HYDRAULIC ANALYSIS OF TAPERED AND INCLINED SPRINKLER IRRIGATION LATERALS

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ABSTRACT

Pipelines with multiple outlets are used extensively for irrigation under various types of surface, sprinkler and trickle irrigation systems. In this research, a lateral pipeline with two diameters (tapered), laid on horizontal, uphill and downhill slopes that usually faced in design were investigated using back stepwise method. Effect of different parameters such as variation of friction factor along the pipeline, variation of discharge with head and velocity head on the total head loss and pressure along the pipeline was studied. Location of average operating pressure head along the pipeline for different slopes and tapering length ratios, which is important for computing the inlet and end pressures head, was estimated. Furthermore, empirical equations for estimating the ratio of head loss at location of average pressure head to the total head loss were developed. Then the pressure head at the upstream end or downstream end can be computed using friction correction factor method. The results showed that neglecting the effect of variation of friction factor underestimate the value of head loss along the lateral and neglecting the effect of variation in discharge overestimate the value of head loss, therefore, neglecting effects of the two parameters at the same time leads to a value of head loss very near to the value of exact solution. The velocity head have small effect on the head loss and can be neglected at sprinkler laterals but it has effect on the total head. Finally, example is given to show how to use the proposed empirical equations at inlet and end pressure computation.

Keywords: laterals, sprinkler irrigation, manifold, uphill, downhill, tapered.

INTRODUCTION

Pressurized pipelines with multiple outlets are used extensively in irrigation (sprinkler, trickle or gated irrigation laterals and manifolds). A detailed hydraulic analysis of multiple outlets pipelines is very important for design and evaluation purposes. Although the increasing progress in computer technology has led to the development of various numerical methods, however, simple but sufficiently accurate analytical methods remain an attractive solution for routing engineering applications.

Numerous procedures have been developed for the design of lateral as presented by Hathoot et al. [1,2]; Scaloppi and Allen [3]; Yitayew et al. [4]; Mizyed [5], and others, considering the flow in a lateral to be steady and spatially varied flow in the downstream direction.

Jain et al. [6] presented a simple method for designing single, paired, and tapered laterals using a lateral discharge equation approach. Vallesquino and Laque-Escamilla [7] presented an alternative approach based on the design concept in which the discharge through the outlets is discrete and non-constant, for hydraulic calculation in simple or set-connected laterals. Valiantzas [8] presented an improved analytical method, which is the extension to the previous modified energy-line approaches, Valiantzas [9]. Anwar [10] developed a friction correction factor for laterals with outlets and outflow at the downstream end of lateral. Anwar [11] demonstrated the application of this friction correction factor to calculate friction head loss in tapered laterals. Recently, Yildirim and Ağiralioğlu [12] presented a simplified analytical solution that takes into account the effect of the emitter discharge exponent on the hydraulic computations of tapered micro-irrigation laterals. Mostafa [13] pointed out that constant pipe diameter is suitable for the design of both lateral and manifold in drip irrigation, where spacing of outlets is relatively small and discharge is small. However, varying pipe diameter is suitable for the design of both lateral and/or manifold in sprinkler irrigation, where, the spacing of outlets is relatively long and discharge is large.

It is essential to compute the inlet pressure head for obtaining the required average outlet pressure head that will result in the design discharge. Keller and Bliesner [14] graphically presented the relationship between the average outlet pressure, frictional head loss and the inlet pressure for an example. Based on the example they suggested that value of the factor K as 0.75 and 0.63 for single-diameter and two diameter pipelines, respectively, where K is the ratio of head loss at position of average pressure head to the total head loss along the lateral. Further, they suggested taking the factor K as 0.5 for pipelines of more than two diameters. Mahar and Singh [15] presented an analytical proof of the factor K. It is normally assumed that the average pressure head lies at about two thirds of the total length from the distal end of the pipeline (Perold [16]), this may be correct for horizontal pipeline. However, much attention has not been paid by the investigators to inclined pipeline and outflow at the end of pipeline. Anwar [17] developed an expression relating lateral inlet pressure head to required average pressure head and friction head losses for horizontal tapered laterals.

The difficulty in design of lateral pipeline arises from the fact that the design criteria of lateral is based on the average operating pressure which its position unknown prior to design, i.e. the upstream and downstream boundary conditions are unknown. This requires trail and error in the analysis. In the previous methods, one or more of the following assumptions are made:

- The outlets are equally spaced and of uniform discharge.
- The pipe friction factor remains constant along the pipe length.
- The velocity head is neglected.
- There is no outflow at the end of pipeline.

In the following analysis, effect of each one of these assumptions on the head loss and pressure head distribution along lateral pipeline will be considered separately and then the global effect of all parameters will be considered. Results obtained from the simulation experiments were later used to calibrate regression equations for the prediction of the location of the average pressure head along the lateral pipeline and values of head loss at that location, which is essential for computing the pressure head at beginning and end of lateral. Also, a tapered pipeline and effect of pipeline inclination will be considered in the analysis, where multiple size design can provide better uniformity since the energy gradient line can be approximated as straight line, Barragan and Wu [18].

HYDRAULIC THEORY

Considering a sprinkler lateral with N outlets, these outlets are numbered from 1 to N along a lateral line (Figure 1). Outlet number 1 lies at the downstream end of the lateral with unknown head (h_{end}). Figure 2 shows definition sketch of variables.

