DYNAMIC MODEL FOR SUBCRITICAL DIVIDING FLOWS IN OPEN CHANNEL JUNCTION

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ABSTRACT

The flow through a channel junction is phenomenon that involves numerous variables, like the number of adjoining channels, the angles of intersection, the shape, the slope of the channel bed, the direction and the discharge of flow. The problem is so complicated that only a few simple and specific cases have been studied.

In the present study, a one-dimensional theoretical new model for subcritical flows in dividing open channel junction is developed. The new model is derived with the aid of the overall mass conservation together with the momentum principle in the streamwise direction to two control volumes through the junction. The component weight of the water and the boundary friction forces acting on the two control volumes and the separation zone shear force acts on the branch channel are included in the analysis.

Given the inflow discharge, depth and a downstream boundary condition the proposed model solves for each discharges and depth in main channel extension and branch channel.

Keywords: *Dividing, Junction, Open channel, Intake*

1. INTRODUCTION

The division of flows in open channels has a direct application in the design of water and wastewater treatment plants and open channel networks of irrigation and drainage system. There have been few attempts at modeling the flow in dividing.

For a specific dividing flow (T-junction), Taylor (1944) [1], has made the first experimental approach to the problem. He was suggested graphical representation in terms of ratio of depths and discharges.

Grace and Priest (1958) [5], presented experimental results covering the cases of dividing flow in a model of rectangular channel in which different ratios of the width of the straight main channel to the branch channel could be obtained and the angle of the branch be varied. Yoshimi and Stelson (1963) [5], investigated the sewage flow from a single inlet conduit to multiple grit channels and proposed a comprehensive description of the flow pattern in the junction region. Rajaratnam and Thiruvengadam (1962-1963) [5], was studied a dividing flow with a subcritical flow in a horizontal main channel and a supercritical flow in a sloping branch channel. Law and Reynolds (1966) [5], used both analytical and experimental investigations to study dividing flow. In Law and Reynolds study, the ratio of the depths in the main channel upstream and downstream of the junction was assumed to be unity. Hagger (1983) [9], proposed a simplified model to evaluate loss coefficients for flow through the branch channel. Ramamurthy and Satish (1988) [8], found that, for a short branch channel with the downstream Froude number exceeding a threshold value, the branch flow exhibits a unsubmerged recirculation region.

Ramamurthy, et al. (1990) [9], were presented an estimate of the discharge ratio in terms of the Froude number in the main channel upstream and downstream of the junction. They declared that experimental data is provided to validate their proposed model. Hager (1992) [4], derived an expression for the energy loss coefficient across a division. He assumed critical flow at the maximum width-contracted section and concluded that the branch discharge coefficient is simply a function of upstream Froude number and discharge ratio. Issa and Oliveira (1994) [7], were carried the first study to conduct a 3D turbulent flow simulation for T-junction geometries. Neary, V. S., and Odgaard, A. J. (1993) [6], presented an experimental of the flow structure at a 90^0 open channel diversion. Neary, et al. (1999)[7], were developed and validate a numerical method for modeling 3D lateral-intake.

Chung-Chieh Hsu, et al. (2002) [3], concluded that the energy heads upstream and downstream of the division in the main channel are found to be almost equal. They gave the energy-loss coefficient of a division is expressed in terms of discharge ratio, upstream Froude number and depth ratio.

Shazy Shabayek, et al. (2002) [10], were developed a one-dimensional theoretical model for subcritical flows in combining flows in open channel.

In the present study analytical solution is developed to solve the dividing flow in an open channel junction. A one-dimensional theoretical model for subcritical flows in dividing open channel junction is developed. The objective of the present study is to verify the two-control volume approach for the subcritical dividing steady flow case as a first step towards the development of a more general formulation.

2. Theoretical Analysis

As the flow approaches the division, the suction pressure at the end of the branch channel accelerates it laterally. This causes the flow to divide so that a portion enters the branch channel with the remainder continuing downstream the main channel. The portion withdrawn by branch is delineated by a curved shear-layer surface, denoted as the dividing stream surface (streamline).

The flow in the junction channels may be subcritical or supercritical, depending on the state of the extension and branch channel. Also, the flow in the junction depends on the backwater effect of the two downstream channels below the dividing junction as well as on the dynamic condition existing at the junction. To solve the dividing problem, six variables must be evaluated; they are the depths and discharges at the three sections enclosing the junction.

