}{D_{i,j}^{\ 4.8704}} = H_i - H_j \tag{6}$$

where *K* is a conversion factor which accounts for the system of units used, ($K = 10.6744$ for $q_{i,j}$ in m^3/s and $D_{i,j}$ and $L_{i,j}$ in m), $C_{i,j}$ is the Hazen-Williams roughness coefficient for the pipe connecting nodes *i*, *j*, $L_{i,j}$ is the length of the pipe connecting nodes *i*, *j* , and H_i, H_j are the pressure heads at nodes *i*, *j*. Then, the flow rate in the pipe is calculated as:

$$q_{i,j} = K^{-0.54}\ C_{i,j} \left[\frac{H_i - H_j}{L_{i,j}}\right]^{0.54} D_{i,j}^{\ 2.63} \tag{7}$$

Eq. (2) represents the law of conservation of mass (continuity equation) which states that the summation of the flow rates in the pipes at node *j* must be equal to the external demand, Q_j, at that node. It has to be noticed that the continuity constraint must be satisfied for each node, *j*, in the network.

Eq. (3) in the model constraints simply states that the algebraic summation of the head loss, h_{f_n}, around each loop $n = 1,\dots, N$ must be equal to zero. The lower limit, $H_{j,\min}$, of the pressure head, H_j, at each node, *j*, is accounted for in the model by Eq. (4). Finally, Eq. (5) defines the constraint on the pipes diameters in the network where $D_{\min}$ and $D_{\max}$ are the minimum and maximum diameters, respectively.

Substitution of the Hazen-Williams formula, Eq. (7) back into Eq. (2) automatically satisfies Eq. (3), and which in turn reduces the deterministic model constraints to equations (4), (5), and (7) in combination with (2).

2- Stochastic (Chance Constraint) Model

The deterministic optimization model described above is transformed into a stochastic (chance constraint) formulation by considering that the future nodal demand, Q_j, is uncertain because of the unknown future conditions of the system and can be considered as an independent random variable.

The chance constraint formulation can now be expressed as:

Objective function:

$$\text{Minimum Cost} = \text{min.} \sum_{i,j \in M} f\left(D_{i,j}\right) \tag{8}$$

Subject to the constraints:

$$P\left[\sum_j K^{-0.54} C_{i,j} \left(\frac{H_i - H_j}{L_{i,j}}\right)^{0.54} D_{i,j}^{2.63} \geq Q_j\right] \geq \alpha_j \tag{9}$$

$$H_j \geq H_{j,\min} \tag{10}$$

$$D_{\min} \leq D_{i,j} \leq D_{\max} \tag{11}$$

Eq. (9) is the probability, P ( ), that the node demands are equaled or exceeded with probability level α_j. The probability level α_j is defined as the constraint performance reliability which accounts for the effect of uncertainty of the future demand.

3- Deterministic Chance Constraint Model

The chance constraint model is now transformed from a stochastic form into a deterministic one through applying the cumulative probability distribution concept by considering the future demand, to be represented by normal random variables with mean, μ, and standard deviation, σ, as:

$$Q \sim N(\mu_Q, \sigma_Q)$$

Similarly, Eq. (9) is transformed into a deterministic form as follows:

$$P\left[\sum_j K^{-0.54} C_{i,j} \left[\frac{H_i - H_j}{L_{i,j}}\right]^{0.54} D_{i,j}^{2.63} - Q_j \leq 0\right] = P\left[W_j \leq 0\right] \leq 1 - \alpha_j \tag{12}$$

where W_j is a normal random variable with mean:

$$\mu_{W_j} = \sum_j K^{-0.54} \mu_{C_{i,j}} \left[\frac{H_i - H_j}{L_{i,j}}\right]^{0.54} D_{i,j}^{2.63} - \mu_{Q_j} \tag{13}$$

and standard deviation:

$$\sigma_{W_j} = \left\{ \sum_j \left[K^{-0.54} \left[\frac{H_i - H_j}{L_{i,j}} \right]^{0.54} D_{i,j}^{2.63} \right] \sigma_{C_{i,j}}^2 + \sigma_{Q_j}^2 \right\}^{1/2} \tag{14}$$

Eq. (12) can be rewritten as:

$$P\left[\frac{W_j - \mu_{W_j}}{\sigma_{W_j}} \le \frac{0 - \mu_{W_j}}{\sigma_{W_j}} \right] \le 1 - \alpha_j \tag{15}$$

or in a simplified form:

$$\phi\left[\frac{-\mu_{W_j}}{\sigma_{W_j}} \right] \le 1 - \alpha_j \tag{16}$$

where ϕ is the cumulative distribution function and ϕ [ ] is the standard normal distribution function.

