# IDENTIFICATION OF PRESSURE AND VELOCITY CORRECTION COEFFICIENTS ALONG BLOCK-STONES RAMP

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**ABSTRACT**

Along a non-uniform flow in a stream or river, the pressure, energy, and momentum coefficients tend to vary from section to section. These coefficients also tend to vary with the water depth and discharge. Although some researchers provided some values of these coefficients for special conditions, their spatial variability in accelerated flow has not been adequately investigated. To simplify calculations, the values of these coefficients are usually assumed to be constants. This implies that the cross-sectional velocity and pressure distributions do not change with space. Values of these constants are often regarded as unity, which implies that the velocity is uniformly distributed over the cross section while the pressure distribution is hydrostatic. However, effect of these coefficients on the solution of non-uniform flow equations is not well understood.

In this study, a new simple method for computing the pressure, energy, and momentum correction coefficients from the water surface profile for accelerating flow over block stones ramp was developed. This method is based on the assumption that the water depth at the beginning of the ramp is equal to the critical depth. The study revealed that the maximum deviation in the pressure is at the ramp crest and decreases in the downstream direction until it reaches to uniform flow condition at some distance from the ramp crest. Values of energy correction coefficient and momentum correction coefficient are nearly constant at the ramp crest and increase by increasing the distance from the ramp crest. Along the ramp, the velocity correction coefficient increases by decreasing the discharge.

***Key words***: accelerating flow, velocity distribution, pressure correction coefficient, velocity correction coefficient, block stones ramp.

## 1. INTRODUCTION

In mountain regions flow and erosion control is a prime task to protect infrastructure, houses and cultivated areas. Due to many interferences in the past, many rivers are recently undergoing strong degradation which forces to take different countermeasures. One standard method is the application of drop structures spaced along the course of a stream. In generally, rigid and concrete constructions were used for this task, while nowadays alternative structures made of loose stones are preferred since the last can better meet natural and ecological requirements.

The flow in short rough steep channels such as block-stones ramp, rock fill dams, weirs, and stepped spillways is different than the flow in long reach channels. It is characterized by developing boundary layer at the crest, large size bed roughness, curvilinear flow, and non-uniform velocity distribution along the structures. Usually, one-dimensional analysis is used in such type of flow, i.e. the Saint Venant equations. The details of distributions of the flow velocity and pressure over the cross section are not explicitly considered. To account for the non-uniform velocity and non-hydrostatic pressure distributions, correction coefficients must be introduced into the cross-sectional averaged flow equation. There are many studies about the cross section velocity correction coefficient (Al-Khatib (1999)[1)], Chen (1992)[2)], Xia (1994)[3)], however, there are few studies about the spatial variation in correction coefficients (Chiu and Murray (1992)[4)]), especially in the case of accelerated flow. Platzer (2000)[5)] and Hassinger (1991)[6)] have deduced some empirical equations for calculating the energy and momentum coefficient along block-stones ramp. However, their equations are not generally applicable.

To study the spatial variation of pressure and velocity distribution coefficients, a non-uniform accelerated flow was established over block stones ramp in a rectangular open channel. Particular experiments were conducted in a flume with a length of 6.0 m, width of 0.3 m, and depth of 0.5 m. Loose stones forming a 20% inclined ramp of 1.0 m length were placed in the mid of the flume. For more details about the stones manufactures, it can be found in Schöberl and Mohamed (2001)[7)]. The bed slope for the approach channel was kept constant at 1.5% and the ramp crest was kept constant at height 0.89 cm above the approach channel bed through all the experiments. Three different discharges were carried out over the ramp. For each discharge, the water surface profile along the ramp was measured at small distance (from 1 to 2 cm) by using an electronic profile-meter. Before carrying out the experiment, the bed roughness height for the ramp was estimated exactly ($k_{sr}$= 8.47mm) and the effective bed level for the ramp was defined. Figure (1) shows schematic representation for the flow over block-stones ramp.