In the present study of subcritical flow, the boundary conditions are specified as the inflow discharge and depth and a third downstream condition that can be either a fixed depth or a rating curve. These boundary conditions define three of the six variables in the problem or two variables and one equation. Applying overall mass conservation to the junction and conservation of streamwise momentum to each of the two control volumes provides the three necessary equations.

In the present study, the junction is divided into two control volumes, as shown in Fig. (1). The considered two control volumes are: one for the main channel flow (C.V.1), and the other for the branch channel flow (C.V.2). The control volumes are bounded by streamlines such that there are no lateral mass fluxes. The separation zone shear force acting on the branch channel control volume, the weight component in the direction of the slope, and the boundary friction force are accounted for in the analysis.

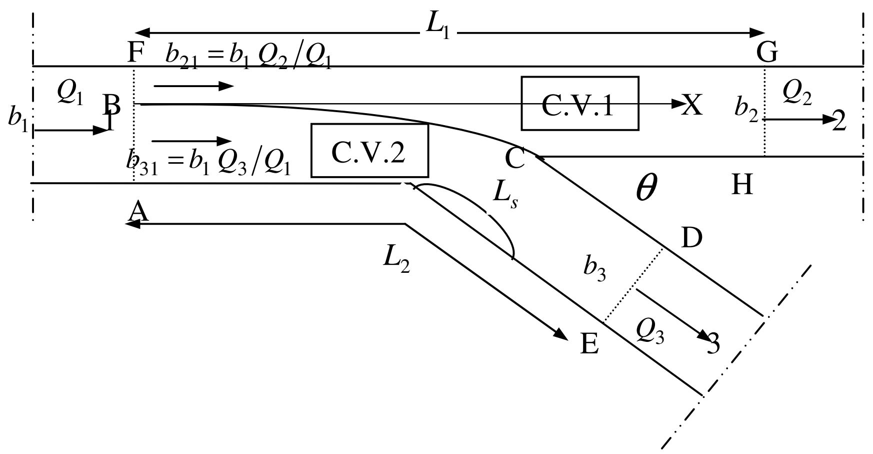

Figure (1) Dividing Flow in Open Channel Junction

The division flow between two channels may be determined with the aid of momentum principle and mass continuity with the following assumptions:

1. The channels are rectangular cross-sectionals;
2. The flow is from channel 1 into channels 2 and 3;
3. Channels 1 and 3 lie in straight line;
4. The flow is parallel to the channel walls, and the velocity is uniformly distributed immediately above and below the junction;
5. The velocities and water surface elevations are constant across the channels at the inflow and outflow sections of the control volumes.
6. The pressure distribution is hydrostatic at sections 1, 2 and 3.
7. Channel widths, control volume lengths, resistance characteristics, and slopes are known;
8. The stream surface curvature is considered small and vertical acceleration negligible; hence the pressure acting along the dividing stream surface between two control volumes is hydrostatic , equal and opposite;

9. The flow at section AB, BF, and AF are nearly uniform $Q_1/b_1 = Q_2/b_{12} = Q_3/b_{13}$, and momentum coefficients at these sections are close to unity; and
10. For the dividing stream surface BC, the stagnation point at C, and hence, the depth y at an indefinitely small section ds, varies from y_1 at B to $y = y_1\left(1 + F_1^{\,2}/2\right)$ at point C. This variation is assumed to follow a very simple relation $y = y_1 + az^2$ where a is a constant and z is the distance from ds to the line BX [6].

According to these assumptions and by applying the momentum principle and mass continuity equation to the dividing junction in the direction of 1 to 2 the required three equations are obtained.
Applying overall mass conservation to the junction gives

$$Q_1 = Q_2 + Q_3 \tag{1}$$

where Q_1 = the discharge in the main channel; Q_2 = the discharge in the extension channel; Q_3 = the discharge in the branch channel.