The final deterministic form of the constraint Eq. (12) is now written as:

$$-\frac{\mu_{W_j}}{\sigma_{W_j}} \le \phi^{-1}\left(1 - \alpha_j\right) \tag{17}$$

where μ_{W_j} and σ_{W_j} are determined using Eqs. (13) and (14).

The final deterministic chance constraint model for water distribution networks is given by the objective function Eq. (8) subject to the constraints Eqs. (17) and (11). The model is nonlinear because of the nonlinear objective function Eq. (8) and the nonlinear constraint Eq. (17) for every node. The other constraint given by Eq. (11) for every pipe is considered to be simple bound. The genetic algorithm (GA) was used as a technique to solve the deterministic chance constraint model for water distribution networks.

GAMC*net* and GACC*net* PROGRAMS

GAMC*net* (Genetic Algorithms Monte Carlo network) and GACC*net* (Genetic Algorithms Chance Constrained network) programs have been written in FORTRAN. GAMC*net* program depends on three main techniques, Yaseen [14]:

1- Genetic algorithm technique to produce the optimal diameters. The GA source code used in this study (FORTRAN GA version 1.7a) after minor modifications is written by Carroll [15].

2- Newton method to simulate hydraulically a network to give the network analysis. The *H*-equations solution method is provided in Larock et al. [16] and it is used after many modifications to be compatible with the whole program.
3- Monte Carlo technique to compute the reliability.

For GACC*net* program, it is consisted of, Ezzeldin [17]:

1. Genetic algorithm technique to produce the optimal diameters. The GA source code used is similar to that used in Abdel-Gawad [18].
2. Newton method to analyze the network using the *H*-equations solution method.
3. Chance Constraint for the uncertainties.
4. Monte Carlo technique to compute the reliability of the optimal set of pipe diameters.

OPTIMIZATION TECHNIQUE

In optimization techniques, external penalty functions have been used to convert a constrained optimization problem into an unconstrained problem. Therefore, for the network optimization, the objective function is given as:

$$Z = C_T + C_P \tag{18}$$

where C_P is the penalty cost.

The design constraints in pipe network optimization that were used in the penalty function are the minimum allowable hydraulic pressures at given nodes as the diameter of each pipe is chosen from a specified set of commercial pipes. Applying the developed adaptive penalty function mentioned in Djebedjian et al. [19], the penalty cost is written as:

$$C_P = \frac{C_T}{M} \cdot \sum_{j=1}^{M} \left(H_{j,\min} - H_j \right) \tag{19}$$

and the objective function is calculated from:

$$Z = \begin{cases} C_T & \text{if } H_{j,\min} - H_j \leq 0 \\ C_T \left[1 + \frac{1}{M} \sum_{j=1}^{M} \left(H_{j,\min} - H_j \right) \right] & \text{else} \end{cases} \tag{20}$$

The penalty cost is applied at the nodes where the pressure head at node is less than the minimum allowable pressure head at the same node. This penalty cost is used in the developed programs GAMC*net* and GACC*net*.

The reliability-based optimization is mainly reliability constrained cost minimization model which is solved using the developed program GAMC*net* to obtain minimum cost solutions for reliability constraints. The selection of the optimized diameters which fulfill reliability constraints is achieved by the objective function for reliability-based optimization *Z*:

$$Z = \begin{cases} C_T & \text{if } H_j \geq 0 \\ C_T\left[1 + \frac{1}{M}\sum_{j=1}^{M}\left(H_{j,\min} - H_j\right)\right] & \text{else} \end{cases} + \begin{cases} 0 & \text{if } R_{estimated} - R_{required} \geq 0 \\ C_T\left(R_{required}^2 - R_{estimated}^2\right) & \text{else} \end{cases} \quad (21)$$

It consists of the total cost of network C_T and two penalty functions: $\frac{C_T}{M} \cdot \sum_{j=1}^{M}\left(H_{j,\min} - H_j\right)$, Djebedjian et al. [19], when the pressure head at node, H_j, is negative (i.e. infeasible solution) and $C_T\left(R_{required}^2 - R_{estimated}^2\right)$; where $R_{required}$ and $R_{estimated}$ are the required and estimated system reliabilities, respectively. The estimated system reliability in this study is defined as the minimal nodal reliability in the system, Bao and Mays [7].

GENETIC ALGORITHMS

Genetic algorithms are search techniques based on the concepts of natural evolution and thus their principles are directly analogous to natural behavior, Gen and Cheng [20]. The brief idea of GA is to select population of initial solution points scattered randomly in the optimized space, then converge to better solutions by applying in iterative manner the following three processes (reproduction/selection, crossover and mutation) until a desired criteria for stopping is achieved.

The micro-Genetic Algorithm (μGA), Krishnakumar [21], is a "small population" GA. In contrast to the Simple Genetic Algorithm, which requires a large number of individuals in each population (i.e., 30 - 200); the μGA uses a small population size.

