A CONCEPTUAL FRAMEWORK FOR MODELING THE WEST GEDAREF SANDSTONE AQUIFER

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ABSTRACT

In this paper a model concept is set up for the simulation of the aquifer system in the over-pumped well fields west of the Gedaref city. Assumptions are made on different system elements that would ultimately be confirmed by numerical simulation. Prior estimates of system parameters as well as calibration targets are established to measure confidence on the model results.

1. Introduction

A typical modeling process starts with data integration and description of the hydrogeological setup, then the definition of the conceptual framework for analysis, and finally comes the numerical approximation. The conceptual framework represents an important phase in defining the quantitative framework within which a numerical scheme works. It identifies and specifies the different steps, which can be taken in the process of formulating, analysis, evaluating and presenting alternative models (**KOUDSTAAL**, 1992). According to SUN (1994) application of sound hydrologic reasoning during the development of an appropriate conceptual model of flow represents a full 90 % of the solution to most hydrogeologic problems.

Three components are discussed in the development of a conceptual framework for flow modeling in the study area. These are the hydrogeologic framework in section 2, the nature of the flow system in section 3, then targets for model calibration and prior estimates of the hydraulic parameters are presented in section 4.

2. The Hydrogeological Framework

The hydrogeological framework includes the outline of model geometry, and the different hydrogeologic units.

2.1. Model Areas confines

The appropriate space and time scales are chosen in relation to the heterogeneity of the system under study and the data available for calibration.

Investigating the subsurface geology of the west Gedaref (**MIRGHANI**, 2002), it became visible that, the basin is divided into three sub-areas of varying sandstone thickness. Groundwater flow in west-Gedaref is dominated by local and sub-regional flow systems (**MIRGHANI,** 2002). Trial to generate regional flow for the whole investigated area is expected to fail in simulating important sub regional features. This is in agreement with RUSHTON'S (1979) conclusion on scaling of regional flow models. He concluded that there is no regional movement of groundwater in hard rocks of transmissivity less than 100 m^2/d. In addition to the slow movement of water, the groundwater gradient in hard rock aquifers is dominated by the topography with groundwater movement mainly towards the nearest valley.

Although data from Azaza and Abu-Naga wellfields show different stratigraphic characteristic, it appeared that they are hydraulically connected. Hence, the two wellfields are tapping a single aquifer and are included in the model area (Fig. 1).