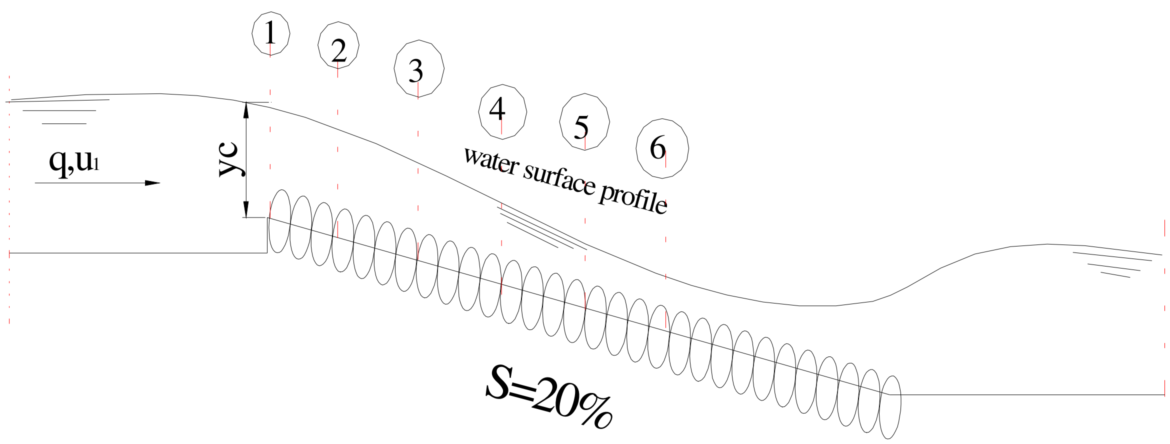


**Fig. (1): Schematic Representation for the flow over block-stones ramp**

## 2. VELOCITY DISTRIBUTION ALONG THE RAMP

Over the past decades, the mean–flow properties and its turbulent structure of uniform open-channel flow have been extensively investigated. However, only a few researchers studied the effect of non-uniformity on the velocity distribution and the turbulence characteristics, i.e. Song and Graf (1994)[8)], Kironoto and Graf (1995)[9)], Tsujimoto, et al. (1990)[10)]. Song and Graf (1994)[8)] showed that the log-law explains the inner-region data and the Coles' law of the wake explains the outer data. However, using the log-law or Coles' law of the wake would make the velocity profile equation very complicated.

For simplicity, the velocity distribution along the ramp was assumed to follow the power law for velocity distribution, which can be expressed as:

$$v / V_{\max} = (\mathrm{y/h})^{\mathrm{m}} \tag{1}$$

where v is the velocity at any height y from the bed; $V_{max}$ is the maximum velocity; h is the water depth; and m is an exponent.

To verify the power law for velocity distribution, the velocity profiles along the ramp was measured at different sections as marked in Fig.(1) for discharges 9.40, 11.40, and 13.40 l/s respectively. By using regression analysis, values of the exponent m in equation (1) were calculated. Table (1) shows the values of m at different distances from the ramp crest.

**Table (1)**: Values of the exponent m in equation (1) along the ramp

| **Section number** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| The distance from the ramp crest (cm). | 0 | 4.5 | 14.5 | 24.5 | 34.5 | 44.5 |
| m exponent in the power law | 0.302 | 0.297 | 0.223 | 0.642 | 0.30 | 0.423 |

## 3. PRESSURE CORRECTION COEFFICIENT

The piezometric pressure correction coefficient λ, which is a function of the pressure distribution and the cross-sectional shape, accounts for pressure variation on the flow cross- section. Yen (1973)[11)] defined it as

$$\lambda = \frac{\int_A p.dA}{\gamma.A.h.\cos\varphi} \tag{2}$$

in which γ = specific weight of the fluid; p= local piezometric pressure, h= mean water depth, and φ= bed slope. For hydrostatic pressure distribution over cross sectional area, A, or constant piezometric pressure over A, λ =1.

Chow (1959)[12] mentioned that the application of the hydrostatic law to the pressure distribution of steady flow in the cross section of a channel is valid only for parallel flow, such as uniform flow, and for practical purposes the hydrostatic law of pressure distribution is also applicable to gradually varied flow as an approximation. For concave or convex curvilinear flow, the pressure distribution over the section deviates from hydrostatic such that the hydrostatic law is invalid. In order to account for this deviation from hydrostatic, the piezometric pressure correction coefficient $\lambda$ must be introduced as a correction for curvilinear flows. It is known that $\lambda > 1$ for concave flow and $\lambda < 1$ for convex flow.