A brief description of the steps in using GA for pipe network optimization, [22], and including reliability is as follows:

1. ***Generation of initial population.*** The GA randomly generates an initial population of coded strings representing pipe network solutions of population size N_G. Each of the N_G strings represents a possible combination of pipe sizes.
2. ***Computation of network cost.*** For each N_G string in the population, the GA decodes each substring into the corresponding pipe size and computes the total

material cost. The GA determines the costs of each trial pipe network design in the current population.

3. ***Hydraulic analysis of each network.*** A steady state hydraulic network solver computes the heads and discharges under the specified demand patterns for each of the network designs in the population. The actual nodal pressures are compared with the minimum allowable pressure heads, and any pressure deficits are noted. In this study, the Newton technique is used.
4. ***Computation of penalty cost.*** The GA assigns a penalty cost for each demand pattern if a pipe network design does not satisfy the minimum pressure constraints. The summation of all pressure deficits throughout the network is used as the basis for computation of the penalty cost. The summation of all pressure deficits is multiplied by the developed penalty coefficient.
5. ***Computation of total network cost.*** The total cost of each network in the current population is taken as the sum of the network cost (Step 2) plus the penalty cost (Step 4).
6. ***Computation of the fitness.*** The fitness of the coded string is taken as some function of the total network cost. For each proposed pipe network in the current population, it can be computed as the inverse or the negative value of the total network cost from Step 5.
7. ***Monte Carlo simulation.*** The demands at the nodes are changed and the hydraulic simulation is achieved. The process continues to N_s iterations and the reliability is calculated.
8. ***Generation of a new population using the selection operator.*** The GA generates new members of the next generation by a selection scheme.
9. ***The crossover operator.*** Crossover occurs with some specified probability of crossover for each pair of parent strings selected in Step 8.
10. ***The mutation operator.*** Mutation occurs with some specified probability of mutation for each bit in the strings which have undergone crossover.
11. ***Production of successive generations.*** The use of the three operators described above produces a new generation of pipe network designs using Steps 2 to 10. The GA repeats the process to generate successive generations. The last cost strings (e.g., the best 20) are stored and updated as cheaper cost alternatives are generated.

MONTE CARLO SIMULATION

Monte Carlo methods are statistical simulation methods that utilize sequences of random numbers to perform the simulation. Assuming that the behavior of a physical (or mathematical) system can be described by probability density functions (pdf's) and are known, the Monte Carlo simulation can proceed by random sampling from the pdf's. Many simulations are then performed (multiple "trials") and the desired result is taken as an average over the number of observations. In this study, the probability distribution function is the nodal demand. The normal random variable (the node demand) $Q = N(\mu_Q, \sigma_Q)$ and the coefficient of variation (*COV*) are written as:

$$Q = \mu_Q + \sigma_Q Y \tag{22}$$

$$COV = \sigma_Q / \mu_Q \tag{23}$$

where μ_Q and σ_Q are the mean and standard deviation of node demand and *Y* is the normally distributed random variable. The standard uniform distribution is transferred to normal distribution using the polar method of Box-Muller transforms (acceptance-rejection algorithm), (Marsaglia and Bray, [23]).

The nodal capacity reliability is calculated as follows (Rao, [1]):

$$R_N = \frac{n_N}{N_s} \tag{24}$$

where N_s is the total number of Monte Carlo simulations performed and n_N number of times that a particular criterion was achieved at node *N*. As the number of simulations increases, R_N converges to its true values. The percent relative error involved with the estimated network reliability *R* has been found to be (Rao, [1]):

$$\%\,Error = 100 \sqrt{\frac{R}{N_s\,(1-R)}} \tag{25}$$

There is an unbounded growth of the relative error for highly reliable networks (i.e. for $R \rightarrow 1$) and this is the main deficiency of the Monte Carlo simulation. A large number of simulations should be performed to ensure accurate results from the reliability study.

Case Study: Two-Loop Network

The case study is a gravity fed two-loop network with 8 pipes, 7 nodes and one constant head reservoir. The layout of the network, the lengths of pipes and the node data are shown in Fig. 1.

The two-loop network problem is originally presented by Alperovits and Shamir [24] and taken as a model network by many researchers. All the pipes are 1000 m long and the Hazen-Williams coefficient is assumed to be 130 for all the pipes. The demands are given in cubic meters per hour and the minimum acceptable pressure requirement for each node is 30 m above the ground level. There are 14 commercially available pipe diameters and Table 1 presents the total cost (in arbitrary units) per meter of pipe length for different pipe sizes.

