PREDICTION OF NON-NEWTONIAN TURBULENT FLOW BEHAVIOR BY A NEWTONIAN APPROACH

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ABSTRACT

One of the most important practical problems in non-Newtonian fluids flowing in pipes is the reliable prediction of the pressure drop accompanying the flow. For laminar flow, this problem is well established as the relation between the pressure drop and mean velocity can be derived by integration of the representative rheological model. However, the prediction of the pressure losses in the turbulent regime is still requiring more theoretical and practical analysis. The analysis in this paper has been focused on the observed phenomena that the turbulent flow behaviour appears unrelated to the laminar rheological characteristics and it could be found similar to Newtonian turbulent flow behaviour.

The aim of study in this paper is to find a theoretical approach that could match the experimentally investigated turbulent flow behavior of non-Newtonian slurries flowing in pipes. The suggested approach is based on the similarity of non-Newtonian turbulent flow behaviour with that of Newtonian one. Dependence of turbulent flow behaviour on the laminar rheological characteristics and also will be discussed. So, the experimental measurements of kaolin slurries at different concentrations have been analyzed and compared with the suggested approach.

Keywords: non-Newtonian, slurries, turbulent, rheology

INTRODUCTION

Slurry of small particles (less than 75 μm, [1]) can behave in a homogeneous fashion, with the particles distributed throughout the whole flow field and essentially no change in solids concentration with height. When increasing solids content the mixture can no longer be regarded as two individual components and the resulting fluid has characteristics which can be quite different from those of Newtonian fluids implying a more complicated relation between the shear stress and shear rate. Non-Newtonian fluids are distinguished from Newtonian fluids in that the viscosity is dependent upon the rate at which the fluid is sheared; hence the use of a single viscosity is no longer appropriate. Instead, empirical relations are fitted to the rheological measurements plotting the rheograms. When mathematical relations are used to approximate the experimentally determined rheograms, they are known as rheological models, and these models can be employed to drive relations linking the pressure gradient along a pipe to the discharge. However, such relations are approximations to the actual behaviour of the fluid and should not be used outside the range of conditions (particularly shear rates) for which they were determined.

One of the most important practical problems in non-Newtonian fluids flowing in pipes is the reliable prediction of the pressure drop accompanying the flow. For laminar flow, this problem is well established as the relation between the pressure drop and mean velocity can be derived by integration of the representative rheological model, see El-Nahhas [2] and El-Nahhas et al. [3]. However, the prediction of the pressure losses in the turbulent regime stills one of the difficult theoretical and practical problems. This difficulty results from that the turbulent flow behaviour could be found independent on the laminar rheological characteristics and it is similar to Newtonian turbulent flow behaviour, see El-Nahhas et al. [4].

The aim of study is to describe the turbulent flow behaviour of non-Newtonian slurries flowing in pipes. The similarity of non-Newtonian turbulent flow behaviour with that of Newtonian one has been taken as a basis. Dependence of turbulent flow behaviour on the laminar rheological characteristics will be discussed. So, the experimental measurements of kaolin slurries at different concentrations (ranging from Cv = 2.8% to 22.6%), tested in a pipeline test loop (D = 17.5 mm) in both laminar and turbulent regimes, have been analyzed. A Newtonian approach for predicting the turbulent flow behaviour of non-Newtonian slurries is suggested and compared with the experimental results.

SLURRY RHEOLOGICAL CHARACTERISTICS

The simplest rheological model is the Newtonian model with a single rheological parameter, the viscosity μ. The viscosity is constant at a given temperature and pressure. It is represented by:

$$\tau = \mu\,\gamma = \mu\,\frac{du}{dy} \tag{1}$$

For a non-Newtonian fluid, there is no single value of viscosity, which is a function of the rate at which the fluid is sheared. Some non-Newtonian fluid rheograms may not be a straight line while attaining the passing from origin (power-law fluids), and other may attain the straightness while doing offset from the origin having a yield stress (Bingham fluids). The yielded pseudo-homogeneous model (Herschel-Bulkley equation) is a combination between the power law and Bingham models and it is often approximates the behaviour of wide range homogeneous non-Newtonian slurries. It is given by:

$$\tau - \tau_Y = k\gamma^n \qquad (2)$$

TURBULENT FLOW OF NEWTONIAN FLUIDS

For hydraulically smooth pipes, the flow mean velocity, V, in the turbulent region of a Newtonian fluid could be approximately obtained by the logarithmic law, considering that the viscous sub-layer usually occupies a very small portion of the pipe area, as:

$$\frac{V}{U_*} = 2.5 \ln\left(\frac{U_* D \rho}{\mu}\right) \qquad (3)$$

NON-NEWTONIAN FLUIDS TURBULENT FLOW

Several theoretical models have been proposed for turbulent flow prediction treating non-Newtonian slurries as continuous-media fluids with properties which can be determined from viscometry or laminar pipe flow measurements. One of these models which has strong analytical approach, is that of Torrance [5]. The Torrance model has been derived for non-Newtonian flow in pipes using the yield pseudoplastic rheological model as starting point. The mean velocity for turbulent flow in smooth pipes is given by:

$$\frac{V}{U_*} = \frac{3.8}{n} + \frac{2.78}{n} \ln\left(1 - \frac{\tau_Y}{\tau_w}\right) + \frac{2.78}{n} \ln\left(\frac{U_*^{2-n} \rho R^n}{k}\right) - 4.17 \qquad (4)$$

Also, the predictive model which has a theoretical basis and is often very successful in predicting pressure drops in non-Newtonian turbulent flow is that proposed by Wilson and Thomas [6] and Thomas and Wilson [7]. This model is based on the hypothesis that the thickness of the viscous sub-layer would increase in non-Newtonian fluids. The thickening of the laminar sub-layer is predicted by a factor called the area ratio, which is the ratio of the integrals of the non-Newtonian and assumed Newtonian rheograms under identical shear conditions. The mean velocity is given by:

$$\frac{V}{U_*} = \frac{V_N}{U_*} + 11.6(\alpha - 1) - 2.5 \ln(\alpha) - \Omega \qquad (5)$$

The term Ω represents any effect of possible blunting of the velocity profile in the logarithmic of core regions of the flow. It depends on the ratio τ_Y/τ_w denoted by ξ, giving:

$$\Omega = -2.5\ln(1-\xi) - 2.5\xi(1+0.5\xi) \tag{6}$$

Examples of situations in which this model succeeds and some cases for which it fails are shown by Xu et al. [8]. Bartosik et al. [9] suggest that when the Wilson-Thomas model fails, the cause may arise in the assumption of a continuous fluid medium, which this method employs. The basis on the assumption that the fluid was a continuum had over the years been questioned by researchers, e.g. Slatter et al. [10].

Slatter [11] developed an alternative theory for non-Newtonian slurries based upon the concept of particle roughness turbulence effect, and is based on the classical Newtonian approach. A roughness Reynolds number was developed based upon a representative particle size, d_{85}, and the Herschel-Bulkley rheological parameters;

$$\mathrm{Re}_r = \frac{8\rho U_*^2}{\tau_Y + k\left[\dfrac{8U_*}{d_{85}}\right]^n} \tag{7}$$

If $Re_r < 3.32$ then smooth wall turbulent flow exists and the mean velocity is given by:

$$\frac{V}{U_*} = 2.5\ln\left(\frac{R}{d_{85}}\right) + 2.5\ln(\mathrm{Re}_r) + 1.75 \tag{8}$$

If $\mathrm{Re}_r > 3.32$ then fully developed rough wall turbulent flow exists and the mean velocity is given by:

$$\frac{V}{U_*} = 2.5\ln\left(\frac{R}{d_{85}}\right) + 4.75 \tag{9}$$

This approach departs from all other approaches to non-Newtonian turbulence modeling in its accommodation of the continuum breakdown in the wall region due to the physical size of the solid particles, which are present.