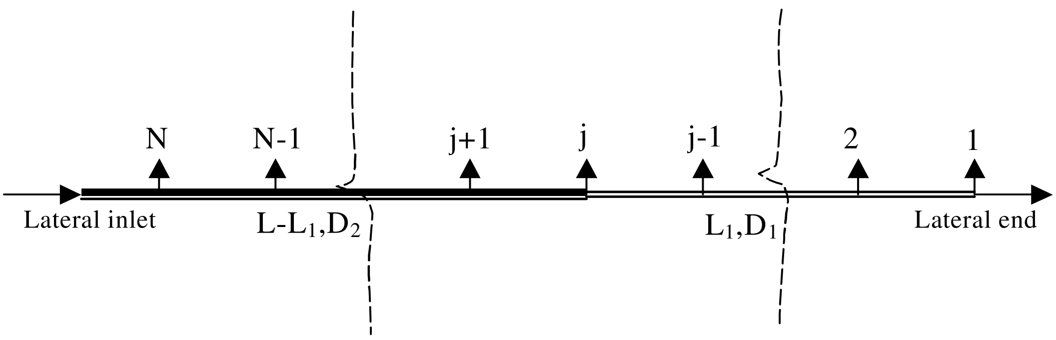

Fig. 1: A sprinkler pipeline with N outlets.

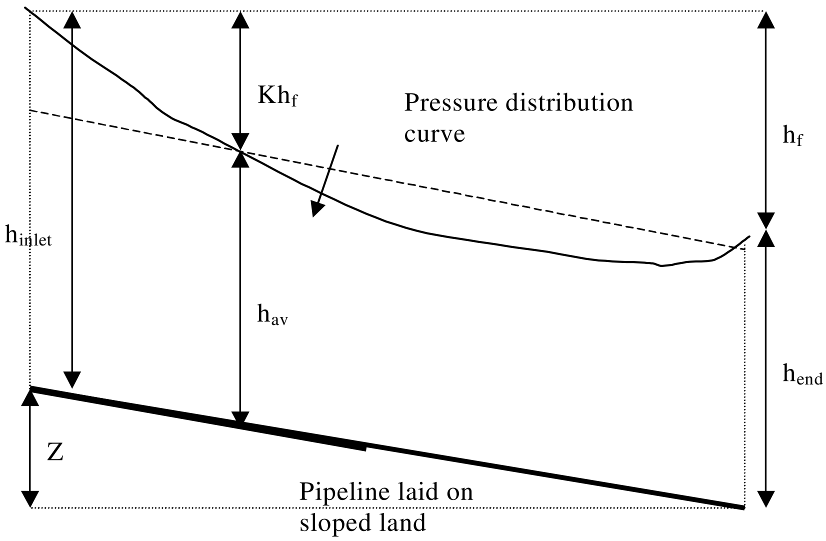

Fig. 2: Definition sketch of variables.

Discharge from any outlet (j) is given by

$$q_j = q_{av}\left(\frac{h_j}{h_{av}}\right)^{0.5} \tag{1}$$

where q_j = outlet discharge, h_j = hydraulic head at outlets, q_{av} = average operating outlet discharge and h_{av} = average operating outlet head.

The flow in lateral segment (j)

$$Q_j = \sum_{i=1}^{i=j} q_j \tag{2}$$

The head at any outlet (j) can be estimated by applying Bernoulli's equation between the end outlet and the outlet (j), which can be written as

$$h_{end} + Z_{end} = h_j + \frac{V_j^2}{2g} + \sum_{i=1}^{i=j} h_f + Z_j \tag{3}$$

The riser height is omitted because it is equal in both sides.

The average pressure head can be approximated by the following relation, Anwar [17];

$$h_{av} = \frac{1}{N}\sum_{1}^{N} h_j \tag{4}$$

The Darcy-Weisbach equation which is more appropriate than the Hazen-Williams one (Liou [19]) is used to calculate head loss caused by friction at segment (j) given by

$$h_{f_j} = f_j \frac{l}{D}\frac{V_j^2}{2g} \tag{5}$$

where $h_{f\,j}$ = head loss caused by friction at segment (j); V_j = mean velocity of flow at segment (j); g = acceleration caused by gravity and l = pipe segment length. The Churchill equation (Churchill [20]) is used to calculate the friction factor f given by

$$f = 8\left[\left(\frac{8}{R_e}\right)^{12} + \frac{1}{(\alpha+\beta)^{1.5}}\right]^{1/12} \tag{6}$$

where R_e = Reynolds number, and α and β = coefficients, given by

$$\alpha = \left\{2.457\ln\left[\frac{1}{\left(\frac{7}{R_e}\right)^{0.9} + 0.27\left(\frac{\varepsilon}{D}\right)}\right]\right\}^{16} \tag{7}$$

$$\beta = \left(\frac{37{,}530}{R_e}\right)^{16} \tag{8}$$

where ε = the pipe roughness height, and D = the pipe diameter.

Procedures of Solution

The following procedures were adopted for solution.

1. Assume a value for the end head ($h_{end}<h_{av}$)
2. Compute the discharge at first outlet at the end of pipeline using Eqn. (1).
3. Compute the discharge in the first segment beginning from the end of pipeline which equal to the discharge of first outlet.
4. Compute the friction factor and mean velocity in the segment using Eqns. from 6 to 8.
5. Compute the head loss through the segment using Eqn. (5).
6. Compute the head at beginning of the segment using Eqn. (3).
7. Compute the outlet discharge in the second outlet using Eqn. (1).
8. Compute the discharge in the following segments using Eqn. (2).
9. Repeat steps from 4 to 8 till arrive the beginning of the pipeline.
10. Check that the computed average head equal to the given value else assume a new value for the pressure head at end of pipeline and repeat steps from 1 to 9.
11. Find the location of the average head along the pipeline and value of head loss at that location.