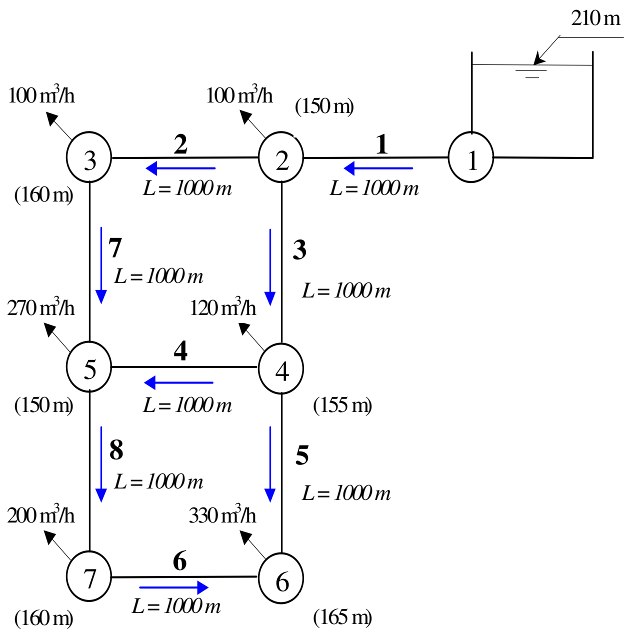

Figure 1. The two-loop network, Alperovits and Shamir [24]

Table 1. Cost data for the two-loop network

Diameter (in)	Cost (units)
1	2
2	5
3	8
4	11
6	16
8	23
10	32
12	50
14	60
16	90
18	130
20	170
22	300
24	550

The GAMC*net* and GACC*net* programs were applied to the case study and the results of the two programs were compared with other optimization methods and GA's. The hydraulic analysis results of the two programs were compared with the EPANET (Rossman, [25]) computer program which is available in the public domain (EPANET, [26]), so it was used to check the hydraulic solution accuracy of the GAMC*net* and GACC*net* programs.

The optimization of this well-known case study has a minimum cost of 419,000 units, (Savic and Walters [5], Abebe and Solomatine [27], Cunha and Sousa [28], Eusuff and Lansey [29] and Liong and Atiquzzaman [30]). The results of the two programs of the present study are similar to the least cost solution.

GAMC*net* Program

The correctness of the optimization algorithms of the two programs was verified by comparing the obtained results with the known optimal solution for this network. Preliminary analyses and various trials of the µGA optimization model for the 2-loop network, Yaseen [14], showed that the most µGA robust parameter values for the two-loop network were: population size of 12, initial random number seed = –1220, and maximum number of generations = 250. The crossover rate was set to 0.5 (uniform crossover). A result of 419,000 units similar to that obtained by many authors was achieved. The number of function evaluations was 741. The details of the program and its arrangement could be found in Yaseen [14].

In this paper, the total number of trials in the Monte Carlo simulations is 3000 based on test results which indicate that further increase in simulation number has very marginal effect on the value of nodal reliabilities. The reliability-based optimization was performed twice at coefficient of variation equal 10% and 20% of nodal demand. The program produced the best solutions for network capacity reliabilities of 50, 60, 70, 80, 90 and 99%.

GACC*net* Program

The chance-constrained GA was used, allowing a reliability target to be considered. For this analysis, a least-cost strategy with target reliabilities (uncertainty of the future demand) α_j = 50, 60, 70, 80, 90 and 99% were created. The strategy was then tested with 3000 of Monte Carlo realizations to verify that the targeted reliability level was achieved.

The following parameters were used in GACC*net* program for solving the two-loop network: population size of 100, maximum number of generations = 500, uniform crossover = 0.5 and mutation ratio = 0.16.

A standard deviation equal to zero refers to the case of no uncertainty, and the larger the standard deviation, the greater the uncertainty. Computer runs were made for various values of α ranging from 0.5 to 0.99. Using $\alpha = 0.5$ is equivalent to using mean values of the nodal demands. Higher values of α refer to more stringent performance requirements so that the likelihood of not meeting future demands is reduced.

RESULTS AND DISCUSSION

The results obtained by GAMC*net* and GACC*net* programs are discussed in the following sections. For GAMC*net* program, the optimal cost and pipe diameters of best solutions of the two-loop network for different reliability requirements, when the coefficients of variation are 10% and 20% of nodal demand are summarized in Tables 2 and 3, respectively. The nodal reliabilities for these solutions are given in Tables 4 and 5 for COV =10% and 20%, respectively. It can be observed that the low nodal reliability occurs mainly at the last two nodes, i.e. at the nodes far from the source.

Figure 2 shows these results of cost and network reliability at coefficient of variation COV = 10% and COV = 20%. It is evident that at constant coefficient of variation, the cost increases with the increase of required network reliability. Also, for the same required reliability, the cost increases with the increase of coefficient of variation. This is expected due to the fact that the higher the reliability requirement, the greater the cost of design. The high reliability of network increases the performance of network at normal conditions.