El-Nahhas [2] studied the similarity of the turbulent flow behaviour of non-Newtonian slurries with that of Newtonians using the logarithmic relation of Newtonian liquids (equation 3) that has been applied to fit the experimental turbulent flow data of the non-Newtonian slurries tested. The Newtonian viscosity, μ, which is meaningless for the non-Newtonian fluids, has been replaced by a characteristic shear-independent parameter C_μ. Equation 3 has been rewritten as:

$$\frac{V}{U_*} = 2.5\ln\left(\frac{U_* D \rho}{C_\mu}\right) \tag{10}$$

El-Nahhas and Vlasak [4] presented comparative plots showing that the experimental turbulent flow data for non-Newtonian (natural and peptized) kaolin slurries are fitted very well by equation 10. They showed that the single input parameter, C_μ encapsulates the effect of the solids presence and is direct proportional to the solids concentration regardless the change in the rheological characteristics and laminar behaviour caused by peptizing effect.

In the present study, the following equation is suggested to express the proportionality of the parameter C_μ on the solids concentration, i.e. mixture density:

$$C_\mu = \mu_{water} + b\,(S_m - 1) \tag{11}$$

where μ_{water} is the water viscosity, S_m is the mixture specific gravity and b is a proportional parameter. This suggested relation would be applicable for the present state of study. More experimental studies should be obtained to generalize the relation studying the effect of other different parameters.

EXPERIMENTAL FACILITY AND TESTED MATERIALS

The extended set of experiments was carried out at the hydraulic laboratory of the Institute of Hydrodynamics, Academy of Science of Czech Republic. An open-loop recirculation pipeline system was employed for studying the slurry flow behaviour. The slurry was forced by a screw pump driven by an electric motor with a speed regulator from an open storage tank to delivery pipe. The test section was located on the back branch of the pipeline and its length to diameter ratio exceeded 400. A stainless steel pipe of internal diameter 17.5 mm was used for measurements. The pipe was equipped with three pressure tappings connected through solids pods to differential Hottingger-Baldvin pressure transducers and the readings were monitored by a computer. At the downstream end of the test pipes a box divider was mounted that allows diversion of the discharge to a plastic container for weigh testing. Since the divider arm was connected to an electric stopwatch, the mass flow rate was precisely measured. The slurry density and hence the volumetric concentration could also be determined. Calibration experiments with clear water, which was periodically run, showed that the pipe used in the test section behaved as a smooth pipe. For more details, see El-Nahhas [2].

The kaolin powder, of density 2549 kg/m^3 and median diameter d_{50} = 2.8 μm, from Horni Briza (Czech Republic) was used for preparing a wide range of concentrations of homogeneous slurries. Table 1 shows mineralogy of this kaolin, which considerably influences the flow behaviour of its suspensions.

Table 1. The tested kaolin chemical composition

Compound	SiO_2	AL_2O_3	Fe_2O_3	TiO_2	CaO	MgO	K_2O	Na_2O	rest
Content %	50.33	35.06	0.67	0.93	0.07	0.2	1.38	0.09	11.27

Kaolin slurries of eight different concentrations, beginning with C_v = 2.8% up to the maximum possible concentration of C_v = 22.6%, were prepared and tested.

RESULTS AND DISCUSSIONS

1. Description of The Slurry Flow Behavior

The slurry flow behaviour can be obviously observed and described by several ways. The easiest way it could be established by analyzing the data measured by viscometry instruments (usually by rotational viscometers). By this way the result data could be used to create direct relation between the shear stress, τ_w and the shear rate, γ, i.e. the rheograms. More efficient method for describing the slurry flow behaviour can be made by analyzing the experimental data measured by a pipeline test loop. The advantage of this method is the similarity of the tested slurry flow structure with that of the actual slurry pipeline. The pipeline test loop data in the laminar region could be used to create the rheograms of the tested slurries by Rabinowitsch-Moony method, [2]. The rheogram determined by this method relates the shear rate and shear stress both calculated at the pipe wall.