ANALYSIS AND DISCUSSION OF RESULTS

A series of accurate simulation experiments generating pressure head distributions along multi-diameter laterals were performed to explore the effect of different parameters on this distribution. An aluminum sprinkler lateral of total length of 324 m; and two diameters D_1=75 mm and D_2=100 mm is considered in the analysis, similar to example given by Anwar [17]. The lateral relative roughness is taken as 0.127 mm for water at 15^{o}c, kinematics viscosity of water 1.14*10^{-6} m^2/s. The required average flow of outlets is changed three times as q_{av} =0.375, 0.5 and 0.75 liter/s, the required operating pressure head is h_{av} =35 m and the spacing between outlets is changed three times 9, 12 and 18 m respectively. Several combinations of pipeline slope (S_o) and tapering length ratio (L_1 / L) were used to simulate the head loss and the pressure head distribution in multi-diameter laterals using a stepwise numerical procedure, where the pipeline slope is changed five times from –1.0% to +1.0% and the small diameter length to total lateral length ratio (L_1 / L) is changed eight times from 0.0 to 1.0. According to the stepwise numerical procedure, the head losses, the changes in discharge and the velocity head were calculated individually for all segments between outlets from the downstream end up to cover the total length. The total number of runs is equal to 88 runs and each run is repeated five times according to the following schemes.

- **Constant outflow and friction factor along the lateral pipeline:**

The outlet discharge is taken constant for all sprinklers and equal to q_{av}, the friction factor is taken as constant for all the length of pipe which have the same diameter and the second term on the right hand side of Eqn. (3) is neglected i.e. the velocity head.

- **Variable outflow discharge along the lateral pipeline:**

The outlet discharge for each sprinkler is computed using Eqn. (1), the friction factor is taken as constant for all the pipe which have the same diameter and the second term on the right hand side of Eqn. (3) is neglected i.e. the velocity head.

- **Variable friction factor:**

The outflow discharge is constant for all sprinklers, the velocity head is dropped from Eqn. (3), and the friction factor for each segment is computed using Eqns. from 6 to 8.

- **Velocity head effect:**

The outlet discharge and friction factor are constant and the velocity head in Eqn. (3) is taken into account.

- **Case of all effects:**

Steps from 1 to 11 mentioned above are applied.

1. Effect of Variation of Different Parameters on Head Loss and Total Head

Figure (3) presents a comparison of the five schemes mentioned above for horizontal one diameter pipeline at number of outlets equal to 27, where value of head loss (h_f) in meter is drawn versus horizontal distance as a dimensionless value. It can be shown that head loss at variable outflow is the lower while the head loss at variable friction factor is the higher. The head loss with velocity head effect coincides with head loss with constant outflow and friction factor. When all the parameters mentioned above are taken into account, gives a value of head loss in between of them.

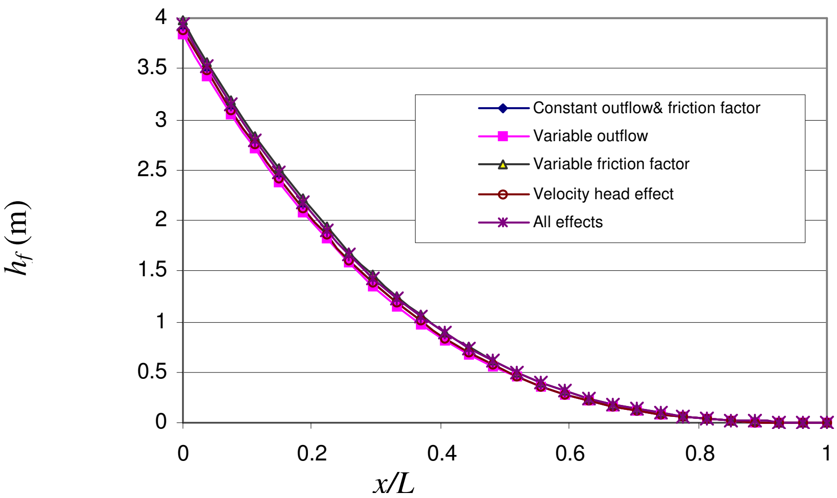

Fig. (3): Values of head loss versus dimensionless horizontal distance for different schemes analysis at horizontal pipeline (S_0=0), number of outlets (N)=27 and L_1/L=0.

Figure (4) shows a comparison between the five schemes for horizontal two-diameter pipeline where L_1/L=0.37. It is noticeable that effect of different parameters on the head loss is significant at two-diameter pipeline than that of one-diameter pipeline and maintains the same trend shown for single diameter pipeline.

To show the effect of variation of outflow discharge, friction factor and velocity head on the pressure head distribution along the pipeline, Figs. (5,6) show the variation of pressure head along the pipeline for horizontal pipe with one and two diameters pipe respectively. It can be shown that these parameters underestimate the total head at the end of pipeline and overestimate the head at the beginning of pipeline. Also, it is noticeable that the pressure head distribution for one diameter pipe differs than that of two diameters pipe, where there is a singularity at change diameter point. From these Figures, it is noticeable that the L_1/L ratio has a great effect on the location of average pressure head.

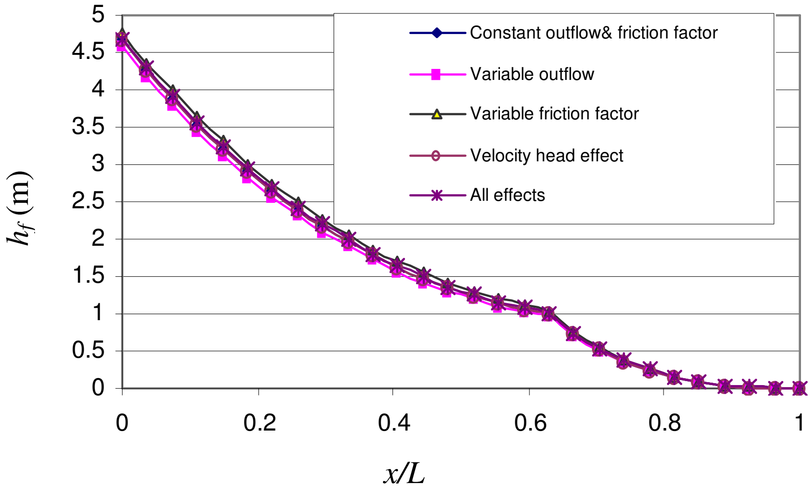

Fig. (4): Values of head loss versus dimensionless horizontal distance for different schemes analysis at horizontal pipeline (S_0=0), number of outlets (N)=27 and L_1/L=0.37.