Table 2. Optimal cost and pipe diameters obtained by GAMC*net* for different required network reliabilities at COV = 10%

Pipe ID	Diameter (in)					
	R = 50%	R = 60%	R = 70%	R = 80%	R = 90%	R = 99%
1	18	18	18	18	18	20
2	14	16	10	12	8	8
3	14	14	16	16	20	18
4	1	1	6	1	10	14
5	14	14	18	16	16	14
6	1	2	10	10	10	1
7	14	14	8	14	1	2
8	12	10	1	4	1	12
Cost	424,000	439,000	455,000	465,000	481,000	500,000

Table 3. Optimal cost and pipe diameters obtained by GAMC*net* for different required network reliabilities at COV = 20%

Pipe ID	Diameter (in)					
	R = 50%	R = 60%	R = 70%	R = 80%	R = 90%	R = 99%
1	18	18	20	20	20	20
2	14	16	10	14	14	8
3	14	14	16	16	14	20
4	1	1	6	1	8	10
5	14	14	16	14	16	16
6	1	1	14	10	6	14
7	14	16	8	12	14	1
8	14	10	1	8	10	1
Cost	448,000	466,000	483,000	487,000	511,000	547,000

Table 4. Node reliability obtained by GAMC*net* for different network reliabilities at COV = 10%

Node ID	Node Reliability, R_N (%)					
	R = 50%	R = 60%	R = 70%	R = 80%	R = 90%	R = 99%
2	100	100	100	100	100	100
3	100	100	99.70	100	100	100
4	100	100	100	100	100	100
5	100	100	81.70	100	100	100
6	83.93	84.73	75.03	80.47	97.90	99.87
7	50.67	60.13	70.00	84.93	91.90	99.80
Network Reliability	50.67	60.13	70.00	80.47	91.90	99.80

Table 5. Node reliability obtained by GAMC*net* for different required network reliabilities at COV = 20%

Node ID	Node Reliability, R_N (%)					
	R = 50%	R = 60%	R = 70%	R = 80%	R = 90%	R = 99%
2	100	100	100	100	100	100
3	99.20	100	99.00	100	100	100
4	100	100	100	100	100	100
5	99.80	100	80.37	100	100	99.9
6	69.47	68.87	80.67	85.23	92.63	99.4
7	60.70	76.60	97.97	95.73	95.43	100
Network Reliability	60.70	68.87	80.37	85.23	92.63	99.4

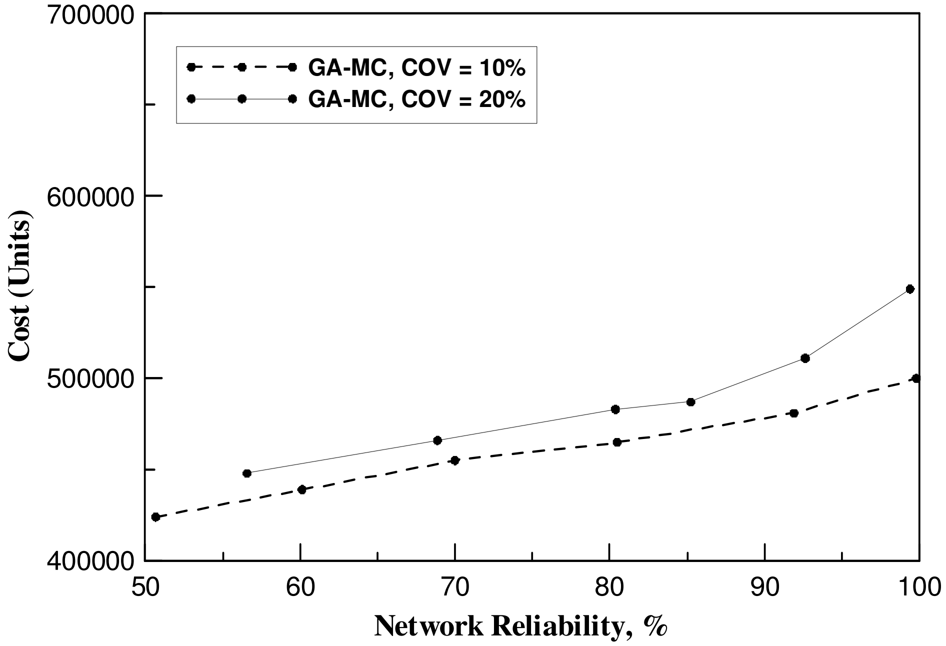

Figure 2. Total cost of network versus network reliability obtained by GAMC*net* for COV = 10% and COV = 20%

Similar to the previous results for the reliability-based optimization, the results obtained here for uncertainty-based optimization using the chance constrained formulation, GACC*net* program, are mentioned. Tables 6 and 7 give the optimal cost and pipe diameters of best solutions for COV = 10% and 20% of nodal demand. The nodal reliabilities at these solutions are shown in Tables 8 and 9. As observed in the previous study, the critical nodes with low reliability are those far from the source.