El-Nahhas [2] used the two methods to present the rheograms of the tested slurries and showed that the lowest two concentrations (2.8%, 5.6%) slurries have wholly turbulent flow indicating Newtonian behaviour. The slurries of volumetric concentration 8.9% and higher showed non-Newtonian behaviour represented satisfactorily by the Herschel-Bulkley model. The rheological characteristic parameters (τ_Y, k and n) were determined. For non-Newtonian slurries, the laminar flow behaviour can be observed at the lower velocities. The laminar flow region is characterized by the low-slope flat curve in the V-i plots. As the mean velocity increases, the turbulent flow occurs having a steep V-i relation. The mean velocity, at which this behaviour transition occurs, increases as the slurry solids volumetric concentration increases and its identification is of great importance in pipeline design at which the flow behaviour change fundamentally. For dense slurries, it can also be noticed that the flow curve at the end of the laminar region could have some points on (or under) the water curve, i.e. drag reduction described by Wilson & Thomas [6] is verified.

2. Predictions of the Flow Behavior

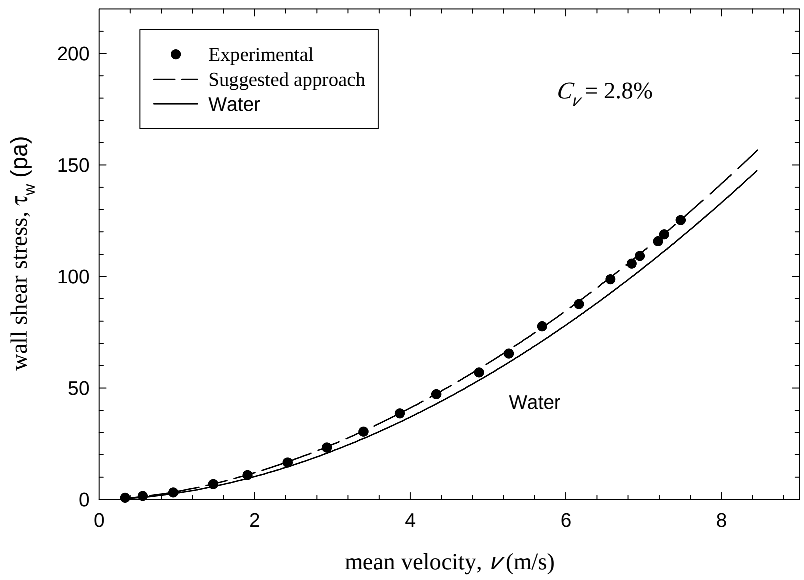
One of the most important practical problems in non-Newtonian fluids flowing in pipes is the reliable prediction of the pressure drop accompanying the flow. Non-Newtonian laminar flow behaviour could be predicted by the relation between the mean velocity and the wall shear stress resulted by successive integration of the representative rheological model. Previous studies on the tested slurries proved that there is a good agreement between the experimental measurements and the theoretical prediction for the laminar flow. The determined rheological parameters are used as input parameters in the theoretical equation; see [2] and [12]. So, the laminar flow prediction is well established, however, the prediction of the turbulent flow stills one of the difficult theoretical and practical problems. One of the major problem areas of the non-Newtonian turbulent flow modeling is that the turbulent flow behaviour of non-Newtonian suspensions could be found independent on their laminar rheological behaviour. This problem arises when using the determined rheological parameters as input parameters for applying the theoretical models of Torrance [5], Wilson and Thomas [6] and Slatter [11] discussed above. It could be obviously found that the rheological parameters that well fit the laminar behaviour are not suitable for turbulent flow predictions by the different models.

3. Turbulent Flow Prediction by the Suggested Approach

The suggested approach that is discussed above (equations 10 and 11) is applied on the tested slurries. In this study, the studied kaolin slurries have been divided to three groups; Newtonian, low-concentrated non-Newtonian and high-concentrated non-Newtonian slurries. Figure 1 presents a comparison between the experimental data and the theoretical predictions by the suggested approach applying equations 10 and 11.

