INVESTIGATION OF B-JUMP NEGATIVE STEP IN RADIAL STILLING BASINS

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ABSTRACT

Presence of steps in radial stilling basins affect positively or negatively the characteristics of the free hydraulic jumps depending upon the type of the step, the height of the step, the position of the step in the basin and the dominant flow conditions. The effects of all these factors on the characteristics of the free radial hydraulic B-jump are addressed through the present experimental investigation. It is found that the depth ratio and the length of jump ratio increase by the increase of the relative step height while the energy loss is decreased. A theoretical model for the computation of the energy loss is developed. Also, statistical models are proposed for predicting the characteristics of the formed free hydraulic B-jump at the negative step in the radial stilling basin. The experimental results are compared with the theoretical ones for energy loss and with the staistical ones for the length of jump. Good agreements are obtained between the results of the developed models and the experimental ones.

Keywords: Hydraulic jumps, Stilling basin, Non-prismatic basins, Expanding channels, Negative steps, Sudden drop, Statistical modeling

1. INTRODUCTION

Hydraulic jumps are one of the most frequently used energy dissipators. It may be free or submerged depending on both the location and the initial depth of the jump relative to the gate. Most of the studies on different types of hydraulic jump are presented in Hager [1]. The hydraulic jump may be also formed in prismatic or in non-prismatic channels, and may be forced or non-forced.

Based on studies of Khalifa and McCorquodale [2] and Abdel-Aal et al. [3], it was found that the relative depth of free radial jump as well as the length of the jump was shorter than those formed in rectangular channels, while the rate of energy loss increases through the the jump in radial basin compared to that in rectangular one.

A drop or negative step is used when the downstream depth is larger than the sequent depth for a classical jump to insure the jump occurrence and to provide more stability of the jump for a wide range of the downstream values. The available studies regarding the formation of hydraulic jumps at steps are for ones formed in rectangular basins. Hager [4] performed experimental and theoretical investigation on B-type jumps at abrupt drops. Hager and Bretz [5] discussed the characteristics of A and B jumps at negative steps. The ranges of relative depth and length representative of these types of jump were analyzed with particular attention to the design of stilling basins. Ohatsu and Yasuda [6] presented a systematic investigation on the characteristics of the hydraulic jump over a wide range of negative steps. Many of the observed cases were studied theoretically by the use of momentum equation. Also, measurements of the pressure distribution over the face of the step were analyzed. Negm [7] studied theoretically and experimentally the hydraulic jump formed in sloping and horizontal rectangular channel with positive or negative step. Armenio et al. [8] investigated the pressure fluctuations beneath a hydraulic jump that developed over a negative step. The study was carried out experimentally using two different drops, an abrupt drop and a rounded one. The inflow and the outflow conditions were varied to obtain B-jump and a wave jump.

Recently, a few studies were conducted on the formation of the submerged hydraulic jump at negative steps in radial basins. Negm et al [9,10,11] investigated experimentally and theoretically the effect of submergence, the relative height of the step, the relative height of end sill and the relative position of negative step on the characteristics of the submerged radial jump. They also studied the effect of the relative height of end sill with or without the negative step. It was found that the ratios of depth and length of the jump slightly decrease with the increase of the relative height of end sill while the energy loss ratio is increased.

This paper presents the results of a theoretical and experimental study on the characteristics of B-jump formed at negative step in radail basin. A theoretical equation is developed (and verified using experimental data) to predict the energy loss ratio of the free radial hydraulic jump in the presence of negative step in the radial basin. The effects of the initial Froude number, the relative height of step and the relative ppositon of step on the characteristics of the B-jump are addressed.

2. THEORETICAL CONSIDERATION

2.1 Dimensonal Analysis

Figure 1 shows a definition sketch for the B-jump at negative step in the radial stilling basin. The following function can be formed:

$$f(\rho, g, \mu, d_1, d_2, d_3, V_1, z, r_1, r_2, r_r, L_j) = 0 \qquad (1)$$

in which ρ is the density of water, g is the gravitational acceleration, μ is the dynamic viscosity of water, d_1 is the initial depth of jump, d_2 is the sequent depth of jump, d_3 is the depth of water at the step, V_1 is the average velocity of flow at the beginning of jump, z is the height of the negative step, r_1 is the radius where the jump begins, r_2 is the radius where the jump ends, r_3 is the radius where the step is constructed and L_j is the length of jump.

