EXPERIMENTAL STUDY OF BACKWATER RISE DUE TO BRIDGE PIERS AS FLOW OBSTRUCTIONS

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ABSTRACT

This paper presents the results of an experimental investigation of backwater rise upstream bridge structure when the stream flow is obstructed by the bridge piers only. The study was carried out under both subcritical and supercritical flow conditions between bridge piers. Also, the study was carried out for three classical types of bridge piers under different values of contraction ratio and discharge values through 180 runs. The study showed that the backwater rise upstream bridges with piers only obstructed the flow depends mainly on the type of the flow, the value of discharge, the geometrical boundaries of the cross-section at bridge site and the effect of pier shape. A comparison between the experimental results and the corresponding values by Yarnell formulas showed small differences. The collected data in different sets of experiments, which involving different types of solid piers under different flow conditions were used in developing general relationships for the backwater rise as a function of the stable flow properties and piers characteristics.

INTRODUCTION

In many cases of bridges on canals, the bridge piers are the only supports that obstruct the flow as in high bridges crossing water ways or due to the reflowing of the cross-section as the bridges on the Nile River and its branches due to the construction of Aswan High Dam. The backwater upstream bridges results from the constriction of flow due to piers are considered the most important problem. The importance of the backwater effect derives from the economic significance of river levels, where the cost of the protective works depends on the predicted flood level. As stated by Henderson [2], the study of backwater rise upstream bridges due to contraction was started by Yarnell in 1934 [5] (see Sturm, 2001 [4]). Yarnell [5] studied the backwater rise upstream bridges for the obstructed stream by piers only. Also, the backwater rise upstream bridges was studied by Soliman et al. [3]. The energy principle is employed to estimate the backwater rise due to setting of piers in the stream as follows:

Applying Bernoulli's equation for horizontal bed between section 2 and section 3 as shown in Fig. 1:

$$(Y_2 + \frac{V_2^2}{2g}).r = Y_3 + \frac{V_3^2}{2g} \qquad (1)$$

in which:

Y_2 flow depth at section 2 (minimum flow depth);
Y_3 downstream flow depth;
V_2 flow velocity at section 2;
V_3 flow velocity at section 3; and
r the residual ratio of energy between sections 2 and 3 ≈ 0.9-1.0.

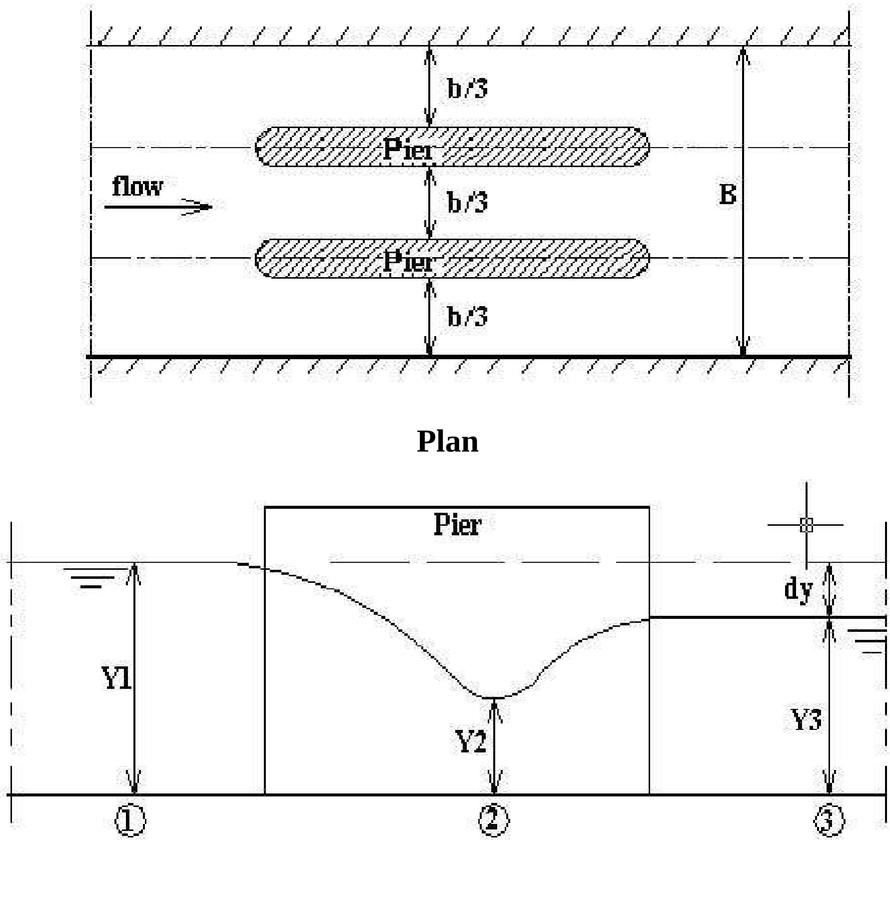


Fig. 1 Flow between bridge piers

From the continuity principle between section 2 and section 3:

$$Y_2\, b\, V_2 = Y_3\, B\, V_3 \qquad (2)$$

in which

b total clear width between piers; and
B canal width.

also,

Cr is the contraction ratio = *b/B* (3)

Rearrangement of Eqs. (1), (2) and (3) results to the following;

$$C_r^2 = \frac{r^3 F_{r3}^2 (2 + F_{r2}^2)^3}{F_{r2}^2 (2 + F_{r3}^2)^3} \tag{4}$$

in which F_{r2} and F_{r3} are Froude numbers at sections 2 and 3, respectively.

The critical value of the downstream Froude number (F_{r3c}) is the value of the Froude number accompanied to the value of Froude number at section 2 equals to unity. As stated by Henderson [2] the limiting value of the contraction ratio for distinguishing between class A (unchoked flow) and class B (choked flow) was used by Yarnell [5] according to the following assumptions $E_1 = E_2$ and $M_2 = M_3$. Assuming critical flow at section 2, the assumption $E_1 = E_2$ leads to:

$$C_r^2 = \frac{27 F_{r1}^2}{(2 + F_{r1}^2)^3} \tag{5}$$

while the second assumption $M_2 = M_3$ leads to

$$C_r = \frac{(2 + 1/C_r)^3 F_{r3}^4}{(1 + 2F_{r3}^2)^3} \tag{6}$$

in which

- E_1 total energy at section 1;
- E_2 total energy at section 2;
- M_2 momentum at section 2; and
- M_3 momentum at section 3.