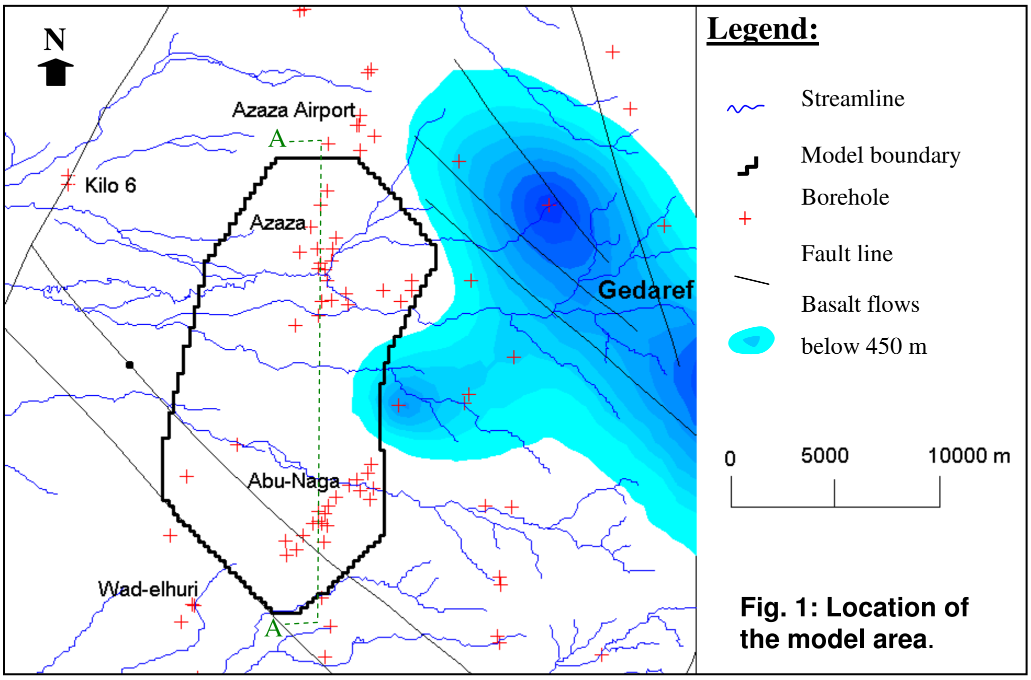

Fig. 1: Location of the model area.

The model boundaries were chosen to coincide closely with the limits of the continuous/regional Nubian sandstone at the east. At the west, the boundary is limited roughly to the maximum possible range for extrapolation of available information. Both to the north and to the south the selected boundaries coincide with clear changes in the Nubian aquifer characteristics, which are believed to mark the sub-basin boundaries. The selected model area covers around 200 km^2. It extends 10 km to the west of the Basalt Nubian contact, and 20 km along a north-south axis from Azaza Airport to the southern edge of the Abu-Naga well field. The selected boundaries shown in figure 1 coincide with some physical features, including a water divide in the north, Basalt thickness contour of 440 m above mean sea level at the northeastern boundary, thin Nubian formation of less than 100 m at the southwest, and inferred fault lines at the south and the southeast.

2.2. Defining the Hydrogeologic units

Two types of hydrogeologic units are identified for the purpose of modeling the Azaza-AbuNage system. The first type is based on the stratigraphic units. The classical method (DE MARSILY et al., 1998) is followed to represent the complex geologic formation within the Azaza-Naga sedimentary basin. With the main purpose to simulate the hydrogeologic behavior, the geologic formation is decomposed into aquifers and aquitards, and then the system is represented schematically as a multi-layered. A second type of hydrogeologic units is introduced in the model to compensate for the lack of data on fractures conductivity. It is used due to the need to characterize the fractures effect using the inverse modeling without prior interpretation of field data. Thus, the fracture system is defined in terms of hydrogeologic units. According to LONG et al. (1997) this will eliminate the need for an intermediate conceptual model to interpret the structural data and may result in a parameter that has little relevance to any flow system.

Drilling logs showed that the geology consists of interbedded sandstone and mudstone of the Gedaref Formation. Using stratigraphic boundaries and regional head data together with the filter position, hydrostratigraphic units of similar properties are identified. The aquifer system is made up of three layers and confining units. The upper aquifer zone lies at 60 – 80 m below ground level (BGL), the middle at 90 – 135 m BGL, and the lower is at 140 – 230 m BGL. Figure 2 shows the layering scheme along a North South direction.