Yasuda and Ohtsu (1999)[13] gave the following simplified formula to estimate the pressure distribution correction coefficient:

$$\lambda = 1 + \frac{1}{\gamma . Q} \int_0^h v . \Delta p . dy \qquad (3)$$

where v = the velocity at any height from the channel bed and can be approximated by the power law for velocity distribution i.e., equation (1) (Heggen (1991)[14]; Chen (1992)[2]; Ohtsu and et al. (2001)[15]) where the maximum velocity at cross-section, $V_{max}$, was assumed equal to 1.1 the mean velocity; y =distance from channel bed; h = water depth; and m =exponent (is taken as average value for m values along the ramp shown in table (1) and equal to 0.36). $\Delta p$ is the deviation from the hydrostatic pressure ($P_s = \gamma.(h-y)$) and is given by $\Delta p = \gamma.(h_p-h).(h-y)/h$ ($h_p$ =pressure head at the channel bottom). By substituting the value of v and $\Delta p$ in eqn. (3) and by integration lead to the following equation.

$$\lambda = 1 + 0.312 \ . \ (h_p - h)\ /b \qquad (4)$$

where b= the channel width, $h_p$ = the pressure head at the channel bottom, and h= the mean water depth. The value of $h_p$ in the above equation must be defined. In this study, the value of pressure distribution along the ramp was calculated using 3D numerical modelling for different discharges and ramp crest height, and from which the value of $h_p$ was defined. Figure (2) shows values of $h_p/h$ versus $x/y_c$ where x is the distance along the ramp and $y_c$ is the critical depth. It is noticeable from Fig. (2), that the large deviation from the hydrostatic distribution at the ramp crest nearly diminish at distance 3~4 $x/y_c$. From Fig.(2), it was found that the variation of the pressure head at the ramp bottom can expressed by the following relation.

$$h_p / h = 0.95 - 0.404 . \mathrm{EXP}(-0.85. \ (x/y_c)^{1.34}\ ) \qquad (5)$$

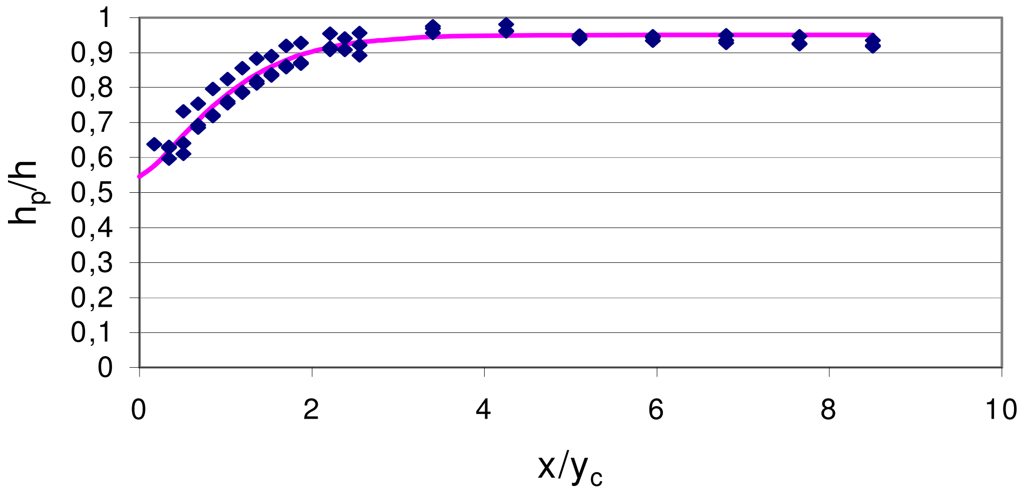


**Fig. (2): Values of $h_p/h$ versus $x/y_c$**

Values of $h_p$ calculated by equation (5) were invoked in equation (4) to calculate λ values. Figure (3) shows the variation of the pressure correction coefficient along the ramp for different dimensionless values of $q/(g.k^3)^{0.5}$ where q is the unit discharge and k is the roughness height. As shown in that Fig., the value of λ deviates from 1.0 for some distance along the ramp, then arrive to a nearly constant value at distance fro 3~4 $x/y_c$ from the ramp crest. Also, the maximum deviation occurs at the ramp crest. In spite of the deviation in λ values from unity is very small (in the range of 3%), but its effect is very high on values of the water depth or the mean velocity. Xia and Yen (1994)[3)] have shown that the error in values of the water depth may arrive to 6 % in the case of neglecting the pressure distribution correction coefficient and the deviation in values of velocity nearly equal to 20%.