Henderson [2] stated that Eq. (6) is more likely to be correct because it does not depend on any assumptions about energy conservation, and in any case it is more useful because its independent variables could be derived from section 3, which are known initially. When the value of Froude number at section 2 equals unity ($F_{r2} = 1$), the flow at section 2 is critical flow and the corresponding value of Froude number at section 3 is the critical value of Froude number in the downstream, which is referred as Fr_{3c} and Equation (4) could be in the following form:

$$C_r^{\,2} = \frac{27 r^3 F_{r3c}^2}{(2 + F_{r3c}^2)^3} \tag{7}$$

As stated by Soliman et al. [3], the type of flow between piers could be determined as follows:

1- Compute the contraction ratio (C_r) = b/B;
2- Calculate Froude number downstream the bridge (F_{r3});
3- Assume the residual ratio of the flowing energy (r); and
4- From Equation (7), the critical value of F_{r3c} can be calculated.

When the value of Froude number in downstream under normal flow conditions (F_{r3}) is less than the critical value of Froude number F_{r3c}, then the flow between piers is subcritical (Fig. 2 class-A or unchoked flow), and vice verse when, the value of Froude number in downstream under normal conditions F_{r3} is greater than the critical value of Froude number F_{r3c}, the flow between piers is supercritical (Fig. 2 class-B or choked flow).

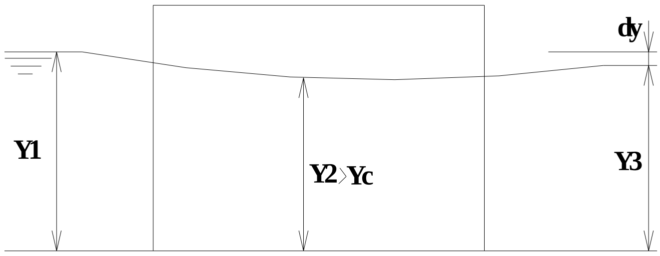
Subcritical flow between piers

As stated by Henderson [2], Yarnell [5] derived the following empirical equation for backwater in subcritical flow (class-A) (see Sturm, 2001 [4]).

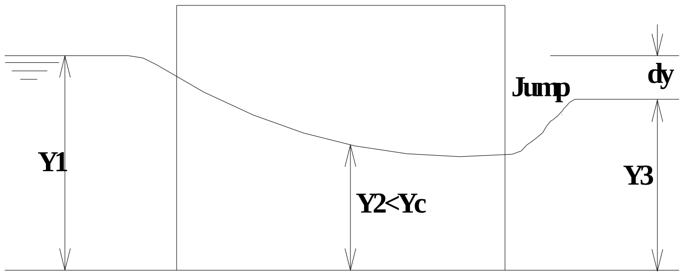
$$\frac{dy}{Y_3} = CF_{r_3}^2[(C + 5F_{r_3}^2 - 0.6)(X + 15X^4)] \tag{8}$$

in which

dy	backwater height;
C	coefficient depending on pier shape = 0.9 for semicircular nose, 1.05 for triangular nose, and 1.25 for rectangular nose [3];
Y_3	normal flow depth in downstream;
F_{r3}	downstream Froude number; and
X	ratio of pier thickness to the distance between piers axis.



Class A – Unchoked flow



Class B – Choked flow

Fig. 2 Types of flow between piers

As mentioned by Henderson [2] the aforementioned Yarnell relationship was derived for pier length-width ratio 4:1. He stated that the backwater (*dy*) increases by 5% and 10% for the ratios 7:1 and 13:1, respectively.

Yarnell [5] carried out few tests on skew piers with inclination angles of 10° and 20° on flow direction. He found that for 10° skew pier the increase in backwater rise is very small, while for 20° the backwater rise increases 230% for unskewed one. This could be explained due to increase of the normal projection width of the pier to the flow.

Supercritical flow

When the value of Froude number downstream bridge site (F_{r3}) is greater than the critical value of Froude number in downstream (F_{r3c}) computed from Equation (7), the flow between piers is supercritical flow. As stated by Soliman et al. [3], Yarnell [5] developed a relationship between the values of dh/Y_3 versus Fr_3/Fr_{3c} as illustrated in Fig. 3. The backwater can be calculating using Yarnell [5] relationship as follows:

1- Calculate the value of F_{r3}/F_{r3c}.

2- From Fig. 3 for each type of pier, the value of dh/Y_3 can be determined.

3- By knowing the value of Y_3 and dh/Y_3, the value of backwater rise, dh, can be calculated.

Soliman et al. [3] stated that the relationship developed by Yarnell [5] was derived for the value of pier length/pier thickness = 4.0. Chow [1] stated that the backwater rise value differs by above or less if the $L_p/t_p > 4.0$ according to pier shape.

In this paper an experimental investigation of backwater rise under both subcritical and supercritical flow conditions between bridge piers is carried out. Also, the effect of the contraction ratio and the pier shape are included in this study.

EXPERIMENTAL WORK

The experimental work was carried out in a re-circulating flume in the Hydraulic Laboratory of Al-Tahady University, Libya through 180 runs. The models of piers were manufactured from timber and painted with a non permeable material to have a smooth surface as shown in Photos 1 and 2. The experiments were carried out under the values of Froude number downstream bridge ranged between 0.19 and 0.93. Six pier models with contraction ratio of 0.42, 0.52, 0.62, 0.71, 0.81 and 0.9 were used. For each pier model, five values of discharge were used. Also, for each model, the downstream depth is changed twice to have both subcritical and supercritical flow between piers. The effect of pier shape was studied through three types of pier noses, namely: rectangular, triangular and circular noses. The discharge was measured by using a flowmeter fitted behind the pump and a calibrated triangular V-notch at flume end. Water depths were measured by using a point gauge supplied with verniers allowing measurements accuracy of ±0.1 mm. The flow velocity was measured by using pitot-tube. In each run, the normal water depth accompanied to each discharge before putting the bridge pier (Y_3) and the flow velocity V_3 were measured. After placing the bridge pier the following parameters were measured:

1- Upstream flow depth Y_1;
2- Approach velocity V_1;
3- Flow depth through vents Y_2; and
4- Flow velocity through vents V_2

Some samples of pier shapes used in the experiments are illustrated in photo 3.