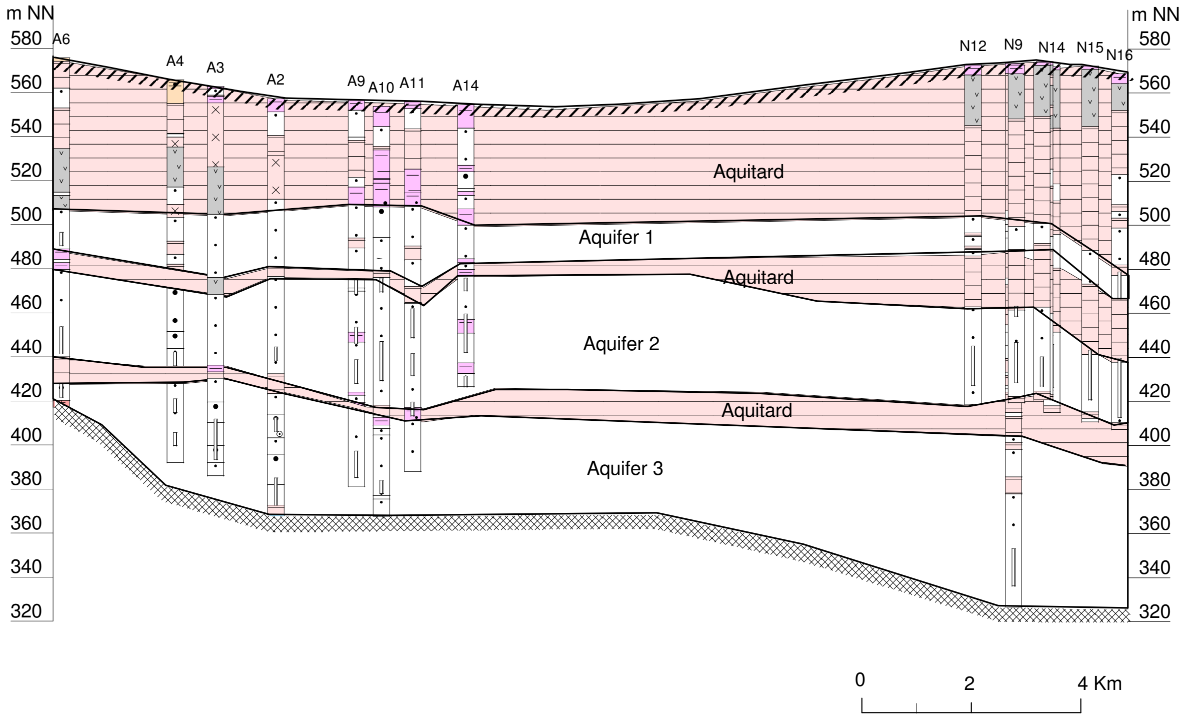

Fig. 2: Scheme of the Hydrostratigraphic units identified in the model area along profile A (fig.1).

The above configuration has taken into account the validity of the governing equations studied by **CONNORTON** (1985) for multi-layered aquifers. In the case of multi-layered aquifers it is recommended to split-up the total range of integration over z (vertically) such that q_x, q_y are sufficiently smooth over each sub-interval of integration. Leakage to and from each layer can be incorporated into the upper and/or lower boundary conditions for each layer.

Three hydrostratigraphic units are considered for the proposed flow model, forming a confined aquifer (the middle layer) and two confining beds. The top and bottom aquifers are not explicitly represented in the model because of lack of information about their properties. The middle aquifer layer is modeled as a leaky confined aquifer. Leakage through the confining Mudstone (which has a vertical hydraulic conductivity much lower than that of the Sandstone aquifer) is modeled with a source/sink term.

The leakage rate from/to the upper/lower aquifer layers depends on the vertical conductivity of the confining beds. Areas with thick mudstone beds (e.g. around Abu-Naga) are assigned zero leakage. However, high leakage is considered at borings screened along two or three layers to account for vertical flow between these layers at well locations.

The configuration of fractures considered as hydrogeologic units in the model is conformed to those identified from satellite imagery (MIRGHANI, 2002). However the final adopted units will depend on their effect on the calibrated numerical model.

3. The flow system conception

The flow system conceptual model is based on the assessment of information available from head and transmissivity data combined with the lithological data.
Focusing on the middle model layer, the scheme below (fig. 3) is meant to describe the flow pattern. The flow model is controlled by the multi-layer aquifer system defined above.