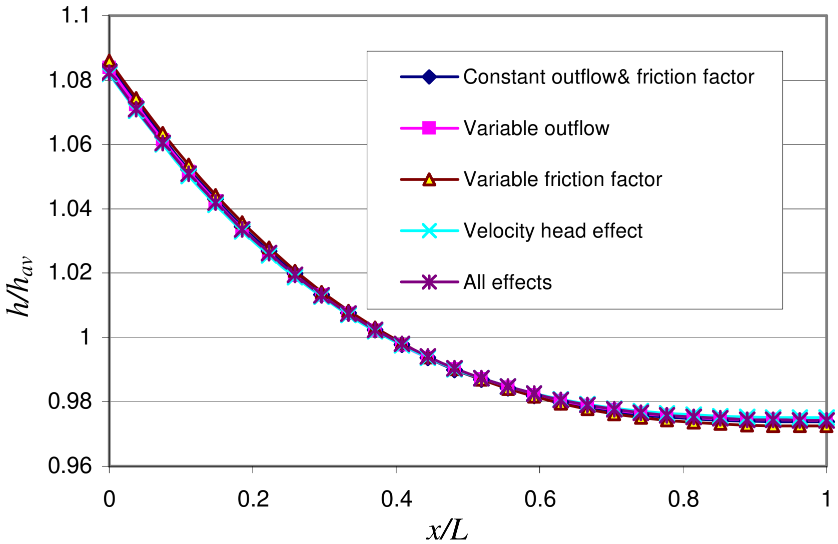

Fig. (5): Values of h/h_{av} versus dimensionless horizontal distance for horizontal pipeline (S_0=0), number of outlets (N)=27 and L_1/L=0.

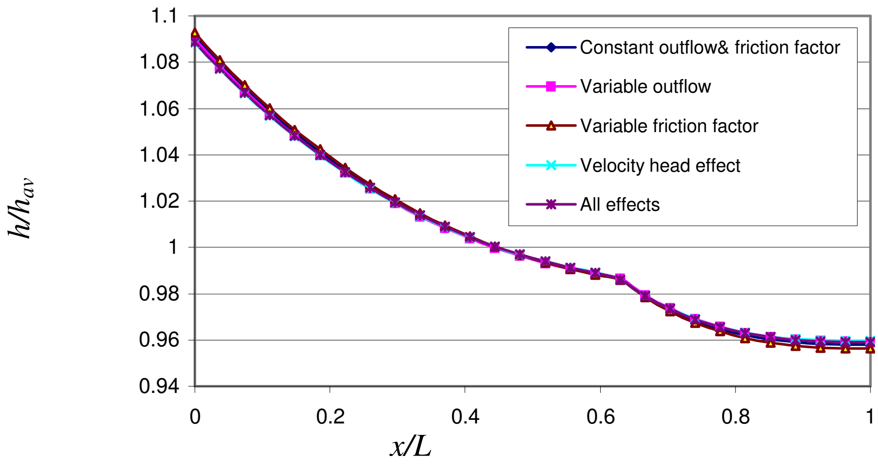

Fig. (6): Values of h/h_{av} versus dimensionless horizontal distance for horizontal pipeline (S_0=0), number of outlets (N)=27 and L_1/L=0.37.

2. Effect of Slope on Head Loss and Total Head along Lateral

To show effect of variation of outflow, friction factor and velocity head on the head loss along the pipe at presence of inclination in the pipeline, Figs. (7,8) show the variation of head loss with distance for downhill (S_0=-0.01) and uphill pipeline (S_0=+0.01) respectively, for two-diameter pipeline (L_1/L=0.52). It can be noticeable that the variation in head loss for the different schemes is large especially for the part of pipe of larger diameter but still have the same trend of horizontal pipeline.

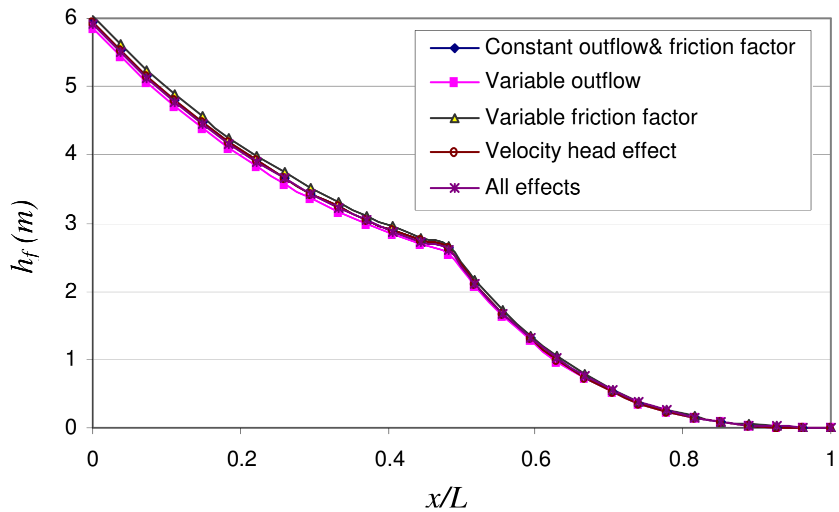

Fig. (7): Values of head loss versus dimensionless horizontal distance at inclined pipeline (S_0=-0.01), number of outlets (N)=27 and L_1/L=0.52.