ANALYSIS AND DISCUSSION

The value of the downstream critical Froude number (F_{r3c}) corresponding to each value of contraction ratio, C_r is calculated from Eq. (7). If the value of Froude number in downstream, F_{r3} is less than F_{r3c}, the flow between piers is subcritical, while the flow between piers is supercritical if the downstream Froude number is greater than F_{r3c}. In this study both the subcritical and the supercritical flows between piers were represented. The subcritical flow between piers is illustrated in Figs. 4 to 10. The figures show that the backwater rise upstream bridge piers under subcritical flow depends mainly on contraction value (C_r). Figures 4 and 5 represent rectangular nose pier, Figs. 6 and 7 represent triangular nose pier, Figs. 8 and 9 represent circular nose pier and Fig. 10 shows a comparison between the backwater rise resulted from the three types of piers. The figuers show that the upstream backwater rise is inversely proportional to the contraction ratio (*b/B*). Figure 4 shows that the backwater rise is directly proportional to F_{r3}/F_{r3c} for the different values of contraction. This could be explained by the increase of the ratio F_{r3}/F_{r3c}, i.e. the flow between piers go towards the choked flow. As the flow between piers go towards the choked flow for a constant contraction, the backwater rise upstream the bridge pier increases. Also, the increase of the ratio F_{r3}/F_{r3c} for constant contraction means an increase of F_{r3}, which results in increasing the backwater rise as confirmed by Eq. (8), in which the backwater depends mainly on the value of F_{r3}. Figure 5 shows a comparison between the computed and calculated values of the dimensionless backwater rise. The figure shows that the comparison between the experimental values of backwater and the corresponding values resulted from Yarnell equation shows that the difference ranges from 14% to 8%.

Figures 6 and 7 show the relationship between the dimensionless backwater and the ratio F_{r3}/F_{r3c} for triangular nose pier (angle of slope = 60°). From Fig. 6 it is found that the backwater rise upstream triangular nose pier follows the same trend as in rectangular nose pier, but the values of backwater for triangular nose pier are smaller than the corresponding values for rectangular one. This could be explained by the sloping face of piers which improves the characteristics of flow streamlines, and results in decreasing the backwater rise. This is confirmed by the former studies, in which one of the important parameters in Yarnell formula [5] is the pier nose shape (*C*). Figure 7 illustrates a comparison between the experimental values of backwater and the corresponding computed ones by Yarnell formula. From the figure it is found that the difference between the measured values and the corresponding calculated ones was about 17%.

Figures 8 and 9 show the relationship between the dimensionless backwater rise and the value of F_{r3}/F_{r3c} for circular nose pier. From Fig. 8 it is found that the relative backwater is directly proportional to the relative value of downstream Froude number. The comparison between measured and calculated values of backwater rise is illustrated in Fig. 9. The figure shows that the difference between the measured and the calculated values ranges between 23% and 19%. Also, it is found that the difference between the measured and calculated values decreases as the flow becomes near to critical flow. The difference between the measured data and the corresponding calculated ones could be referred to the fact that Yarnell formula [5] was derived for pier length-width ratio 4:1 [2], while the pier length-width ratio in this study ranged between 5:1 and 30:1. Figure 10 shows the effect of front of the pier on the upstream backwater for subcritical flow between piers. The figure illustrates that the backwater rise in case of circular front of the pier is less than that for both triangular and rectangular noses piers, respectively.

By using the analysis regression of data, the following relationships between the dimensionless backwater rise and the contraction ratio and downstream Froude number are developed as follows:

1- Rectangular pier

$$\frac{dy}{Y_3} = 0.217 - 0.367C_r + 0.389\ F_{r3}$$

2- Triangular pier

$$\frac{dy}{Y_3} = 0.205 - 0.338C_r + 0.322\ F_{r3}$$

3- Circular pier

$$\frac{dy}{Y_3} = 0.178 - 0.315C_r + 0.314\ F_{r3}$$

in which

dy backwater upstream bridge;
Y_3 downstream flow depth;
F_{r3} downstream Froude number; and
C_r the contraction ratio b/B.

The foregoing formulas could be applied for the following ranges:

F_{r3} = 0.2 -0.62, C_r = 0.42 – 0.9 and pier length-width ratio 5:1 to 30:1.

When the downstream Froude number (F_{r3}) is greater than the downstream critical Froude number (F_{r3c}), the flow between piers is supercritical flow. Figures 11 to 13 show the relationships between backwater and the downstream Froude number for rectangular, triangular and circular piers, respectively, at supercritical flow between piers at different values of contraction. The figures show that the backwater rise upstream three types of piers depends mainly on the value of contraction ($C_r = b/B$). Also, from figures it is found that for the same value of contraction, the backwater rise increases as the ratio Fr_3/Fr_{3c} increases for the three types of piers. This could be explained as the downstream Froude number increases due to the increase of discharge flows between piers. The increasing flow discharge between piers for the same contraction ratio results in increasing the backwater rise. Figures 14 to 16 illustrate the comparison between both the measured and calculated values of backwater rise for the three types of piers. The figures show that the difference between the measured and calculated values of backwater rise ranges between 5 %, 7% and 3% for rectangular, triangular and circular piers, respectively.

Figure 17 shows the comparison between the measured values of backwater rise resulted from three types of piers. The figure shows that the backwater rise at using circular pier is less than that when using both triangular and rectangular piers. Also, the backwater for triangular pier is less than that for the rectangular pier. This could be explained due to the fact that the circular front of pier make the streamlines to volplane more easily than that in both triangular and rectangular nose pier, respectively. From Figs. 11, 12 and 13 the following relationships could be developed for computing the backwater upstream bridge for supercritical flow between piers at downstream Froude number ranges between 0.69 to 0.93 and contraction ratio between 0.42 to 0.9 as follows:

1- Rectangular pier

$$\frac{dy}{Y_3} = 0.1013 \left(\frac{Fr_3}{Fr_{3c}}\right)^{2.586}$$

2- Triangular pier

$$\frac{dy}{Y_3} = 0.0699 \left(\frac{Fr_3}{Fr_{3c}}\right)^{2.556}$$

3- Circular pier

$$\frac{dy}{Y_3} = 0.0537 \left(\frac{Fr_3}{Fr_{3c}}\right)^{2.554}$$

SOLVED EXAMPLE

A three vents bridge is constructed on a lined canal with bed width and flow depth equal to 13 m and 1.2 m, respectively. The width of each vent is 3 m and pier width equals to 2 m. If the discharge is 31 m^3/sec, calculate the backwater after the bridge construction.