Using the dimensional analysis principle based on the three repeating variables ρ, d_1 and V_1, Eqn. (1) becomes

$$f\left(\frac{L_j}{d_1}, \frac{d_2}{d_1}, \frac{d_3}{d_1}, \frac{z}{d_1}, \frac{r_1}{d_1}, \frac{r_2}{d_1}, \frac{r_3}{d_1}, F_1, R_1\right) = 0 \qquad (2)$$

In Eqn. (2), both d_2/d_1 and d_3/d_1 are function of F_1, r_2/d_1 and r_1/d_1 gives r_2/r_1 and r_3/d_1 and r_1/d_1 gives r_3/r_1 while the effect of R_1 is neglected as the viscosity has a negligible effect on the hydraulic jump characteristics in the present study because the temperature was fixed during the course of the experimental work. Also, r_2/r_1 is a function of the length of the jump. Equation (2) becomes

$$\frac{L_j}{d_1} = f\left(\frac{z}{d_1}, \frac{r_3}{r_1}, F_1\right) \qquad (3)$$

Similar relationships for the depth ratio and for the energy loss ratio could be obtained. The nature of the function presented in Eqn. (3) will be determined based on the experimental data using the multiple linear regression analysis.

2.2. Momentum Approach

Based on the use of the 1-D momentum and continuity equations as applied on the control volume shown in Figure 1 for B- jump formed at negative step in radial basin. Negm et al. [12] developed the following theoretical equation for B –jump based on the shown forces in Figure 1.

$$\begin{aligned} &d_o^3(2r_o^2 + r_o r) - d_o^2[(d+Z)(r_o^2 - r_o r)] - d_o r_o[2 + Z^2(2r + r_o) + \\ &Z(4rd + 2r_o d) + d^2(r_o - 1) + d(r-1) + r] - 6F_1^2(d_o r_o - 1) = 0.0 \end{aligned} \qquad (4)$$

in which $d_o = d_2/d_1$, $r_o = r_2/r_1$, $d = d_3/d_1$, $r = r_3/r_1$ and $Z = z/d_1$.

Equation (4) can be solved easily if rearranged to take the following explicit forms as in Eqn. (5)

$$F_1 = \sqrt{\frac{d_o^3\,(2r_o^2 + rr_o) - d_o^2\left[(d+Z)(r_o^2 - rr_o)\right] - d_o r_o\left[2 + Z^2(2r + r_o) + Z(4rd + 2r_o d) + d^2(r_o - 1) + d(r-1) + r\right]}{6(d_o r_o - 1)}} \tag{5}$$

These equations are verified by Negm et al.[12] using experimental collected data on the same laboratory flume, Habib [13].