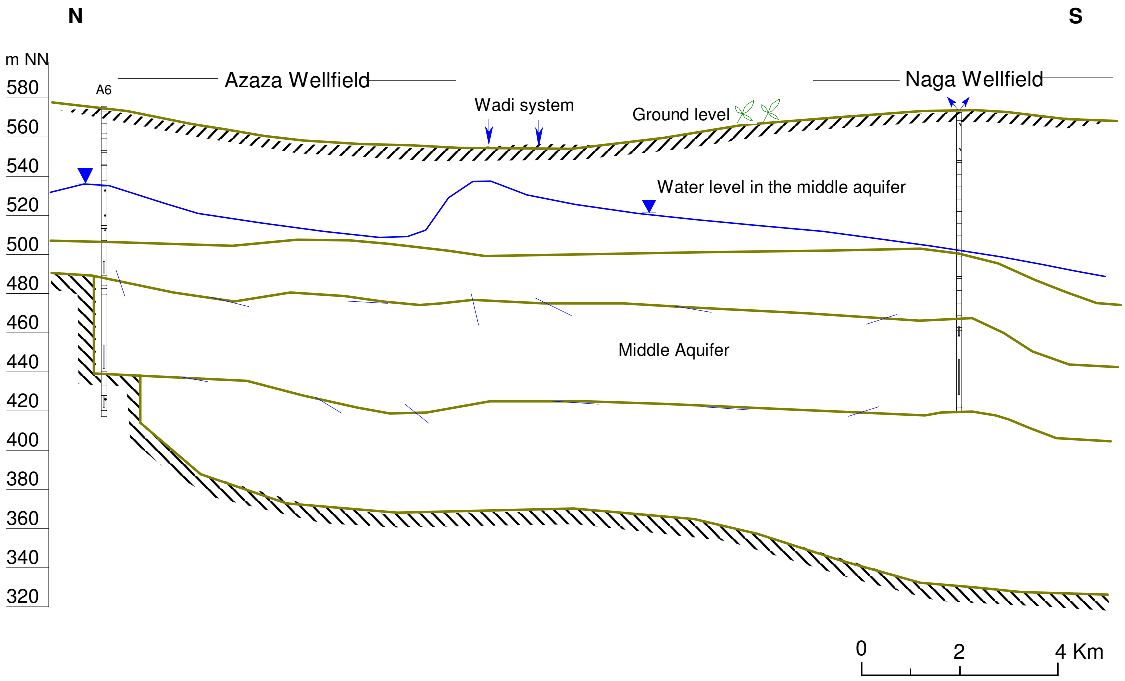

Fig. 3: Schematic diagram showing horizontal and vertical flows in the middle layer of the model.

The following assumptions are considered to enable the subsequent quantification of flow components:

1- The flow occurs under confined to leaky condition, with leakage varying in space;
2- The aquifer is under steady conditions before 1992 (base year), unsteady flow regime prevails in the aquifer after 1992, due to the extensive pumping, and to seasonal variation of natural gradients;
3- The flow in the aquifer is horizontal, and is represented with a planar depth averaged, two-dimensional model:

$$S\frac{\partial h}{\partial t}+\frac{\partial \overline{q}_i}{\partial x_i}=\overline{Q}_\rho$$

$$\overline{q}_i=-T_{ij}\frac{\partial h}{\partial x_i}$$

Where:

S = storativity,

h = hydraulic head,

q_i = Darcy velocity vector,

Q_ρ = source/sink term,

T_{ij} = transmissivity tensor,

t = time,

x_i = spatial coordinates.

4- The interaction between the model and the upper/ lower aquifers is approximated by a source/ sink leakage on top/bottom of the model;
5- The aquifer is heterogeneous and can be represented by zoned or continuous heterogeneity;
6- Horizontally, the flow direction in the porous matrix is probably affected by the prevailing structural pattern;
7- Fractures in the sandstone formation are modeled as discrete units or as equivalent continuum.

4. Parameterization and uncertainty analysis

This step deals with the estimation of different parameters characterizing the system model described above.

Two types of information are considered, namely:
1- Sample information: These are state variables such as head distribution and fluxes estimated from available field measurements. Uncertainty analysis is carried out to establish the plausible range of errors in the estimated parameters. (to be used as calibration targets for numerical approximation).
2- Prior information: These are estimates of the system parameters. Such parameters are separated into distributed ones in space such as transmissivity and storativity; and discrete parameters such as well discharge and constant values of head and recharge at the boundaries.

As a rule, the above information are not known accurately and their values are affected by uncertainty. Uncertainty associated with different parameters will be handled as described in the following sections.