Solution

Total width = 3 x 3 + 2 x 2 = 13 m; then the canal is obstructed only by two piers.

C_r = 9/13 = 0.69

$$C_r^{\ 2} = \frac{27r^3F_{r3c}^2}{(2+F_{r3c}^2)^3}$$

By trial and error, Fr_{3c} = 0.54

A_{canal} = 15.6 m^2

V = 31/15.6 = 1.98 m/sec

$Fr_3 = 1.98/(9.81*1.2)^{0.5} = 0.577$

As Fr_3 is greater than Fr_{3c} the flow between piers is supercritical.

$dy/Y_3 = 0.101\ (0.577/0.54)^{2.58} = 0.12$ cm dy = 14.3 cm for rectangular pier.

$dy/Y_3 = 0.069\ (0.577/0.54)^{2.55} = 0.081$ cm dy = 9.8 cm for triangular pier.

$dy/Y_3 = 0.053\ (0.577/0.54)^{2.553} = 0.062$ cm dy = 7.5 cm for circular pier.

CONCLUSIONS

From this study it could be concluded that:

1- The backwater rise upstream bridges for subcritical flow between piers is inversely proportional to the value of the contraction (C_r) for all types of piers.
2- The backwater rise upstream bridges for subcritical flow between piers is directly proportional to the value of the downstream Froude number (F_{r3}) for all types of piers and different values of contraction.

3- The difference between the backwater rise resulted from experimental work and the corresponding values resulted from Yarnell equation [5] for subcritical flow between piers could be refered to the difference in pier length-width ratio.
4- The values of backwater rise for circular pier is smaller than that for both triangular and rectangular piers, respectively, for the same flow conditions.
5- By using the regression of data analysis the following relationships between the dimensionless backwater rise and the contraction ratio and downstream Froude number were developed as follows:
   1- Rectangular pier
$$\frac{dy}{Y_3} = 0.217 - 0.367 C_r + 0.389\ F_{r3}$$
   2- Triangular pier
$$\frac{dy}{Y_3} = 0.205 - 0.338 C_r + 0.322\ F_{r3}$$
   3- Circular pier
$$\frac{dy}{Y_3} = 0.178 - 0.315 C_r + 0.314\ F_{r3}$$
   The foregoing equations could be applied for the following ranges:
   F_{r3} = 0.2 -0.62, C_r = 0.42 – 0.9 and pier length-width 5:1 to 30:1.
6- The dimensionless value of the backwater rise is directly proportional to the term Fr_3/Fr_{3c} for three types of piers in supercritical flow between piers.
7- In supercritical flow, the difference between the measured and calculated values of backwater rise by Yarnell [5] equation ranges between 5 %, 7% and 3% for rectangular, triangular and circular piers, respectively.
8- The following relationships were developed for computing the backwater rise upstream bridges for supercritical flow between piers at downstream Froude number ranges between 0.69 to 0.93 and contraction ratio between 0.42 to 0.9:

   a- Rectangular pier
$$\frac{dy}{Y_3} = 0.1013\ \left(\frac{Fr_3}{Fr_{3c}}\right)^{2.586}$$
   b- Triangular pier
$$\frac{dy}{Y_3} = 0.0699\ \left(\frac{Fr_3}{Fr_{3c}}\right)^{2.556}$$
   c- Circular pier
$$\frac{dy}{Y_3} = 0.0537\ \left(\frac{Fr_3}{Fr_{3c}}\right)^{2.554}$$

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2- Henderson, F.M. "Open channel flow", Macmillan Publishing Co., Inc., New York, 1966.

3- Soliman, M.A. “Design of irrigation structures and works”, Al-Azhar University, Cairo, Egypt, 1989.
4- Sturm, T.W. “Open channel hydraulics”, McGraw Hill Inc., New York, 2001.
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NOTATION

The following symbols are used in the present study

B	total clear width between piers;
B	canal width;
C	coefficient depending on pier shape;
C_r	contraction ratio = b/B;
dy	backwater upstream bridge;
E_1	total energy at section 1;
E_2	total energy at section 2;
F_{r1}	upstream Froude number;
F_{r2}	Froude number between piers;
F_{r3}	downstream Froude number;
F_{r3c}	critical Froude number at downstream;
M_2	momentum at section 2;
M_3	momentum at section 3;
r	residual ratio of energy between sections 2 and 3;
V_2	flow velocity at section 2;
V_3	flow velocity at section 3;
X	ratio of pier thickness to the distance between piers axis;
Y_2	flow depth at section 2 (minimum flow depth);
Y_3	normal flow depth in downstream;
Y_c	critical flow depth between piers.

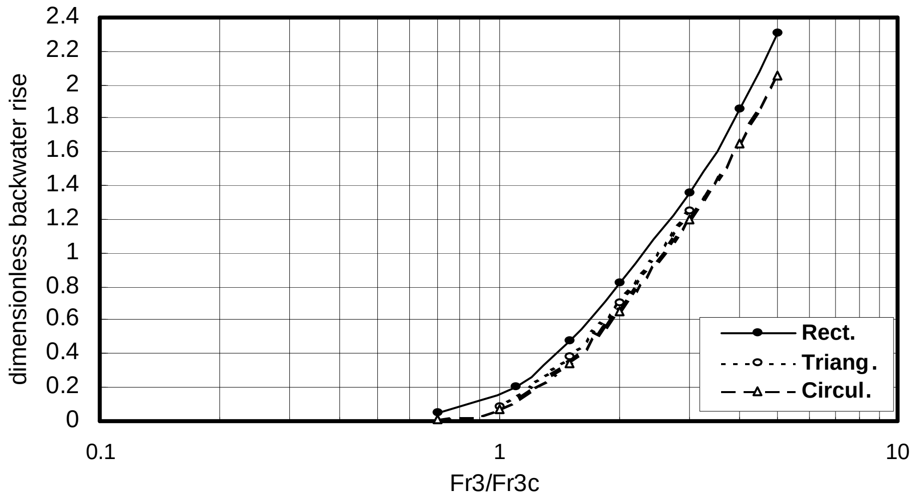

Fig. 3 backwater calculaion in case of supercritical flow between piers [9]. after Soliman et al . [5].