It could be noticed that the water curve in all presented figures is plotted using the same approach and verified by experimental measurements. Figure 2 presents the same comparisons for low-concentrated non-Newtonian slurries. For both Newtonian and low-concentrated non-Newtonian slurries the experimental data are very well fitted by the suggested approach giving the parameter, b, in equation 11 the value 0.003. However, the parameter, b, takes the value 0.0047 for high-concentrated slurries presented in Figure 3. Because the laminar/turbulent transition velocity increases as the slurry concentration increases, it is noticed that the turbulent flow experimental data lied in different velocity range according to the slurry concentration.

Referring to the Figures 1-3 the suggested approach, based on the similarity of the turbulent flow behaviour of both Newtonian and non-Newtonian slurries, is successfully predict the turbulent flow of slurries with wide concentration range implying both Newtonian and non-Newtonian behavior.



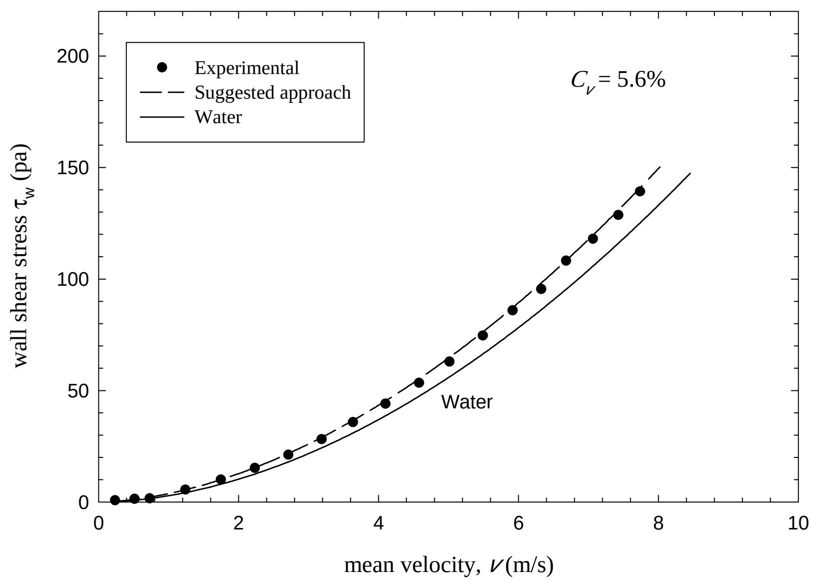


Figure 1. Turbulent flow prediction of Newtonian kaolin slurries

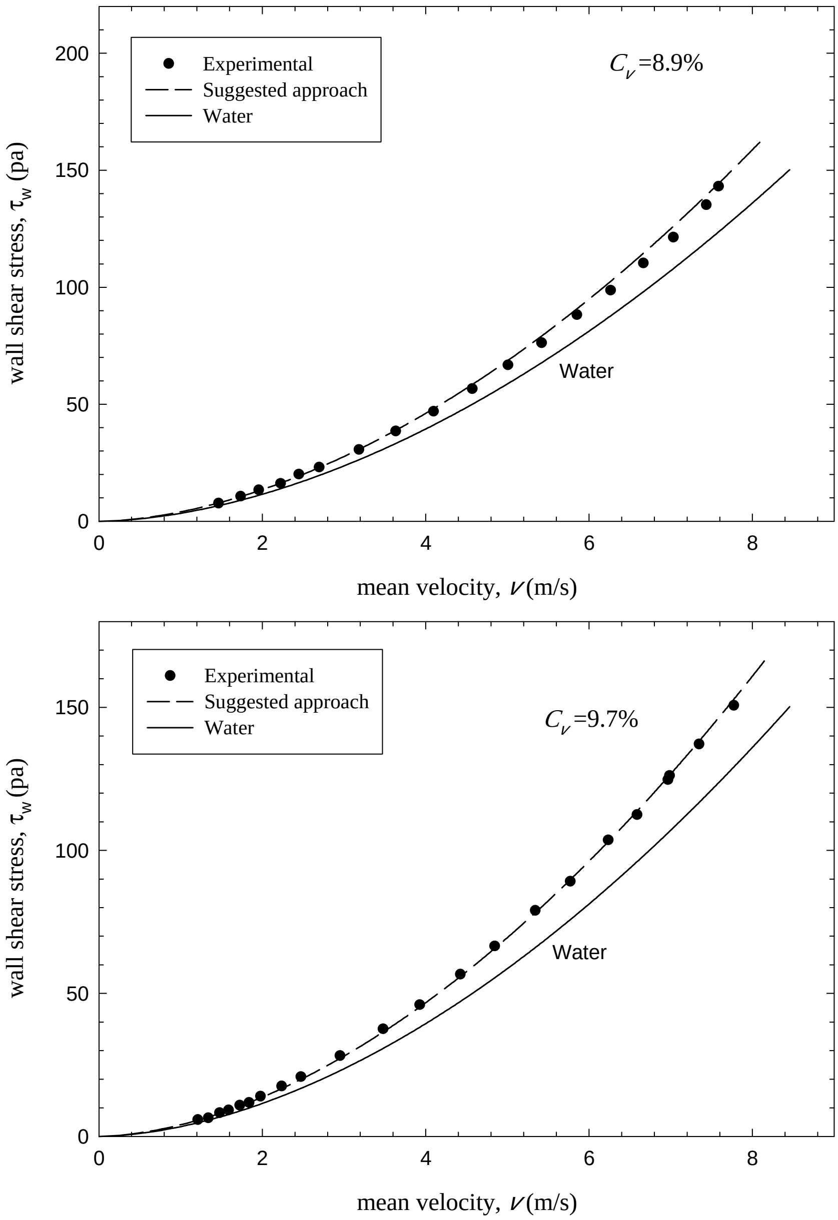


Fig. 2. Turbulent flow prediction of low-concentrated non-Newtonian kaolin slurries