4.1. Head

The error expected in head values could be attributed to various sources, such as measurement errors, the accuracy of the topographic levels, the well design and the source of the data. Additionally, discrepancies between measured and simulated head arise from un-modeled small scale heterogeneity, discretization and interpolation errors. Errors typically coming from different sources are considered random and normally distributed (SUN, 1994, CHILES, 1999, CHRISTENSEN and COOLY, 1999, MEYERS, 1997); and are hence accounted for by the head variance. An estimate of the head variance for 2D steady state is provided by MINZEL (1982) as:

$$\sigma_h^2 = \frac{8}{\pi^2} J^2 \lambda^2 \sigma_{\ln T}^2$$

where σ_h^2 is the head variance, J is the mean gradient, λ is the correlation scale, and $\sigma_{\ln T}^2$ is the variance of $\ln T$.

The above variance value provides a measure of the accuracy or the closeness of measurements to the true unknown head. The square root of the calculated variance is used to indicate the range of plausible/target head. Below are estimates of the values considered for calculating MINZEL's head-variance in the study area.

$J = 0.00154$.

$\lambda = 8\ km$ according to the variogram-model fitted to the head measurements.

$\sigma_{\ln T}^2 = 0.34$.

Therefore, head residuals of the intended 2D simulation results at measurement locations are tolerated within a range of between $\pm 6.30\ m$.

Sources of errors contributed to the typical large variance of the head values in Gedaref case include:

1- Well design (called scale effect) caused by the varying length and position of the filter. Here wells tapping more than one aquifer layer don't reflect the true head due to enhanced leakage,

2- Transient effect due to pumping or seasonal recharge showed a variance of 0.55 to 2.22 m^2 (standard deviation of 0.74 – 1.80 m) reaching an extreme value of 3.20 m at borehole A16 in 1996 close to two khor lines,

3- Accuracy of the reference ground elevation, 4- Measurement error in the range of $\pm 0.05\ m$ is a possible human/ instrument error,

5- and finally, interpretation errors due to the sample configuration (**MIRGHANI,** 2002).

4.2. The Water Budget

The flow balance is an essential feature of any groundwater problem. The appropriate balance equation must be satisfied at zero time of simulation. As the starting conditions in groundwater simulation refer to a particular time in a continuous process, the base year 1992 is considered for the balance calculations.

Assuming 2D isotropic regional groundwater flow conditions, the flow balance is described by the differential equation:

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right)=S\frac{\partial h}{\partial t}-Q(x,y)$$

Where, h is the hydraulic head [L], T is the transmissivity [L^2T^{-1}], S is the dimension-less storage coefficient and Q is source/sink term per unit area of the aquifer [LT^{-1}]. The left-hand terms represent the lateral flow, and the right-hand terms represent inflows and water release from storage. Assuming that before 1992 the groundwater system in the model area existed in a state of dynamic equilibrium with negligible head variation, a steady state model can be used to simulate the starting conditions. This implies a long-term balance between natural recharge and discharge in the area. Therefore, the term $\frac{\partial h}{\partial t}$ in the balance equation above is neglected.

Using the above equation, a tentative water budget is calculated with the help of a computerized method recommended by **STOERTZ** and **BRADBURY** in 1989 (**ANDERSON,** 1992). Using the model grid (**MIRGHANI,** 2002), all nodes are given specified head values interpolated from available head measurements. The water balance module of FEFLOW code (**DIERSCH, 1998**) is used to calculate the total in/out flow through the boundaries, and to roughly define the recharge/ discharge zones. The flux analyzer is then used to estimate the vertical component of the flux.

The total in/out flow in the modeled aquifer is calculated at 7183 m^3/d. However, this budget is highly dependent on the estimated average transmissivity of $4.29\times10^{-4}\ m^2/s$ and on the interpolated heads taken from the water level contour map (**MIRGHANI,** 2002). They are also dependent on the scale of the model. Therefore, the total in/outflow in the calibrated model is accepted within $\pm10\%$ of the estimated value.