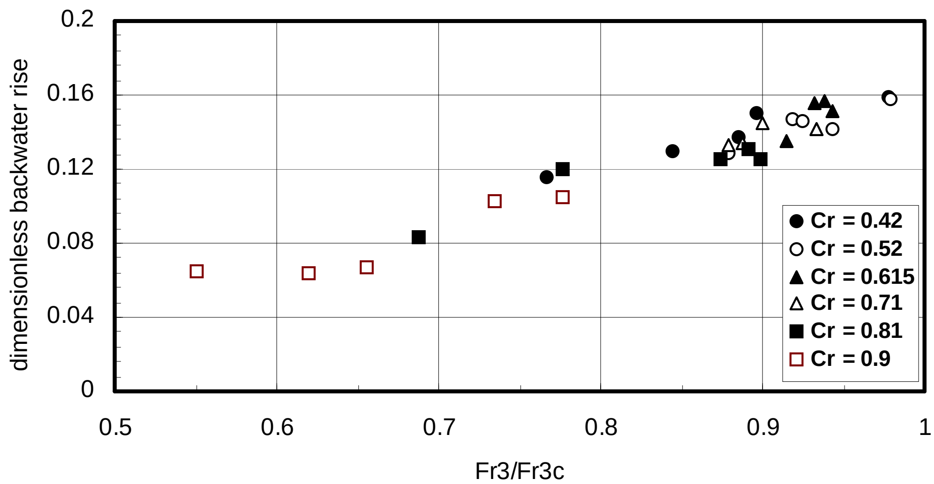

Fig. 4 The relationship between the dimensionless backwater and the ratio Fr3/Fr3c at rectangular nose pier for subcrtical flow between piers .

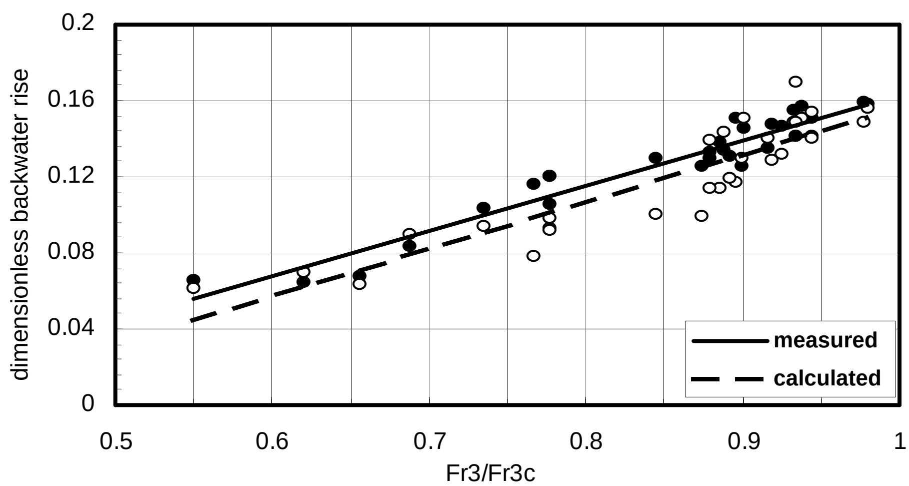

Fig. 5 A Comparison between measured and calculated dimensionless values of backwater for subcritical flow at rectangular nose pier .

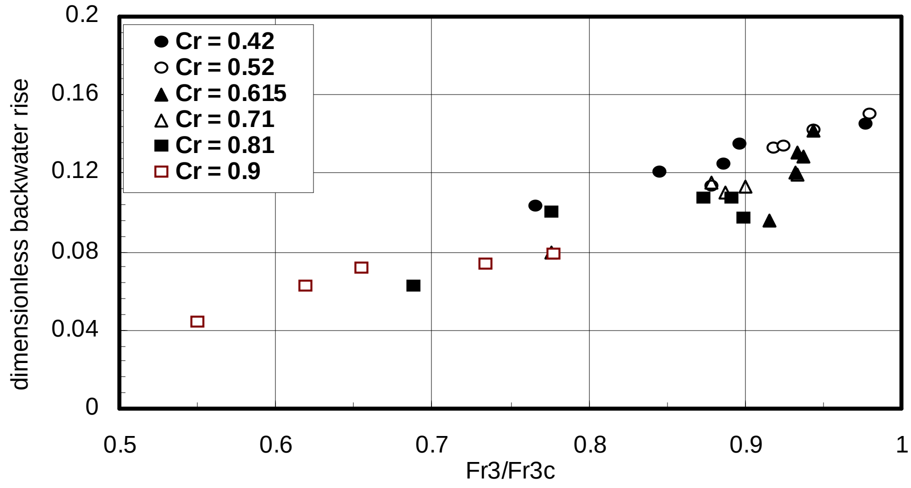

Fig. 6 The relationship between the dimensionless backwater and the ratio Fr3/Fr3c at triangular nose pier for subcrtical flow between piers .

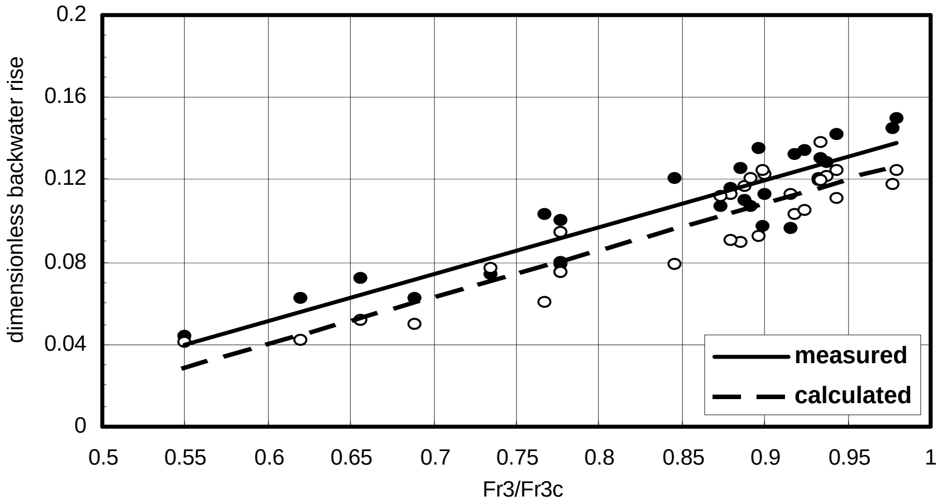

Fig. 7 A Comparison between measured and calculated dimensionless values of backwater for subcritical flow at triangular nose pier .