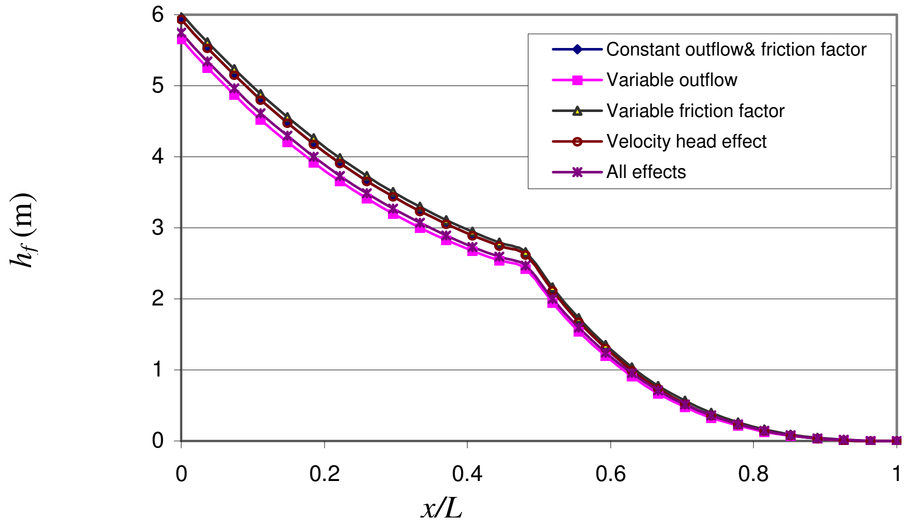

Fig. (8): Values of head loss versus dimensionless horizontal distance at inclined pipeline (S_0=+0.01), number of outlets (N)=27 and L_1/L=0.52.

Figures (9, 10) show the variation of pressure head as dimensionless value (h/h_{av}) along the pipeline laid on slopes S_0=-0.01 and +0.01, respectively, at L_1/L=0.52. It is indicated that for downhill sloping laterals the errors in pressure head computed by the simplified method is significant at sloped lateral, while, the error for uphill-sloping laterals is small. Also, the location of maximum, minimum and mean pressures depends on the slopes of the lateral lines and the effect is large for downhill pipeline.

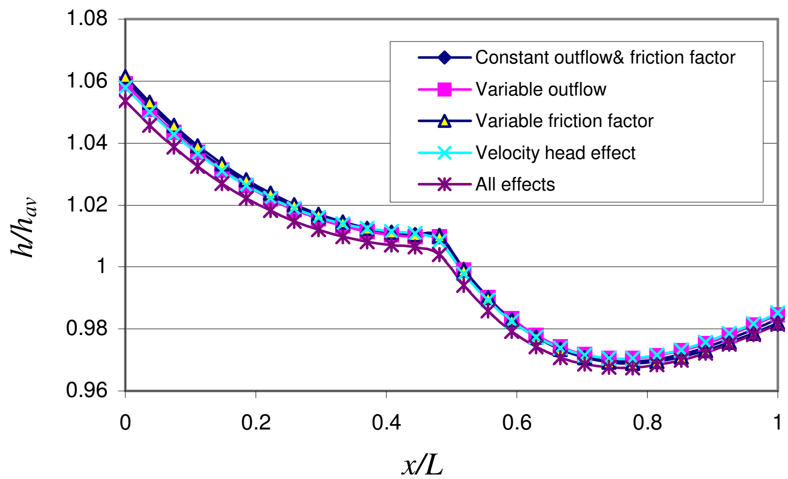

Fig. (9): Values of h/h_{av} versus dimensionless horizontal distance at inclined pipeline (S_0=-0.01), number of outlets (N)=27 and L_1/L=0.52.

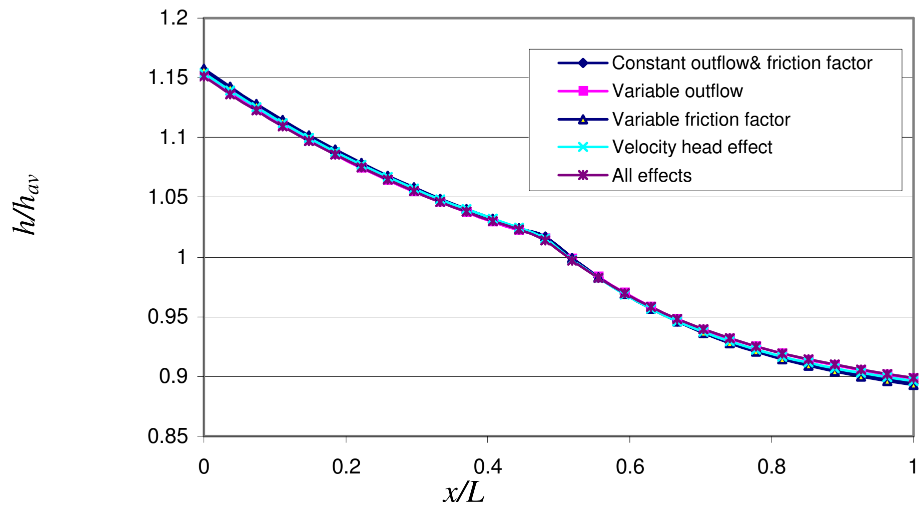

Fig. (10): Values of h/h_{av} versus dimensionless horizontal distance at inclined pipeline (S_0=+0.01), number of outlets (N)=27 and L_1/L=0.52.

3. Location of Average Pressure Head

The location of average pressure head from the pipeline inlet, minimum pressure head and maximum pressure head are important in design purposes. The uniformity of sprinkler flow or water pressure is determined using simple pressure ratios such as minimum to maximum $h_{\min}/h_{\max}$ and minimum to average $h_{\min}/h_{av}$. Besides, simple pressure ratios or sprinkler flow ratios can be used as a design criterion for hydraulic design for sprinkler irrigation system design. In the following, the factors which affect location of average pressure head will be discussed and a method for estimating it will be developed.