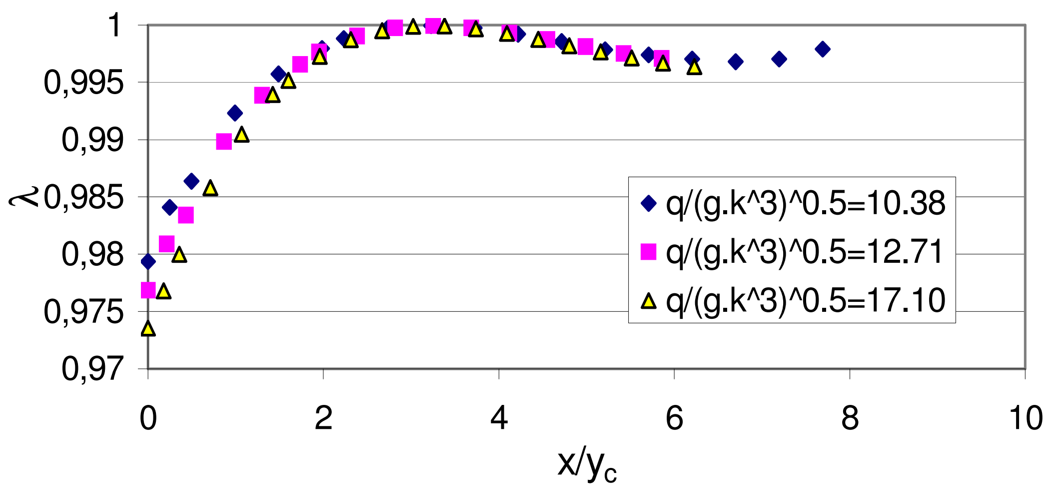


**Fig. (3): Values of the pressure correction coefficient (λ) along the ramp for different discharges**

## 4. ENERGY AND MOMENTUM CORRECTION COEFFICIENT

The flow acceleration over the ramp with developing boundary layer causes strong variation in the shape of the velocity profiles. Also, the high bed roughness makes the velocity profile very far from the normal logarithmic shape. For this, it is very important to be taken into account the correction factors for the velocity distribution. The value of the Coriolis-coefficient must be taken into account, when we calculate the kinetic energy height. Also, the value of the Boussinesq-coefficient must be taken into account, when we calculate the momentum force.

### (1) Critical Depth Definition

If the control section at the ramp crest was established at critical conditions, the water depth would be equal to the critical depth. Heggen (1991)[14] gave three definitions for the critical depth.

- The depth at which fluid velocity (celerity) of a gravity wave of infinitesimal amplitude or the depth at which specific energy and specific force, neither corrected for velocity profile, are minimised. This is satisfied when

$$F_e = \sqrt{\frac{V^2}{gD}} = 1 \qquad (6)$$

where $F_e$ = Froude number; D = hydraulic depth A/B; and B = flow width at depth y. This definition of critical depth, common in design practice, is valid only for a uniform profile, a condition never met (French (1985)[16]).

- The depth at which specific force corrected for velocity profile is minimised. This is satisfied when

$$F_e = \sqrt{\frac{1}{\beta}} \qquad (7)$$

where $\beta = \dfrac{\int_0^A v^2 .dA}{V^2 .A}$

- The depth at which specific energy corrected for velocity profile is minimised. This is satisfied when

$$F_e = \sqrt{\frac{1}{\alpha}} \qquad (8)$$

where $\alpha = \dfrac{\int_0^A v^3 .dA}{V^3 .A}$

This energy minimisation definition is employed by Chow (1959)[12], Henderson (1964)[17], and French (1985)[16], and is commonly used for water surface profiles.