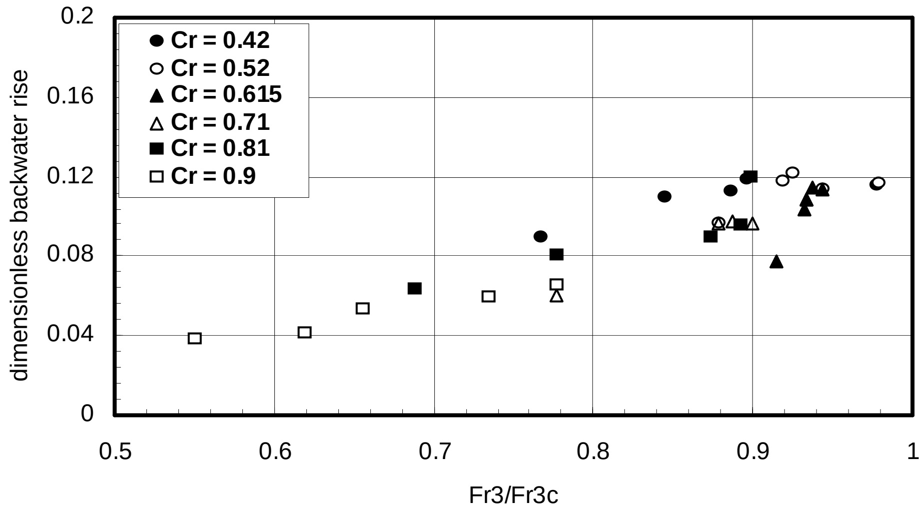

Fig. 8 The relationship between the dimensionless backwater and the ratio Fr3/Fr3c at circular nose pier for subcrtical flow between piers .

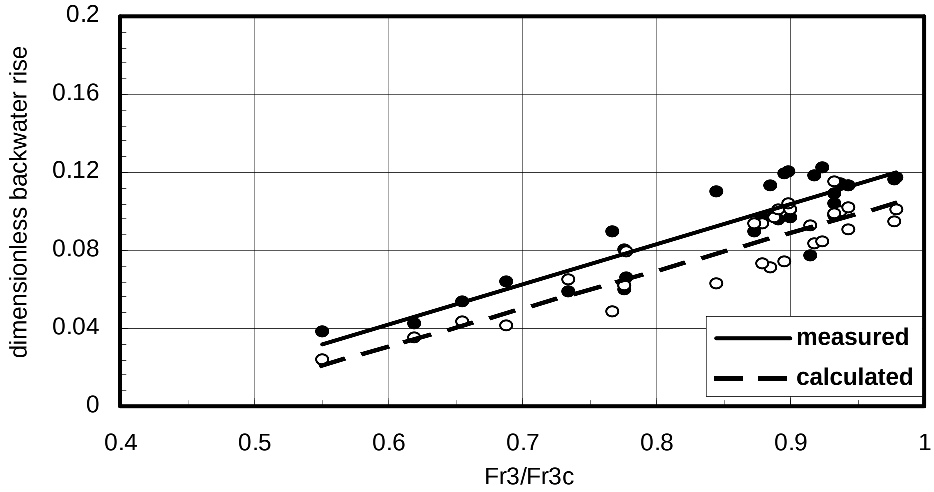

Fig. 9 Comparison between measured and calculated dimensionless values of backwater for subcritical flow at circular nose pier .

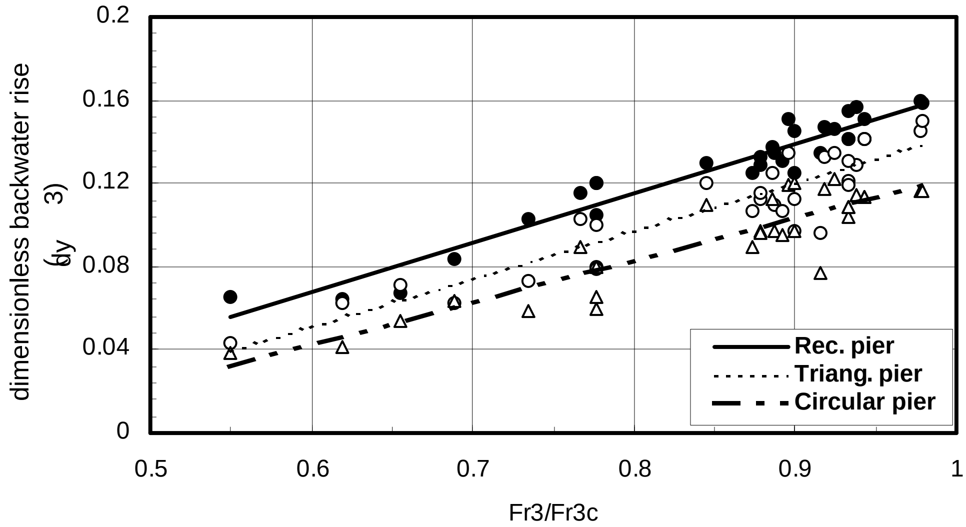

Fig. 10 A comparison between the dimensionless backwater rise for three types of piers.

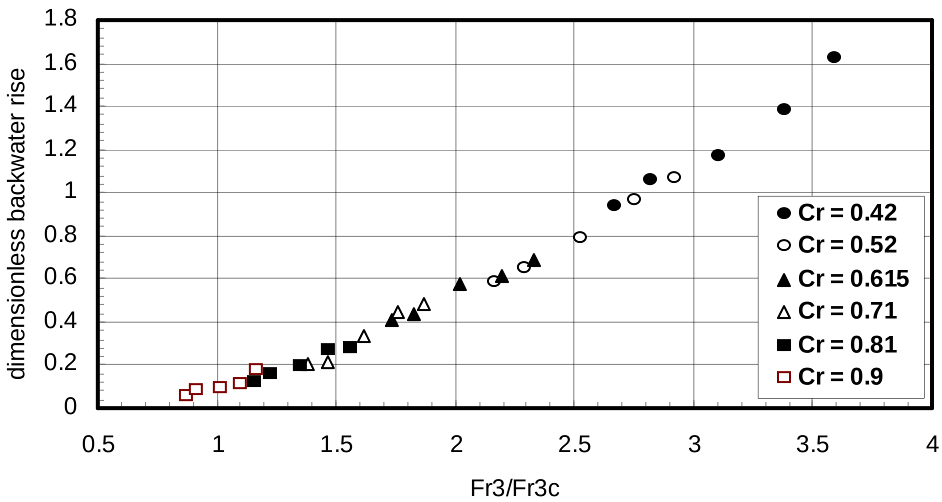

Fig.11 The relationship between backwater and Fr 3/Fr3c at rectangular pier for supercritical flow between piers .