Horizontal and Uphill Sloped Pipelines ($S_0 \geq 0$)

Figure (11) shows values of location of average pressure head from the inlet of pipeline $(x/L)_{hav}$ versus ratio of small diameter pipe length to the total length of the pipe (L_1/L) at horizontal pipeline for different number of outlets. It is noticeable that values of $(x/L)_{hav}$ increases by increasing of (L_1/L) ratio till arrive a maximum value then begin to decrease till arrive a value nearly equal to the beginning value at (L_1/L) equal to 1.0. Also, the number of outlets has a small effect on the location of average pressure head.

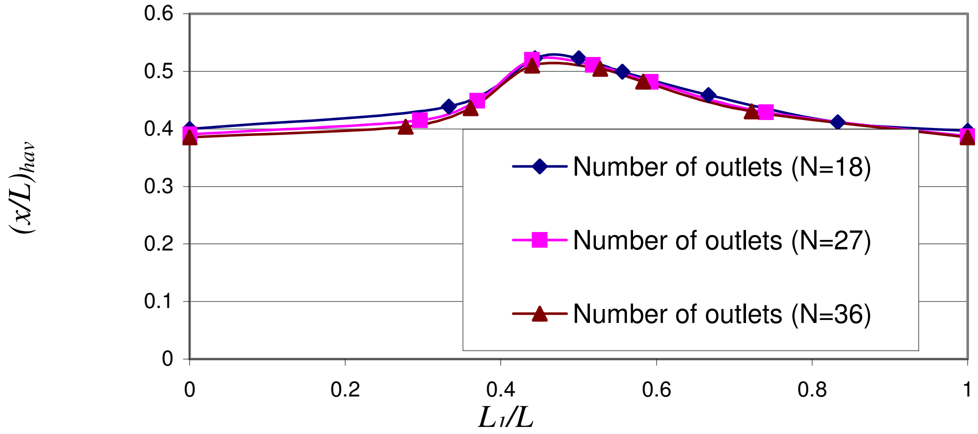

Fig. (11): Variation of location of average pressure head with L_1/L ratio at horizontal pipe (S_0=0.0) for different number of outlets.

Figure (12) shows values of $(x/L)_{hav}$ versus L_1/L ratio at uphill inclined pipeline (S_0=+1.0%) for different number of outlets. It is noticeable that the relation takes the same trend as horizontal pipe but with higher values i.e. the location of average pressure depends on the slope of lateral lines and number of outlets have small effect on the location of average pressure head which can be neglected.

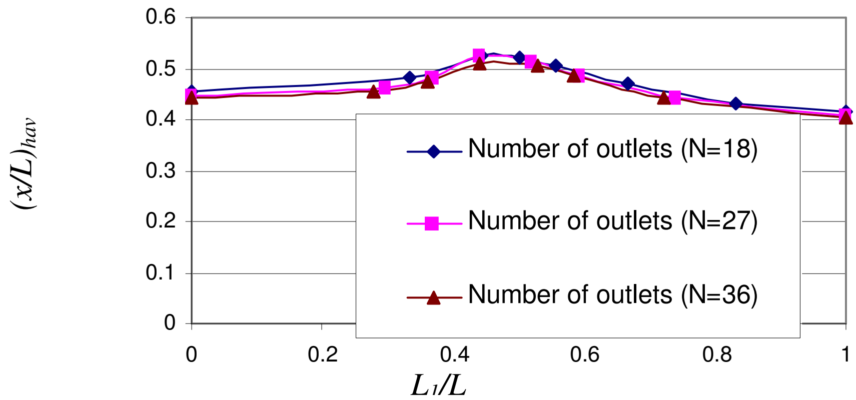

Fig. (12): Variation of position of average pressure head with L_1/L ratio at uphill inclined pipe (S_0=+1.0%) for different number of outlets.

The numerical results for each slope (S_0) are given in Fig. (13) where $(x/L)_{hav}$ values are plotted versus S_0 values. The data for each value of S_0 clustered around a curve which may be expressed in the form;

$$(x/L)_{hav} = c_1 + c_2\left(\cos(L_1/L - c_3)180/\pi\right)^{c_4} \quad (9)$$

where c_1, c_2, c_3 and c_4 are coefficients depending on pipeline inclination S_0. The coefficients c_1, c_2, c_3 and c_4 obtained are listed in Table (1).

Table (1): Values of the coefficients c_1, c_2, c_3 and c_4 in Eqn. (9)

S_0	c_1	c_2	c_3	c_4	R^2
0.0	0.39	0.124	0.51	66.13	0.97
+0.5%	0.40	0.119	0.50	45.41	0.98
+1.0%	0.43	0.089	0.48	60.47	0.94

Apparently the coefficients c_1, c_2, c_3 and c_4 are proved to be dependent mainly on the inclination of pipeline S_0. So, the correlation between c_1, c_2, c_3 and c_4 and S_0 my be given in the form;