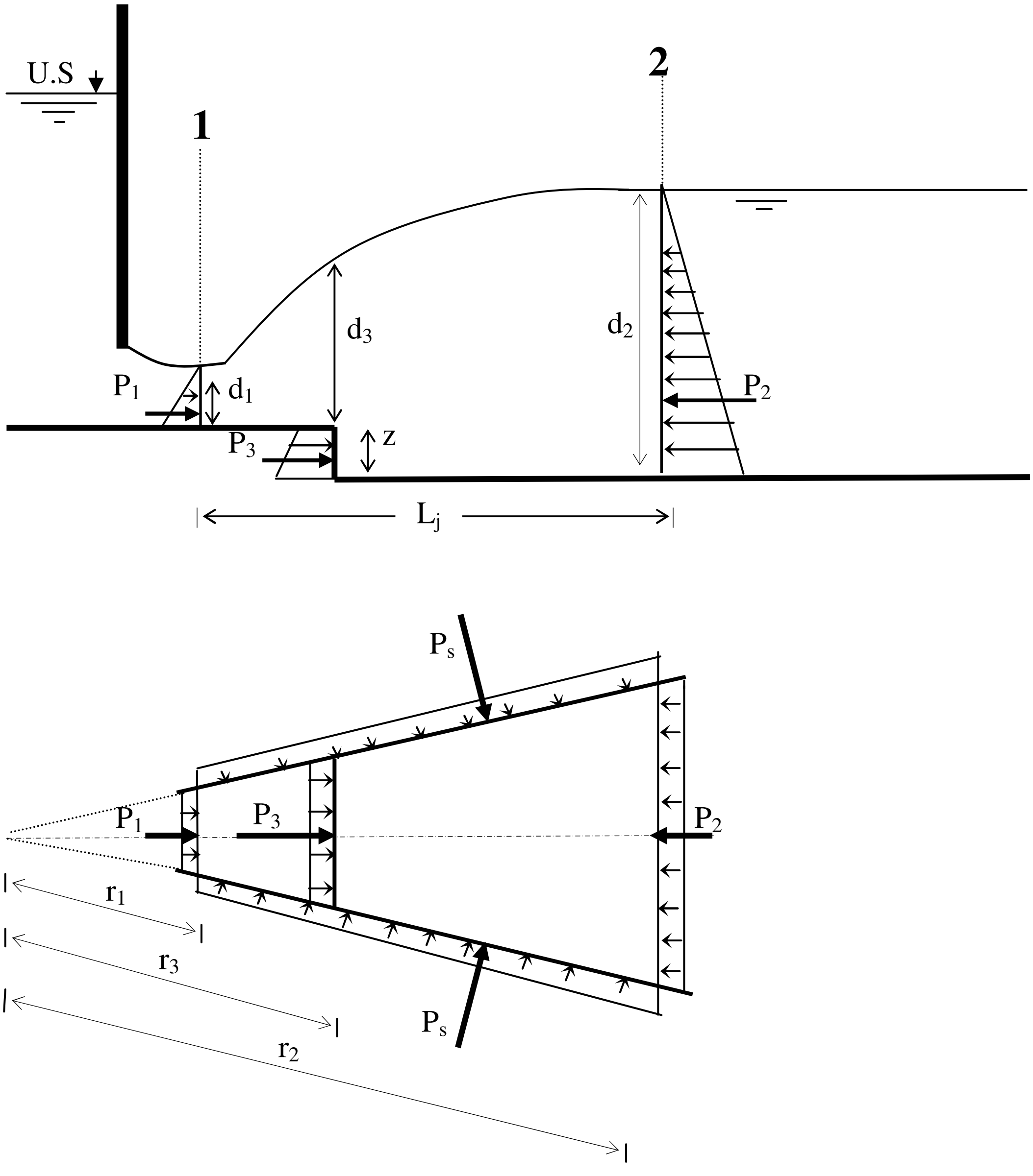

Figure 1. Definition sketch showing the formation of the B-jump at negative step in radial stilling basin. Shown also the forces used in applying the momentum equation by Negm et al. [12]

2.3. Energy Approach

Applying the energy equation at sections 1 and 2 where the jump begins and ends and assuming uniform velocity distribution and hydrostaic pressure distriution, one obtains

$$E_L = E_1 - E_2 = \left(d_1 + \frac{V_1^2}{2g} + z \right) - \left(d_2 + \frac{V_2^2}{2g} \right) \tag{6}$$

Keeping in mind that $F_1=V_1/(gd_1)^{0.5}$ and $V_2=V_1/r_o d_o$, one obtains:

$$E_1 = d_1\left(1 + \frac{1}{2}F_1^2 + Z \right) \text{ and } E_2 = d_1\left(\frac{2r_o^2 d_o^3 + F_1^2}{2r_o^2 d_o^2} \right) \tag{7}$$

Knowing that $E_L/E_1=1-E_2/E_1$, one gets

$$\frac{E_L}{E_1} = \frac{r_o^2 d_o^2 (2 + F_1^2 - 2d_o + 2Z) - F_1^2}{r_o^2 d_o^2 (2 + F_1^2 + 2Z)} \tag{8}$$