Figure 3 illustrates the impact of increasing the coefficient of variation of the nodal demands. As mentioned before, the higher the reliability requirement, the greater the cost of the design. The same is true for different coefficients of variation. It is interesting to note that the two curves coincide at $\alpha = 0.5$. At this value, the nodal demands are the mean value, i.e. the program performs optimization. The obtained value for the cost is 419,000 units which is similar to the previously mentioned optimal cost.

Table 6. Optimal cost and pipe diameters obtained by GACC*net* for different required network reliabilities at COV = 10%

Pipe ID	Diameter (in)					
	α = 0.50	α = 0.60	α = 0.70	α = 0.80	α = 0.90	α = 0.99
1	18	18	18	20	20	20
2	10	12	16	10	14	14
3	16	16	14	16	14	16
4	4	1	1	4	1	8
5	16	16	14	16	14	14
6	10	10	1	10	6	1
7	10	10	14	10	14	14
8	1	1	12	1	12	12
Cost	419,000	428,000	454,000	459,000	478,000	515,000

Table 7. Optimal cost and pipe diameters obtained by GACC*net* for different required network reliabilities at COV = 20%

Pipe ID	Diameter (in)					
	α = 0.50	α = 0.60	α = 0.70	α = 0.80	α = 0.90	α = 0.99
1	18	20	20	20	20	20
2	10	14	12	14	12	14
3	16	14	16	16	18	20
4	4	10	1	1	1	3
5	16	14	16	14	16	18
6	10	4	10	8	12	12
7	10	12	10	14	10	10
8	1	10	1	10	1	1
Cost	419,000	454,000	468,000	497,000	526,000	622,000

Table 8. Node reliability obtained by GACC*net* for different network reliabilities at COV = 10%

Node ID	Node Reliability, R_N (%)					
	α = 0.50	α = 0.60	α = 0.70	α = 0.80	α = 0.90	α = 0.99
2	100	100	100	100	100	100
3	58.23	100	100	93.40	100	100
4	100	100	100	100	100	100
5	85.77	99.80	100	97.50	100	100
6	61.67	73.20	85.23	99.40	99.00	100
7	60.10	68.47	96.80	95.40	99.93	100
Network Reliability	58.23	68.47	85.23	93.40	99.00	100

Table 9. Node reliability obtained by GACC*net* for different network reliabilities at COV = 20%

Node ID	Node Reliability, R_N (%)					
	α = 0.50	α = 0.60	α = 0.70	α = 0.80	α = 0.90	α = 0.99
2	100	100	100	100	100	100
3	53.63	99.47	99.97	100	99.97	100
4	100	100	100	100	100	100
5	69.90	100	97.10	100	97.20	99.97
6	55.57	70.33	92.53	96.90	99.43	100
7	54.53	73.60	83.57	98.53	99.97	100
Network Reliability	53.63	70.33	83.57	96.90	97.20	99.97

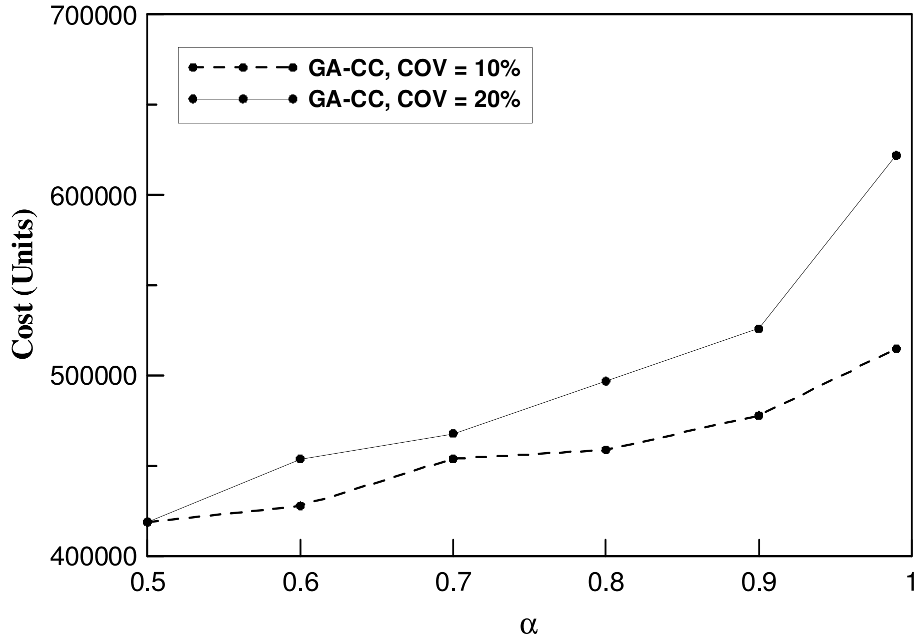

Figure 3. Total cost of network versus reliability α obtained by GACC*net* for COV = 10% and COV = 20%

To verify the effectiveness of the optimum cost provided by the chance constrained optimization algorithm, the Monte Carlo simulations were used in which the simulation generated the reliability according to the assumed normal distribution. The system reliability is evaluated for two different coefficients of variations (10 and 20%) and a range of required reliability levels from 50 to 99%.