$$c_1 = 400(S_0)^2 + 0.39 \quad (10a)$$

$$c_2 = -500(S_0)^2 + 1.5(S_0) + 0.124 \quad (10b)$$

$$c_3 = -200(S_0)^2 - (S_0) + 0.51 \quad (10c)$$

$$c_4 = 715600(S_0)^2 - 7722(S_0) + 66.13 \quad (10d)$$

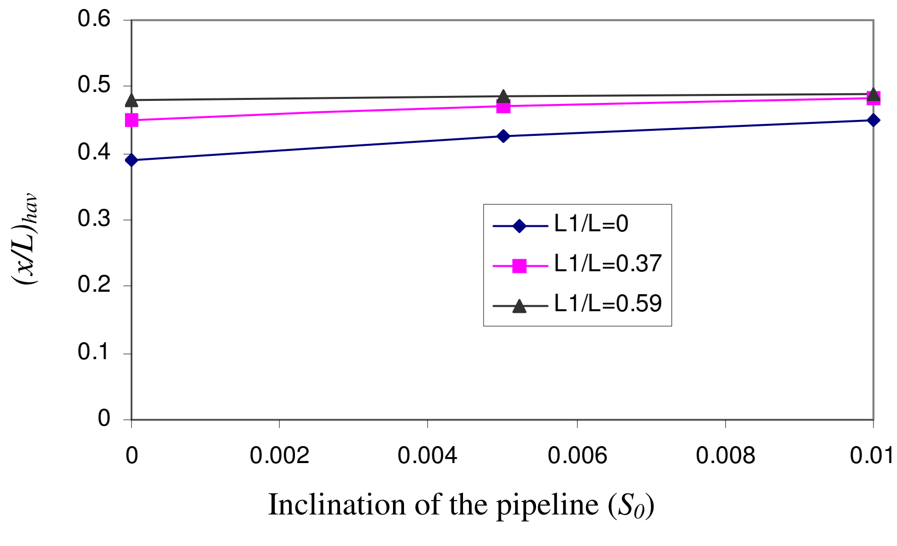

Fig. (13): Values of $(x/L)_{hav}$ versus S_0 for different values of L_1/L ratio.

Downhill Sloped Pipeline ($S_0 < 0$)

For lateral pipeline laid on downhill slope the minimum pressure head is not necessary lie at the end of the pipeline, so the distribution of pressure head along the pipeline is different than that of horizontal and uphill slopes. Figure (14) shows the relation between location of average pressure head and L_1/L ratio for downhill-sloped pipeline (S_0=-1.0%) at number of outlets equal to 18, 27 and 36 respectively. It can be shown from this figure that there are unsymmetrical in relation i.e. value of $(x/L)_{hav}$ at L_1/L equal to zero differs than that at L_1/L equal to one. However, in practice, values of L_1/L usually are higher than 0.4. So, location of average pressure head versus L_1/L at L_1/L> 0.4 can be represented in a power relation shown in Fig. (15) and Eqn. (11).

$$(x/L)_{hav} = 0.3635(L_1/L)^{-0.48} \qquad R^2=0.975 \qquad (11)$$

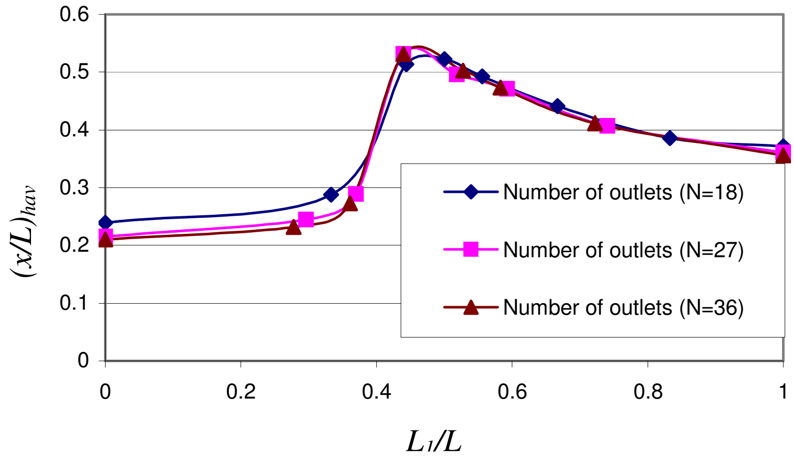

Fig. (14): Variation of Location of average pressure head $(x/L)_{hav}$ with L_1/L ratio for downhill inclined pipeline (S_0=-1.0%) at different number of outlets.

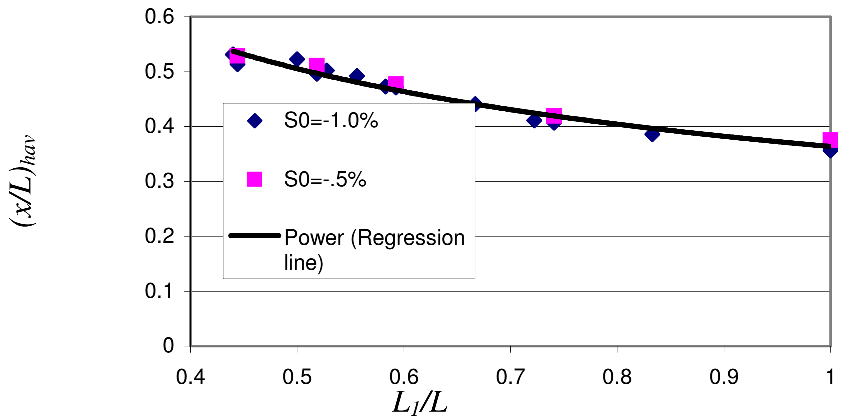

Fig. (15): Variation of Location of average pressure head $(x/L)_{hav}$ with L_1/L ratio for downhill inclined pipeline (S_0=-1.0, -0.5%).

4. Head Loss Value at Location of Average Head

The inlet pressure of a lateral is an input parameter for the design of submain/main lines of sprinkler systems. This section presents the development of expressions for the factor K, which is the ratio of head loss at location of average pressure head along a lateral to the total head loss at the end of the lateral as shown in Fig. (1), for any number of outlets, inclination slope and ratio of tapered length to the total length of lateral L_1 / L.