Eqs. (6), (7), and (8) can be unified into a single expression:

$$V^{2-k}\int_0^A v^k .dA = \frac{g.A^2}{B}$$

where k=1 for the first $y_c$ definition, 2 for the second definition, and 3 for the third.

The velocity profile in two-dimensional flow is generally assumed to follow the power law, i.e., equation (1). While the exponent coefficient m lends itself to subsequent mathematical manipulation, it is informative to relate m to the coefficient α. Comparing fluid power determined from integrated (1) to the power determined by mean V, it could be shown that:

$$\alpha = \frac{(m+1)^3}{3m+1} \tag{9}$$

and

$$\beta = \frac{(m+1)^2}{2m+1} \tag{10}$$

A value of m = 0.3, at the ramp crest corresponding to measured velocity profile, yields α =1.156, an experimentally verified mean value for many natural channels (Jarrett 1984[19)]; Hulsing et al. 1966[20)]). Chow (1959)[12)], Henderson (1964)[17)], and French (1985)[16)] note that in some open channel cases α can exceed 2, corresponding to a uniform gradient profile having a value of m of 1.0.

**(2) Values of Energy Correction Coefficient along the Ramp**

Along a non-uniform flow in a stream or a river, the energy coefficient tends to vary from section to section. It also tends to vary with water depth and discharge. In the previous section, it has been shown how to calculate the energy correction coefficient at the ramp crest (α) from Eq. (9). Once the value of α at the ramp crest has been known, we can calculate α at any distance along the ramp by applying the energy conservation equation between the ramp crest section and any section along the ramp. The energy conservation equation between two sections (1&2) as shown in Fig. (4) can be written as follow;

$$\Delta Z + \lambda_1 .h_1 + \alpha_1 .\frac{v_1^2}{2.g} = \lambda_2 .h_2 + \alpha_2 .\frac{v_2^2}{2.g} + S_f .\Delta X \tag{11}$$

where the different variables as defined in Fig. (4) and $S_f$ is the frictional slope which can be calculated as follow;

$$S_f = \frac{f}{8}.\frac{1}{h_{av}}.\frac{v_{av}^2}{g} \tag{12}$$

where f was taken as the friction factor at normal flow, $h_{av}=h_1+h_2/2$, and $v_{av}=v_1+v_2/2$.