Applying the conservation momentum in the respective streamwise direction for the channel control volume, C.V.1, gives

$$\rho Q_2 V_2 - \rho Q_2 V_1 = P_{21} - P_2 + B_1 + W_1 - F_{b2} \tag{2}$$

where ρ = density of water; V_1 = the mean velocity in the main channel ; and V_2 = the mean velocity in the extension channel; P_{21} = the hydrostatic force at section 1; P_2 = the hydrostatic force at section 2; B_1= the hydrostatic force acting across the dividing stream surface; W_1= the weight of the water; and F_{b2} = the friction force to the bed and walls of the channel.
Also, conservation momentum in the respective streamwise direction for the channel control volume, C.V.2, gives

$$\rho Q_3 V_3 - \rho Q_3 V_1 = P_{31} - P_3 + B_2 + W_2 - F_{b3} - F_s \tag{3}$$

where V_3 = the mean velocity in the branch channel; P_{31} = the hydrostatic force at section 1; P_3 = the hydrostatic force at section 3; B_2= the hydrostatic force acting across the dividing stream surface; W_2= the weight of the water; F_{b3} = the friction force to the bed and walls of the channel; and F_s = the separation zone shear force.
The forces in the two Equations (2) and (3) can be detailed as:

1- The momentum forces $\rho Q_2 V_2$, $\rho Q_2 V_1$, $\rho Q_3 V_3$ and $\rho Q_3 V_1$ are due to the flux of the two control volumes. These force are computed as

$$\rho Q_2 V_2 = \gamma b y_2^{\ 2} F_2^{\ 2} \tag{4-a}$$

$$\rho Q_2 V_1 = \gamma b y_1^{\ 2} \frac{Q_2}{Q_1} F_1^{\ 2} \tag{4-b}$$

$$F_2^{\ 2} = F_1^{\ 2} \frac{Q_2^{\ 2} y_1^{\ 3}}{Q_1^{\ 2} y_2^{\ 3}} \tag{4-c}$$

$$\rho Q_3 V_3 = \gamma b_3 y_1^{\ 2} F_3^{\ 2} \tag{4-d}$$

$$\rho Q_3 V_1 = \gamma b \frac{Q_3}{Q_1} y_1^{\ 2} F_1^{\ 2} \tag{4-e}$$

$$F_3^{\ 2} = F_1^{\ 2} \frac{Q_3^{\ 2} y_1^{\ 3}}{Q_1^{\ 2} y_3^{\ 3}} \tag{4-f}$$

where F_1= the Froude number in the main channel ($F_1^{\ 2} = V_1^{\ 2} / g y_1$), F_2= the Froude number in the extension channel ($F_2^{\ 2} = V_2^{\ 2} / g y_2$), and F_3= the Froude number in the branch channel ($F_3^{\ 2} = V_3^{\ 2} / g y_3$).

2- The hydrostatic forces P_{21}, P_2, P_{31}, and P_3 are due to the water pressure on the upstream and downstream boundaries of the two control volumes. These forces are given by

$$P_{21} = \frac{1}{2} \gamma b_{21} y_1^{\ 2} \tag{5-a}$$

$$P_2 = \frac{1}{2} \gamma b_2 y_2^{\ 2} \tag{5-b}$$

$$P_{31} = \frac{1}{2} \gamma b_{31} y_1^{\ 2} \tag{5-c}$$

$$P_3 = \frac{1}{2} \gamma b_3 y_3^{\ 2} \tag{5-d}$$

where γ = specific weight of water; b_{21}= the width of main channel breadth which passing Q_2; b_2 = the width of the extension of main channel at section 2; b_{31} = the width of main channel breadth which passing Q_3; b_3 = the width of the branch channel at section 3; y_1 = the depth of the water in the main channel; y_2 = the depth of the water in the extension channel; and y_3 = the depth of the water in the branch channel.

3- The pressure forces B_1 and B_2 are acting in the dividing stream surface of each control volume. This pressure forces can be computed as

$$B_1 = B_2 = \int \frac{1}{2} \gamma y^2 ds. \cos \varphi \tag{6}$$

where φ= the angle between stream surface and the branch channel axis.
For the dividing streamline BC, the stagnation point at C, and hence, the depth y at an indefinitely small section ds, varies from y_1 at B to $y = y_1\left(1 + F_1^2/2\right)$ at point C. This variation is assumed to follow a very simple relation $y = y_1 + az^2$ where a is a constant and z is the distance from ds to the line BX ($dz = ds.\cos\varphi$).
Integrating equation (6) gives

$$B_1 = B_2 = \int_0^{(b_2-b_{21})} \frac{1}{2}\gamma\left(y_1 + \frac{F_1^2 y_1}{2(b_2-b_{21})^2} z^2\right)^2 dz \qquad \text{(7-a)}$$