The flux analyzer of FEFLOW indicated significant vertical flow component estimated at 3623 m^3/d at low-lying areas around Al-Laya wadi system. This vertical component is represented in the source term $(Q(x,y))$ of the 2D water budget equation above. Upward (-ve) flux typically shows up around the two wellfields indicating the capture zones of pumping wells, and leakage to the underlying aquifer. This is calculated as 3855 m^3/d.

4.3. Prior Information

To reduce the uncertainty in the model estimated parameter, prior information should be incorporated (**YEH**, 1986, **ANDERSON** and **WOSSNER**, 1992, **GOMEZ,** 1989). The coefficient of variation $\left(\frac{s\tan dard \quad deviation}{mean}\right)$ is recommended by **ANDERSON** and **WOSSNER** (1992) to quantify the uncertainty associated with prior information. This will allow the estimation of the plausible range of parameter values and hydrologic stresses prior to calibration.

Transmissivity

Dealing with a fractured porous media, two approaches are followed to produce the transmissivity distribution within the aquifer.

The first approach is a deterministic one. Here identified discrete units (sub-aquifer units and/or fractures) assumed to have unique (homogeneous) transmissivity values to be provided by the inverse solution. The plausible Transmissivity range for each unit is estimated from available pumping tests. T is averaged over the identified homogenous zones using the geometric mean according to the effective transmissivity concept. This gives rise to a very low T-zone, a low, middle and a high one. The orders of magnitude of the four values are 2.8, 3.7, 5.0 and 10.0 $10^{-4}\ m^2/s$ respectively. The overall coefficient of variation of T estimates (equals ±0.05) indicates the plausible range for transmissivity estimates.

In the second approach, T of the porous sandstone matrix is regarded as continuous (i.e. spatially correlated) random variable affected with uncertainty. Therefore, its spatial variability can be described through geostatistical methods. Here, standard deviation maps produced by Kriging and stochastic simulation account for the range of certainty in the interpolated field.

4. Hydrogeologic Stresses

Hydrogeologic stresses include natural and man-made induced recharge and discharge. No measurements of the stresses are available in the Gedaref area. Estimates of natural recharge and discharge are provided in the previous section by calculating the steady state water budget based on T and h data. Discharge due to well abstraction in the model area is estimated from the capacity of the water tanks and the approximate pumping duration. The total abstraction in the area before construction of Azaza well-field in 1992 is estimated at around 3600 m^3/d. After 1992 pumping increased to around 7200 m^3/d (2.60 Million m^3/y, or about 2% of the annual precipitation in the area).
Due to the lack of measured flux data, 10 to 15% accuracy in the estimated water budget is considered a suitable target for the Gedaref model.

6. Conclusion

In this paper, a complete design of a flow model is provided within a prescribed framework. Decisions are made on the model area and the boundary conditions. Estimates of the state variables (head and water budget) as well as prior estimates of transmissivity and hydrologic stresses are given based on continuous or discrete structural analysis of available data.

To assess the calibration, and the underlying assumptions associated with the conceptual models, two principal points has to be emphasized, namely:

1- A model cannot be more accurate than the data used to build it. Thus, the first step is to analyze the certainty range and the available data limitations. This step will enable the evaluation of the inverse modeling effort.
2- Without adopting specific objective criteria one (the modeler or the decision maker) is never satisfied with whatever modeling effort.

Based on the above points, the problem was not the "flow" portion of the model, but carried on to error analysis and uncertainty evaluation processes.

To the end of this discussion, some shortcomings are discussed for future studies in the study area. In the present study the adopted assumptions and methods of analysis will govern the modeling outcome. The 2D-model representation of the west Gedaref hydrogeology was a necessary simplification of reality. However, dealing with heterogeneous porous fractured sandstone, strong spatial anisotropy is expected to dominate. Such complicated aquifer system would require advanced modeling concept based on 3D information and mapping, which is only possible with further investigations. Interaction between aquifer layers needs proper account of the spatial variability of the vertical hydraulic conductivity, which in turn requires proper pumping tests. Monitoring of seasonal variation in groundwater levels should pay attention to nearby pumping, and should continue throughout the year to allow adequate recharge estimation.

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