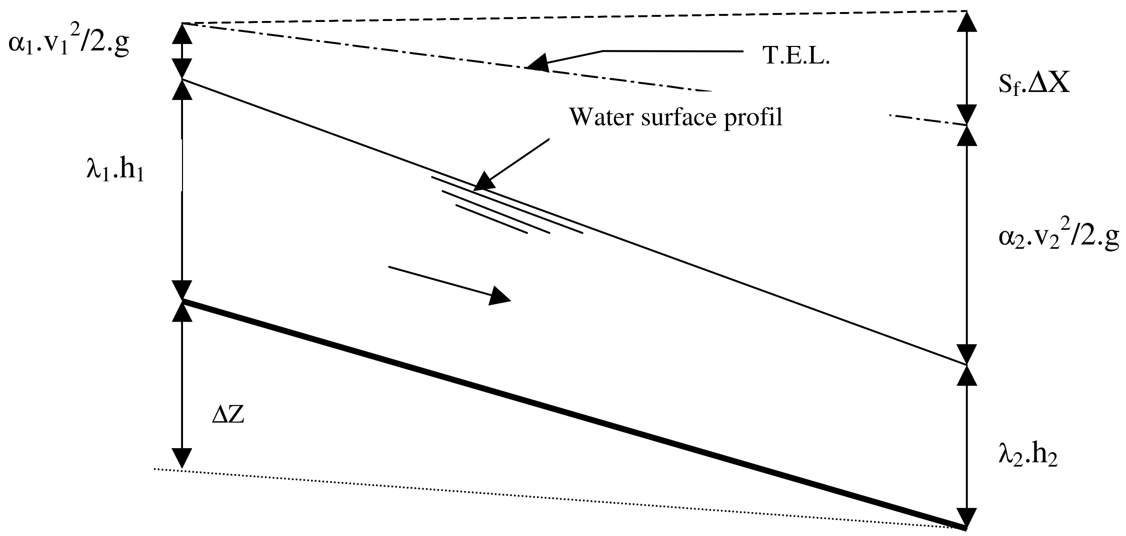


**Fig. (4): Schematic sketch for the variables**

Figure (5) shows values of the energy correction coefficients calculated from Eq. (11) along the ramp for different values of the dimensionless parameter $q/(g.k^3)^{0.5}$. As shown from that Fig., values of α increase by decreasing $q/(g.k^3)^{0.5}$ values. Also, at the same $q/(g.k^3)^{0.5}$ the value of α increases by increasing the distance from the ramp crest until it reaches a nearly constant value.

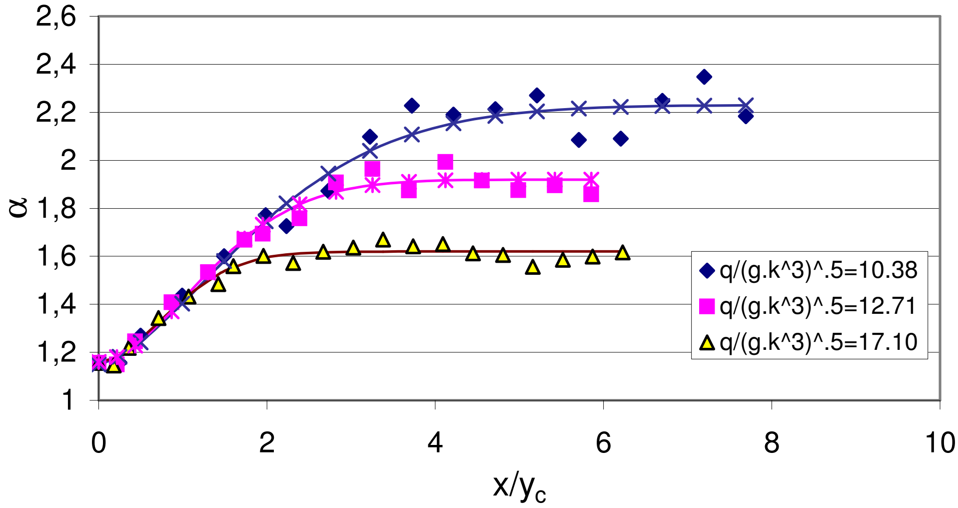


**Fig. (5): Values of the energy correction coefficient (α) along the ramp for different discharges**

### (3) Values of the Momentum Correction Coefficient (β) along the Ramp

As explained in Sec. (4.1), the momentum correction coefficient at the ramp crest can be calculated from the second definition for the critical depth. In equation (10), if the exponent m at the ramp crest is considered equal to 0.3 as calculated from the velocity distribution, then β value will be equal to 1.056. Once the value of β at the ramp crest has been known, we can calculate β at any distance along the ramp by applying Bernoulli's equation between the ramp crest section and any section along the ramp. The Bernoulli's equation between two sections (1&2) can be written as follow;

$$\Delta Z + \lambda_1.h_1 + \beta_1.\frac{v_1^2}{2.g} = \lambda_2.h_2 + \beta_2.\frac{v_2^2}{2.g} + S_f.\Delta X \qquad (13)$$

where the different variables as defined before.

Figure (6) shows values of the momentum coefficient calculated from Eq. (13) along the ramp for different values of the dimensionless parameter $q/(g.k^3)^{0.5}$. As shown from that Fig., values of β increase by decreasing $q/(g.k^3)^{0.5}$ values. For the same $q/(g.k^3)^{0.5}$ value, β values increase by increasing the distance from the ramp crest.