$$B_1 = B_2 = \int_0^{(b_2-b_{21})} \frac{1}{2}\gamma\left(y_1^2 + \frac{F_1^2 y_1^2}{(b_2-b_{21})^2} z^2 + \frac{F_1^4 y_1^2}{(b_2-b_{21})^4} z^4\right)dz \qquad \text{(7-b)}$$

$$B_1 = B_2 = \frac{1}{2}\gamma(b_2 - b_{21})y_1^2\left(1 + \frac{F_1^2}{3} + \frac{F_1^4}{20}\right) \qquad \text{(7-c)}$$

4- The component of the weight of water in each control volume can be computed as

$$W_1 = \gamma\left(\frac{A_{21} + A_2}{2}\right)L_1 S_{o1} \qquad \text{(8-d)}$$

$$W_2 = \gamma\left(\frac{A_{31} + A_3}{2}\right)L_2 S_o \qquad \text{(8-e)}$$

where A_{21} = the cross-sectional area of the main channel at section 1; A_2 = the cross-sectional area at the extension of main channel, section 2; A_{31} = the cross-sectional area of the main channel at section 1; A_3 = the cross-sectional area of the branch channel at section 3; L_1 = the length of the control volume C.V.1; L_2 = the length of the control volume C.V.2; S_{o1} = the average of bed slopes of the main and extension channel; and S_{o2} = the average of bed slopes of the main and branch channel.

5- The friction forces on the two control volumes due to the bed and the walls of the channels can be given as

$$F_{b2} = \rho\left(\frac{V_1}{C}\right)^2\left[\frac{b_{21} + b_2}{2} + y_1\right].\frac{L_1}{2} + \rho\left(\frac{V_2}{C}\right)^2\left[b_2 + 2y_2\right].\frac{L_1}{2} \qquad \text{(9-a)}$$

$$F_{b3} = \rho\left(\frac{V_1}{C}\right)^2\left[b_{31} + y_1\right].\frac{L_2}{2} + \rho\left(\frac{V_3}{C}\right)^2\left[b_3 + 2y_3\right].\frac{L_2}{2} \qquad \text{(9-b)}$$

where ρ = density of water, and C = dimensionless Chezy coefficient.

6- The separation zone shear force acts on the branch channel control volume, C.V.2, only, due to the recirculating at the entrance. It is computed as

$$F_s = C_f \frac{\rho V_1^2}{2} . y_1 . L_s \tag{10}$$

where C_f = coefficient of friction, and L_s = the length of the separation zone interface. Substituting all the above-mentioned forces into the momentum equation for the two control volumes, Equations (2) and (3), the resulting equation for the C.V.1 is

$$\begin{aligned}\gamma b_2 y_2^2 F_2^2 - \gamma b_{21} y_1^2 F_1^2 &= \frac{1}{2}\gamma b_{21} y_1^2 - \frac{1}{2}\gamma b_2 y_2^2 - \frac{1}{2}\gamma (b_2 - b_{21}) y_1^2 \left(1 + \frac{F_1^2}{3} + \frac{F_1^4}{20}\right) \\ &+ \gamma \left(\frac{A_{21} + A_2}{2}\right) L_1 S_{o1} - \rho \left(\frac{V_1}{C}\right)^2 \left[\frac{b_{21} + b_2}{2} + y_1\right] . \frac{L_1}{2} \\ &- \rho \left(\frac{V_2}{C}\right)^2 \left[b_2 + 2 y_2\right] . \frac{L_1}{2}\end{aligned} \tag{11}$$

For C.V.2, this gives

$$\begin{aligned}\gamma b_3 y_3^2 F_3^2 - \gamma b_{31} y_1^2 F_1^2 &= \frac{1}{2}\gamma b_{31} y_1^2 - \frac{1}{2}\gamma b_3 y_3^2 - \frac{1}{2}\gamma (b_2 - b_{21}) y_1^2 \left(1 + \frac{F_1^2}{3} + \frac{F_1^4}{20}\right) \\ &+ \gamma \left(\frac{A_{31} + A_3}{2}\right) L_1 S_{o1} - \rho \left(\frac{V_1}{C}\right)^2 \left[b_{31} + y_1\right] . \frac{L_2}{2} \\ &- \rho \left(\frac{V_3}{C}\right)^2 \left[b_3 + 2 y_3\right] . \frac{L_2}{2} + C_f \frac{\rho V_1^2}{2} . y_1 . L_s\end{aligned} \tag{12}$$

It should be noted that Equations (8) and (9) do not explicit include the angle of the branch channel.