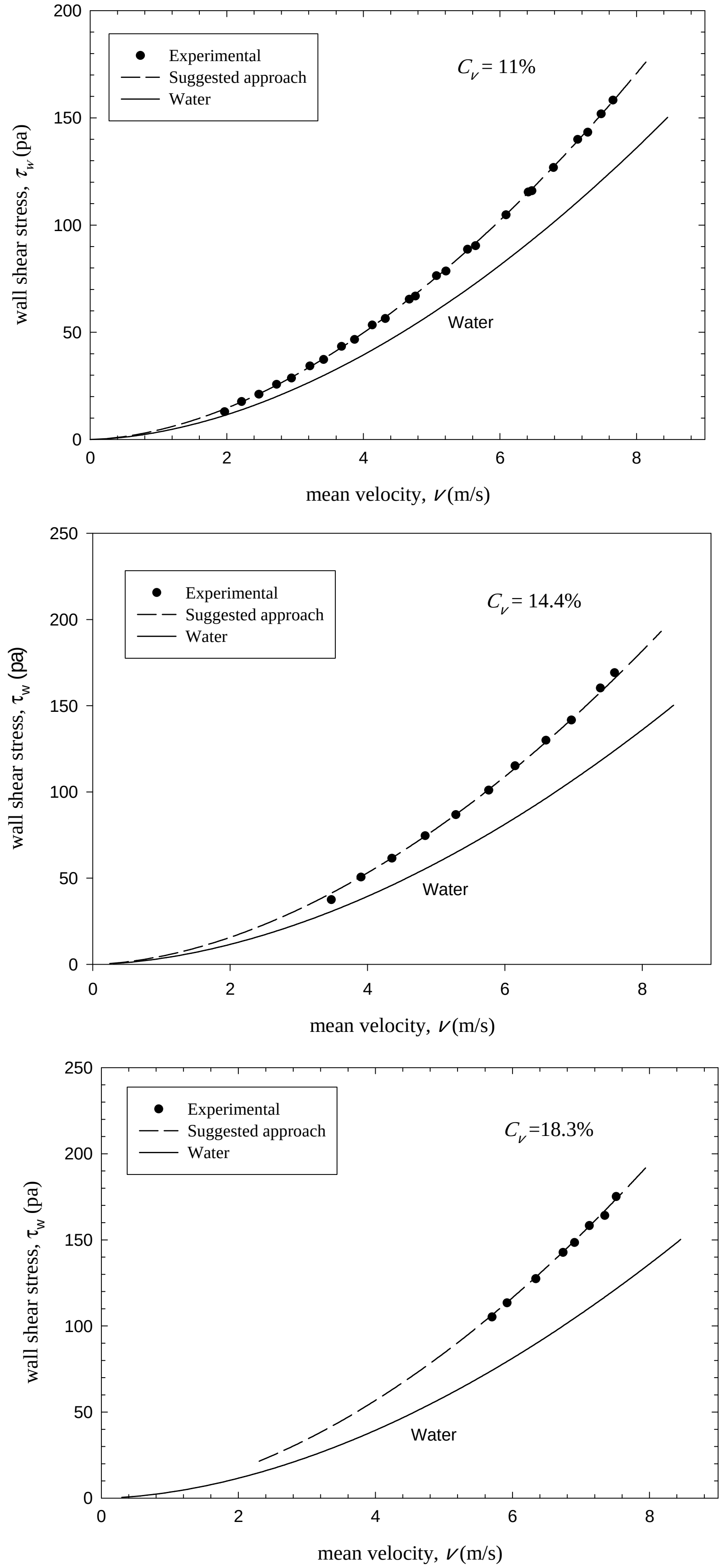


Fig. 3 Turbulent flow prediction of high-concentrated non-Newtonian kaolin slurries

CONCLUSIONS

The tested dense kaolin slurries behave as non-Newtonian liquids according to the Herschel-Bulkley model. The rheological parameters, which well match the laminar flow characteristics, are not suitable for the predictions of turbulent flow behaviour by the models of Torrance, Wilson and Thomas, and Slatter, which are different in their adoptive concepts. A theoretical approach has been suggested to predict the non-Newtonian flow behaviour. It is based on the similarity of non-Newtonian turbulent flow behaviour with that of Newtonian one. The suggested approach well matches the experimentally investigated turbulent flow behaviour for both Newtonian and non-Newtonian slurries. This suggested predictive approach would be applicable for the present state of study. More experimental studies should be obtained to generalize it studying the effect of other different parameters.

NOMENCLATURE

b	proportional parameter	[kg/m.s]