Figure 4 shows the Total cost of network versus network reliability obtained by GACC*net* using the Monte Carlo simulation. The trends are similar to that of Fig. 2 but there is a high increase in the optimal cost as the reliability approaches 99%.

It is well known that the accuracy of the Monte Carlo simulation decreases as the required reliability increases. For this reason, the performance of the chance constraint approach is compared with that of the more accurate Monte Carlo simulation approach, based on the 3,000 realizations required under a Monte Carlo Simulation for this network. To make this comparison between the two approaches, Fig. 5 illustrates the reliability-cost curves resulted from the two programs at COV = 20%. The data in Fig. 5 show that the cost estimated by the Monte Carlo approach is higher than that of the Chance Constraint and that the differences between the Chance Constraint and Monte Carlo simulation estimates of reliability are moderate with the maximum difference at 99% due to the big number of simulations required by Monte Carlo simulation to decrease the error of reliability estimation, Eq. (25). The difference between the two approaches can be attributed to the value of Y in Eq. (22) which could take positive and negative values according to the normal distribution and depending on the required reliability, it could be greater than $\phi^{-1}(1-\alpha_j)$ in Eq. (17).

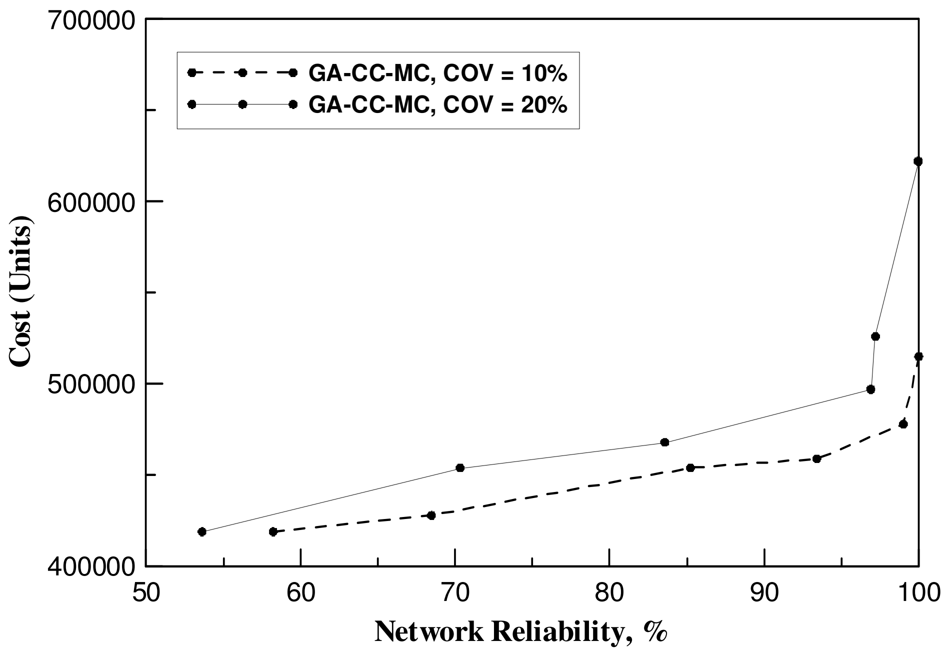

Figure 4. Total cost of network versus network reliability obtained by GACC*net* for COV = 10% and COV = 20%

Although the comparison between the results from these two approaches, Figs. 2 and 3, show a difference depending on the network reliability, the difference in computational time is rather significant. Based on an Intel ® Pentium 4 (1.7 GHz) personal computer, the GA linked with Monte Carlo simulation requires approximately 30 minutes to develop the reliability-based optimization, whereas the GA linked with chance constraint requires only 2 min.