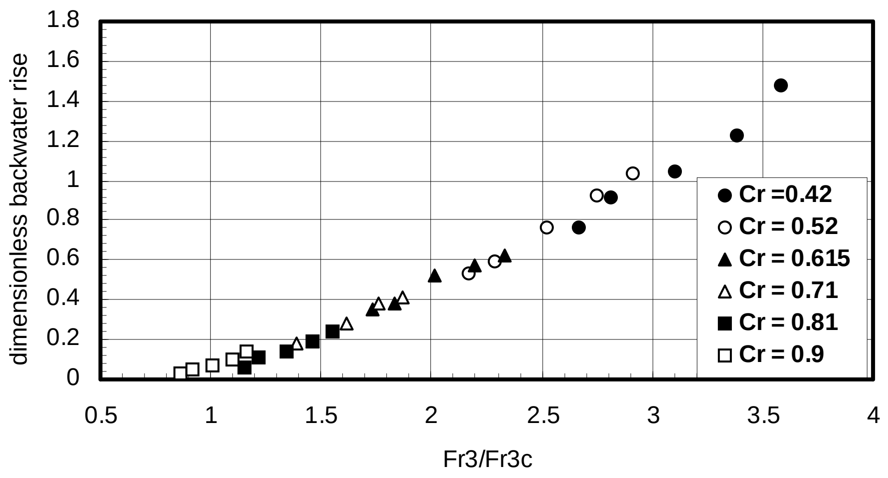

Fig.12 The relationship between backwater rise and Fr 3/Fr3c at triangular pier for supercritical flow between piers .

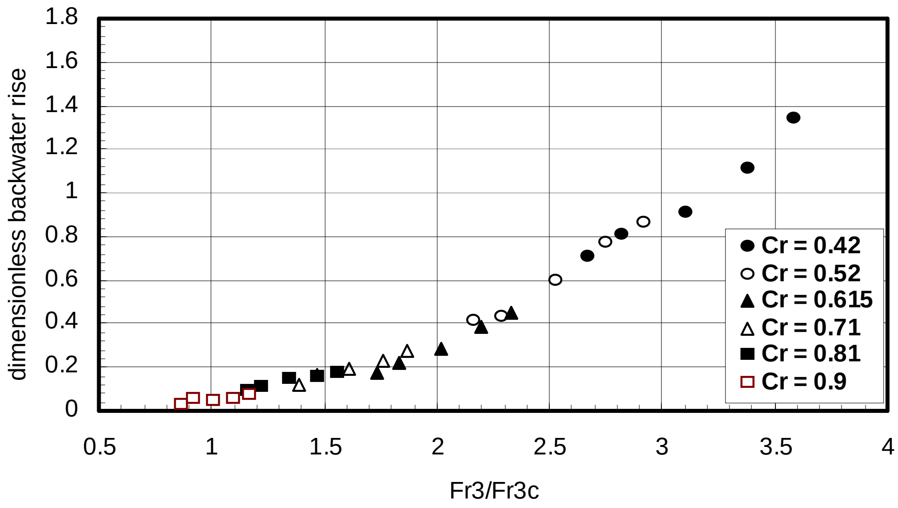

Fig.13 The relationship between backwater rise and Fr 3/Fr3c at circular pier for supercritical flow between piers .

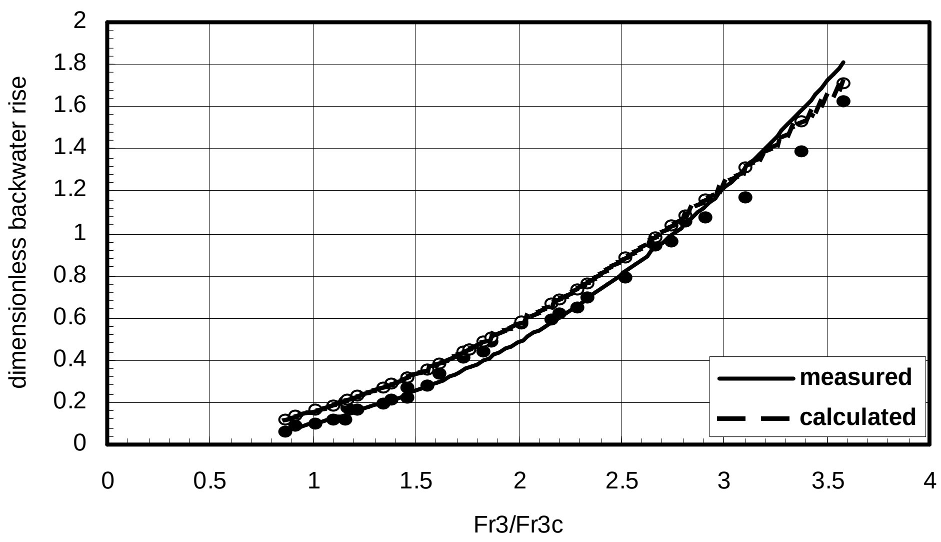

Fig.14 A Comparison between measured and calculated values of backwater rise at rectangular pier for supercritical flow .

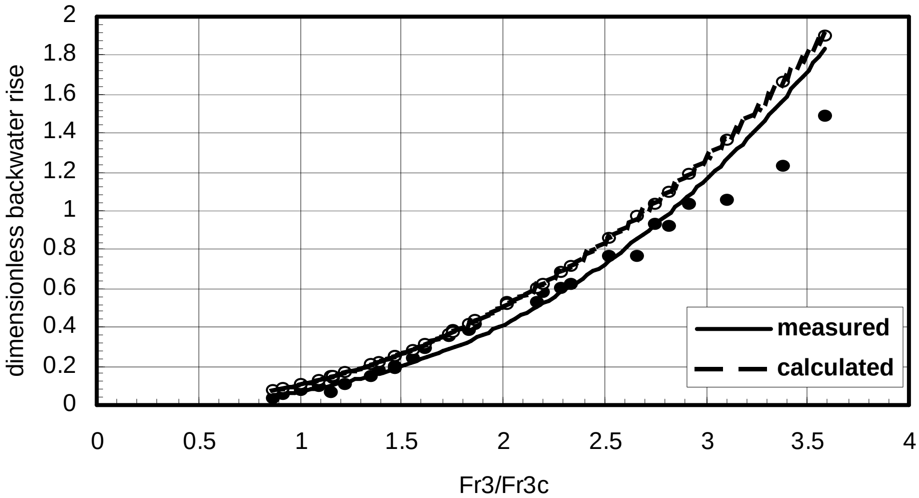

Fig.15 A Comparison between measured and calculated values of backwater rise at triangular pier for supercritical flow .