ξ	ratio τ_Y/τ_w	[-]
C_v	volumetric concentration	[-]
C_μ	characteristic parameter	[kg/m.s]
D	pipe internal diameter	[m]
d_{50}	mass median particle diameter	[m]
d_{85}	diameter of which 85% (by mass) of the particles are finer	[m]
k	consistency index	[$\mathrm{kgm^{-1}s^{n-2}}$]
n	flow behavior index	[-]
R	pipe radius	[m]
Re	Reynolds number	[-]
$\mathrm{Re_r}$	roughness Reynolds number	[-]
S_m	mixture specific gravity	[-]
u	local fluid velocity in the pipe-axis direction	[m/s]
U_*	shear velocity $(\tau_w/\rho)^{0.5}$	[m/s]
V	mean velocity	[m/s]
V_N	mean velocity for the equivalent smooth-wall flow of a Newtonian fluid with viscosity μ_a	[m/s]
y	vertical distance in a pipeline cross–section	[m]
Ω	a parameter defined by equation (6)	[-]
α	ratio of areas under the non-Newtonian and Newtonian rheograms	[-]
γ	shear rate	[1/s]
γ_w	wall shear rate	[1/s]
μ	dynamic viscosity	[kg/ms]
μ_a	apparent viscosity	[kg/ms]
ρ	density	[kg/m^3]

τ	shear stress	[kg/ms^2]
τ_w	wall shear stress	[kg/ms^2]
τ_Y	yield stress	[kg/ms^2]

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