Horizontal and Uphill Sloped Pipelines ($S_0 \geq 0$)

Figure (16) shows the Variation of the factor K with L_1 / L for different values of pipeline inclination slope, S_0, higher than or equal to zero. The data for each value of S_0 are grouped around a curve that may be expressed in relation of the form;

$$K = C_1\left(\frac{L_1}{L}\right)^3 - C_2\left(\frac{L_1}{L}\right)^2 - C_3\left(\frac{L_1}{L}\right) + C_4 \tag{12}$$

where C_1, C_2, C_3 and C_4 are coefficient depending on the inclination of the lateral pipeline. Values of these coefficients are shown in Table (2) and may be given in the following forms;

$$C_1 = -3600 S_0^{\,2} + 42 S_0 + 0.77 \tag{13a}$$
$$C_2 = -3600 S_0^{\,2} + 26 S_0 + 0.73 \tag{13b}$$
$$C_3 = 16 S_0 + 0.04 \tag{13c}$$
$$C_4 = -400 S_0^{\,2} + 12 S_0 + 0.75 \tag{13d}$$

Table (2): Values of the coefficients C_1, C_2, C_3 and C_4 in Eqn. (12)

S_0	C_1	C_2	C_3	C_4	R^2
0.0	0.77	0.73	0.04	0.75	0.95
+0.5%	0.89	0.77	0.12	0.80	0.94
+1.0%	0.83	0.63	0.20	0.83	0.93

Downhill Sloped Pipeline ($S_0 < 0$)

Figure (17) shows the Variation of the factor K with L_1 / L (larger than 0.4) for different values of pipeline inclination slope, S_0, at downhill sloped lateral. The data for all values of S_0 are grouped around a curve which may be expressed in relation of the form;

$$K = 0.9\left(\frac{L_1}{L}\right)^3 - 0.83\left(\frac{L_1}{L}\right)^2 - 0.14\left(\frac{L_1}{L}\right) + 0.8 \qquad R^2=0.89 \tag{14}$$

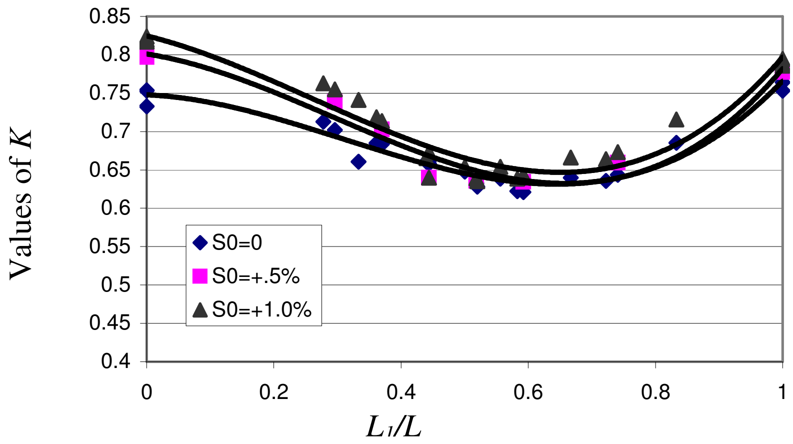

Fig. (16): Variation of K with L_1/L for different values of pipeline inclination slope.

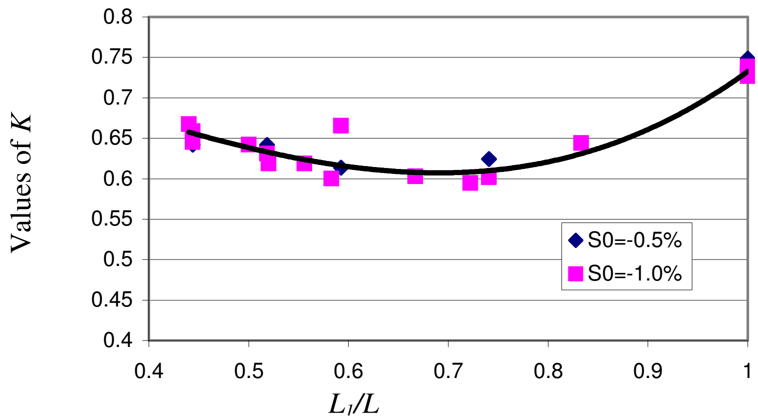

Fig. (17): Variation of K with L_1/L for different values of downhill pipeline inclination slope.

To show the applicability of the above equations, practical example is given in the following to compute the inlet pressure head of lateral pipeline.

Example

Compute the inlet pressure head using the present method for a two diameter aluminum sprinkler lateral laid uphill with the given data as q_{av} =0.8 liter/s, N=24, l=12 m, Z=3.6 m, h_f=2.609, h_{av} =31 m, D_1=100 mm, L_1=60 m, D_2=125 mm and h_r (riser height)=1.0 m.

Solution

Lateral pipeline inclination (S_0) = Z/L=0.0125

L_1/L=60/288=0.208

Using Eqns. 9 and 10; $(x/L)_{hav}$=0.494

Using Eqns. 12 and 13; K **=0.77**

$$h_{inlet} = h_{av} + S_0 L \left(\frac{x}{L} \right)_{hav} + Kh_f + h_r = 35.787 \text{ m}$$

This value is very near to the value computed by Mahar and Singh [15] for the same example which equal to 35.731. The difference is mainly because of considering the location of average pressure head at the mid of pipeline at [15].

CONCLUSIONS

A need for more understanding of the contribution of variation of discharge along lateral pipeline, variation of friction factor, velocity head to the total head loss is important. Therefore, an effort is made in this study to investigate the significance effect that will result. Investigations of effect of variation of outlets discharge, friction factor and velocity head along a lateral pipeline on the total head loss have shown that there is a significant effect due to these variations. Generally, variation of outlets discharge decreases the total head loss, while variation of friction factor increases the total head loss. As a result of this investigation, a simple method to estimate the location of average pressure head is developed for both horizontal and inclined tapered pipelines. Expressions were developed to compute the head loss at location of average pressure head at horizontal, uphill and downhill tapered pipeline. For horizontal and uphill pipeline, the location of average pressure head and head loss are affected by the inclination and tapering length ratio. For downhill pipeline, the location of average pressure head and head loss at it are independent of inclination of lateral pipeline. A practical procedure to estimate inlet and end pressures for horizontal, laid on uphill slop and laid on downhill slope pipeline is suggested.

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