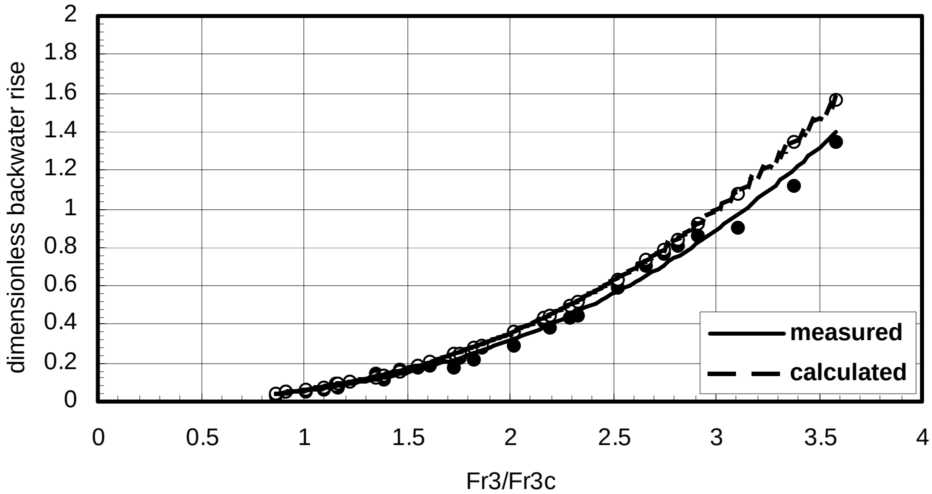

Fig.16 A Comparison between measured and calculated values of backwater rise at circular pier for supercritical flow .

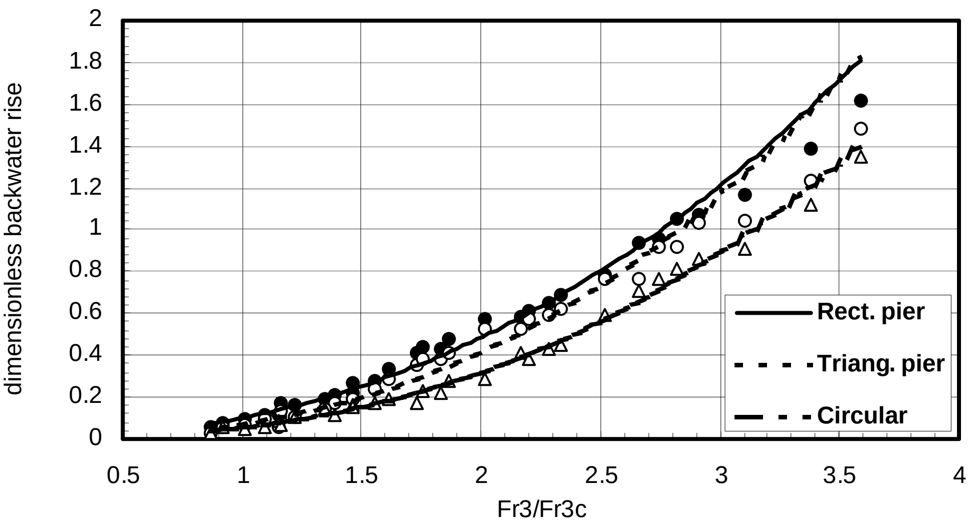

Fig.17 The relationship between backwater rise and Fr 3/Fr3c at different types of piers for supercritical flow between piers .

Photo 1 The rectangular and triangular pier models

Photo 2 The circular pier models

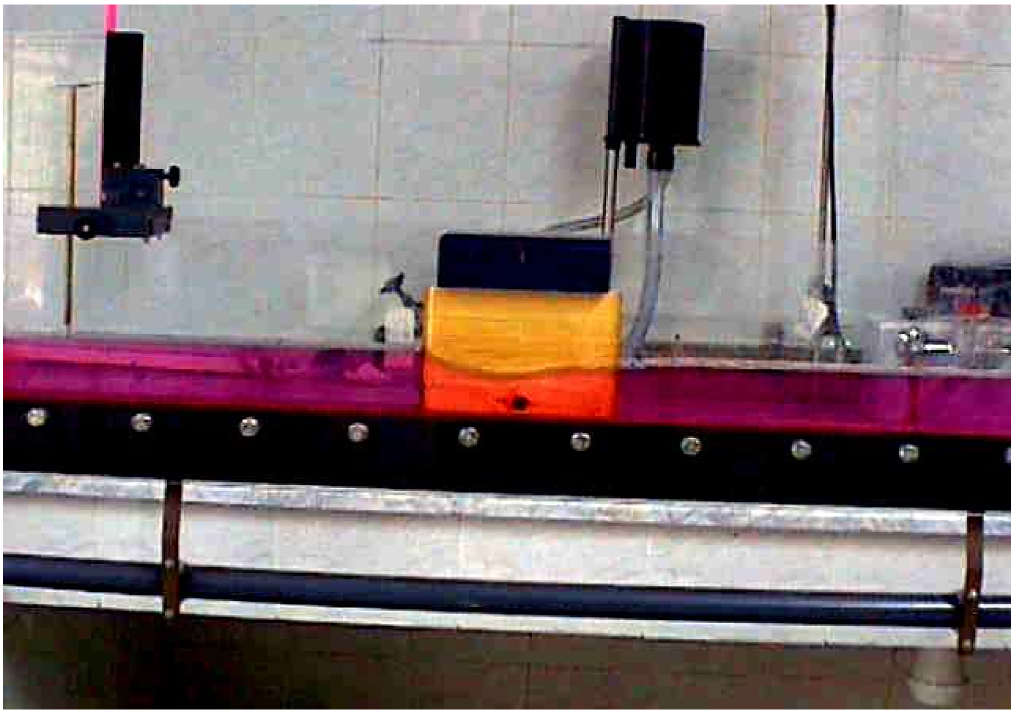

Photo 3 Backwater upstream circular pier