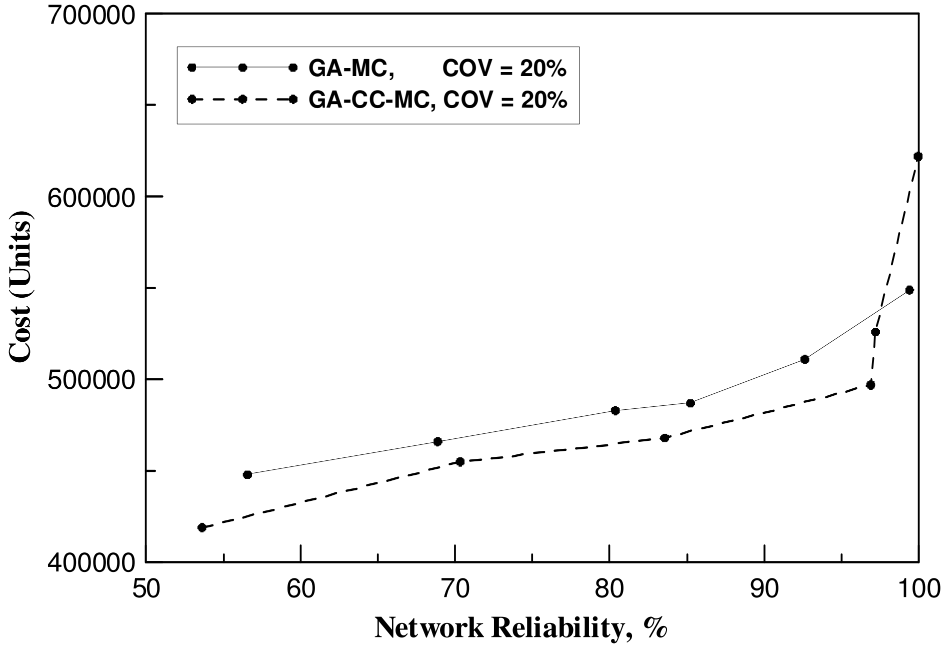

Figure 5. Total cost of network versus network reliability obtained by GAMC*net* and GACC*net* for COV = 20%

CONCLUSIONS

Uncertainty-based optimization and reliability-based optimization approaches were presented. The first approach uses the Chance Constraint (CC) formulation while the second applies the Monte Carlo simulation (MC). CC and MC are used to estimate uncertainty or reliability, respectively. Genetic Algorithms (GAs); which have an advantage over classical optimization techniques in that they may be used to find optimal or near optimal solutions to nonlinear, discrete, and discontinuous problems; were used as an optimization tool. Linking GA with CC or MC, GAs may be used to estimate optimal values of uncertainty- or reliability-based optimization of water distribution systems.

As long as the number of random variables (diameters) is moderate, the chance constrained approach is more attractive than Monte Carlo Simulation in estimating of reliability due to the very little computational time.

The use of the CC optimization approach is restrictive in the sense that the reliability levels specified are generally based on subjective judgments. As a consequence the actual network reliability can be calculated only once the optimization process has converged and the final solution is obtained.

The results of the two approaches at two values of coefficient of variation reveal the well-known relation between the total cost and network reliability, that the higher the reliability requirement, the greater the cost of design. The high reliability of network increases the performance of network at normal conditions.

Uncertainty-based optimization has been shown to provide an approximation to the reliability-based optimization using the Monte Carlo simulation. Generally, it requires less computational time when compared with Monte Carlo simulation. Also, the optimal cost estimated by the Monte Carlo approach is higher than that of the Chance Constraint and that these differences are moderate and can be attributed to the value of Y in Eq. (22) which could take positive and negative values according to the normal distribution and depending on the required reliability, it could be greater than $\phi^{-1}(1-\alpha_j)$ in Eq. (17).

NOMENCLATURE

$C_{i,j}$	Hazen-Williams roughness coefficient for pipe connecting nodes i, j
C_P	penalty cost
C_T	total cost
$D_{i,j}$	diameter of pipe connecting nodes i, j in the system (m)
$D_{\max}$	maximum diameter, (m)
$D_{\min}$	minimum diameter, (m)
E_p	energy supplied by a pump, (m)
H_j	pressure head at node j, (m)
$H_{j,\min}$	minimum required pressure head at node j, (m)
h_f	head loss due to friction in a pipe, (m)
K	conversion factor which accounts for the system of units used, Eq. (6)
$L_{i,j}$	length of pipe connecting nodes i, j, (m)
M	total number of nodes in the network
N	total number of pipes
N_s	total number of Monte Carlo simulations
$P(\ )$	probability
Q_j	discharges into or out of the node j, (m^3/s)
$q_{i,j}$	flow in pipe connecting nodes i, j, (m^3/s)
R	network reliability
$R_{estimated}$	estimated system reliability
R_N	nodal capacity reliability:
$R_{required}$	required system reliability
Z	objective function

Greek Symbols

α_j	probability level for the node demands
ϕ	cumulative distribution function
μ_Q	mean of random variable Q, (m^3/s)
σ_Q	standard deviation of random variable Q, (m^3/s)

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