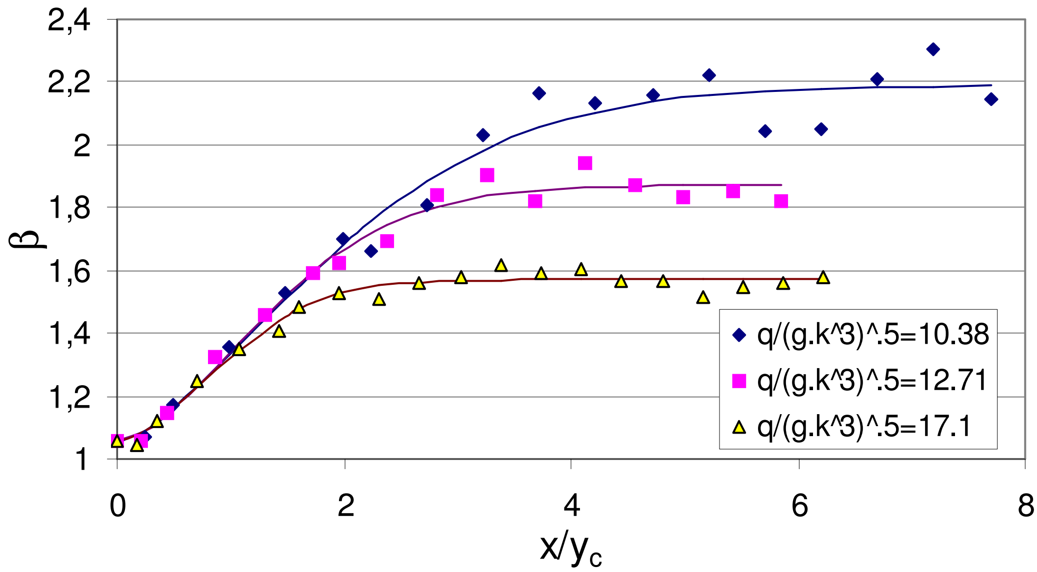


**Fig. (6): Values of the momentum correction coefficient (β) along the ramp for different discharges**

## 5. MODIFIED SHALLOW WATER EQUATIONS

In the field of river engineering and open channel flow, many calculations are made to predict the water levels and flow velocities, the computer programs used are based on the two-dimensional shallow-water equations (SWE). The similarity between the predicted and measured water levels and flow velocities is good in uniform flow cases. However, in the case of non-uniform flow, the predicted water levels are different than the measured.

These deviations may be due to limited applicability of the usual form of the SWE in the computations. These equations are commonly derived by depth-integrating the continuity equation and the Navier-Stokes equation in flow direction, assuming vertical profiles of pressure (hydrostatic) and velocity (e.g. logarithmic) as in uniform flow. However, in accelerating or decelerating flow the profiles deviate from the hydrostatic pressure. Simplifications made in the conventional SWE are responsible for the differences between the measured and predicted water levels and flow velocities. Therefore, the previous corrections are invoked in the SWE. The depth-integrated continuity equation for steady two-dimensional flow is kept unchanged:

$$\frac{dUh}{dx} = 0 \tag{14}$$

in which $U$ is the depth-averaged velocity and $h$ the water depth.
Depth-integrating the Navier-Stokes equation in flow direction yields the momentum equation of the SWE:

$$\frac{d}{dx}(\beta \rho U^2 h) + \lambda \rho g h \frac{d\eta}{dx} + (f \rho U^2) = 0 \tag{15}$$

in which $\beta$ and $\lambda$ are the correction coefficients for momentum flux and pressure respectively, $\rho$ is the water density, $\eta$ is the water level, g is the gravitational acceleration and f is the friction factor. In the above equation, the convection coefficient $\beta$ describes the influence of the non-uniformity of the velocity profile on the depth-integrated momentum flux. The pressure coefficient $\lambda$ describes the influence of the deviation from the hydrostatic pressure.

## 6. CONCLUSION

In this study, a method for computing the pressure and velocity correction coefficients from the water surface profile for accelerating flow along block stones ramp was developed. It was found that the pressure correction coefficient is less than 1.0 in accelerated flow and the maximum deviation occurs at the ramp crest. The value of the energy correction coefficient is nearly constant at the control section at the beginning of the chute and equal to 1.156 and increases along the ramp until it reaches a constant value at the normal depth. Also, the value of the energy correction coefficient along the ramp decreases by increasing the discharge. The value of the momentum correction coefficient has the same trend for the energy correction coefficient but with smaller values. Based on these results, it can be concluded that the numerical techniques simulating flows over rough steep channels should include these corrections. This simple method for calculating the velocity distribution coefficients still needs validation in the future from velocity measurements and formulation in empirical equations.

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