Nondimensionlizing Equations (8) and (9), in terms of the discharge ratio $R_q = Q_2/Q_1$, the depth ratios $R_y = y_1/y_2$ and $R_{y3} = y_1/y_3$, the widths ratios $R_b = b_2/b_1$ and $R_{b3} = b_3/b_1$, and the upstream Froude number F_1, results in the following equations:

$$R_b F_1{}^2 R_q{}^2 R_y - R_q F_1{}^2 = \frac{R_q}{2} - \frac{R_b}{R_y{}^2} - \frac{1}{2}\left(R_b - R_q\right)\left(1 + \frac{F_1{}^2}{3} + \frac{F_1{}^4}{20}\right)$$
$$+\left(\frac{R_q + \left(R_b / R_y\right)}{2}\right)\frac{L_1}{y_1} S_{o1} \tag{13}$$
$$-\frac{F_1{}^2 L_1}{2C^2 y_1}\left\{\left(\frac{R_q + R_b}{2} + \frac{y_1}{b_1}\right) + R_q{}^2 R_y{}^2\left(R_b + \frac{2y_2}{b_1}\right)\right\}$$

$$R_{b3} F_1^2 (1-R_q)^2 R_{y3} - (1-R_q) F_1^2 = \frac{(1-R_q)}{2} - \frac{R_{b3}}{R_{y3}{}^2} - \frac{1}{2}\left(R_b - R_q\right)\left(1 + \frac{F_1^2}{3} + \frac{F_1^4}{20}\right)$$
$$+\left(\frac{(1-R_q) + \left(R_b / R_y\right)}{2}\right)\frac{L_1}{y_1} S_{o2}$$
$$-\frac{F_1^2 L_2}{2C^2 y_1}\left\{\left(\frac{(1-R_q) + R_{b3}}{2} + \frac{y_1}{b_1}\right) + (1-R_q)^2 R_{y3}{}^2\left(R_{b3} + \frac{2y_3}{b_1}\right)\right\}$$
$$-C_f F_1{}^2 \frac{L_s}{2b_1} \tag{14}$$

Equations (1), (13) and (14) are the three required equations to solve the subcritical steady dividing flows in open channel junctions.

3. ANALYSIS OF RESULTS

The derived Equations (11) and (12) do not include the angle of the branch channel. However, the angle may have an indirect influence through the magnitude of the separation zone shear coefficient C_f, and the separation zone shear length, L_s.

Equations (13) and (14) are two nonlinear equations that can be solved for the values of R_q and R_{y3}, given R_q and F_1. The component of the water weight in the direction of the slope and the friction forces acting on the two control volumes and the separation zone shear force acts on the branch channel are included in the present analysis.

Given the inflow discharge, depth and a downstream condition, the proposed model calculates the downstream and branch discharges and depths. Channel widths, control volume lengths, shear coefficient, and slopes are known.

The advantage of the proposed model is in its capability to be scaled up to prototype applications, since it includes most of the physical effects.

4. CONCLUSIONS

A one-dimensional theoretical new model for the case of steady subcritical dividing open channel junction flows, which can be incorporated as an enhancement in current open channel network models, is introduced.

For dividing flow in rectangular open channels a theoretical model is developed to relate the discharge ratio $R_q = Q_2/Q_1$ with the Froude number F_1 and the depth ratios $R_y = y_1/y_2$ and $R_{y3} = y_1/y_3$.

The present model has wider applications, since it includes most of the physical effects neglected in other previous theories.

The advantage of the proposed method is its capability to be scaled up to prototype applications, since it includes most of the physical effects in other theories, such as the boundary friction force.

The application of the momentum principle in the streamwise direction to two control volumes in the junction allows the model to be easily implemented in network models and makes handling of the junctions consistent with that of the channel reaches.

A model test is necessary for each channel shape, if an accurate prediction of junction performance is required. The empirical coefficient of shear should be calibrated at a specific site, with guidance from experimental or field data. Unfortunately, no experimental data is available at this time for verification of the present model.

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