3. EXPERIMENTAL WORK

The experimental work of this study was conducted using a re-circulating adjustable flume of 15.0 m long, 45 cm deep and 30 cm wide, Habib [13]. The discharges were measured using pre-calibrated orifice meter fixed in the feeding pipeline. The tailgate fixed at the end of the flume was used to control the tail-water-depth of flow. The radial basin was made from a clear prespex to enable visual inspection of the phenomenon being under investigation. The model length was kept constant at 130 cm and the angle of the divergence was kept constant to 5.28°. The model was fixed in the middle third of the flume between its two side-walls as shown in Figure 2. A smooth block of wood was formed to fit well inside the basin model extending from upstream the gate by 5.0 cm to the position where the step was desired. The wood was painted very well by a waterproof material (plastic) to prevent wood from changing its volume by absorbing water. A fixed height of the step of 2.5 cm was used at two different positions of the step ($r_3=r_1$ and $r_3=1.17r_1$) dwnstream from the gate opening were tested under almost the same flow conditions. Testing the other positions of the step at this height (2.5 cm) did not produce B-jump. The range of the experimental data were as follows: Froude numbers (2.0-7.0), r_o (1.2-1.4), relative position of the step, r (1.0-1.17), and relative height of the step, z/d_1 (0.0 – 1.35).

Each model was tested using five different gate openings and five discharges for each gate opening. The measurements were recorded for each discharge. A typical test procedure consisted of (a) a gate opening was fixed and a selected discharge was allowed to pass. (b) the tailgate was adjusted until a free hydraulic jump is formed. (c) once the stability conditions were reached, the flow rate, length of the jump, water depths upstream and at the vena contracta downstream of the gate in addition to the tail water depth and the depth of water above the step were recorded. The length of jump was taken to be the section at which the flow depth becomes almost level. These steps were repeated for different discharges and different gate openings and so on till the required ranges of the parameters being under investigation were covered.

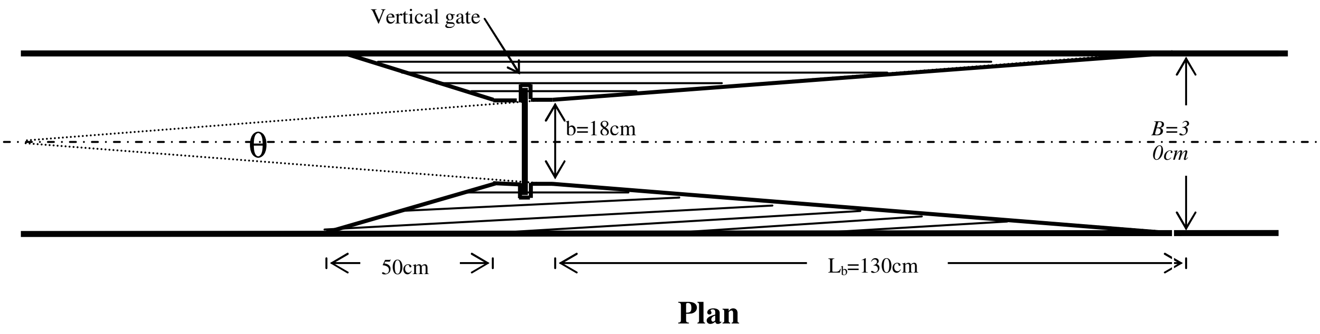

Figure 2. General sketch of the smooth radial stilling basin model

4. VERIFICATION OF THEORETICAL EQUATIONS

Figure 3a presents the comparison between the theoretical values of the energy loss and the experimental ones for all data at the two different relative positions of the step (r=1.0 and r=1.17). The correlation coefficient between both values is 0.979 and the mean relative absolute error, MRE, is 0.034. Figure 3b shows the comparison for typical values of the relative height of the step at the first position (r=1.0) as an example. Clearly, good agreement is obtained in both cases.

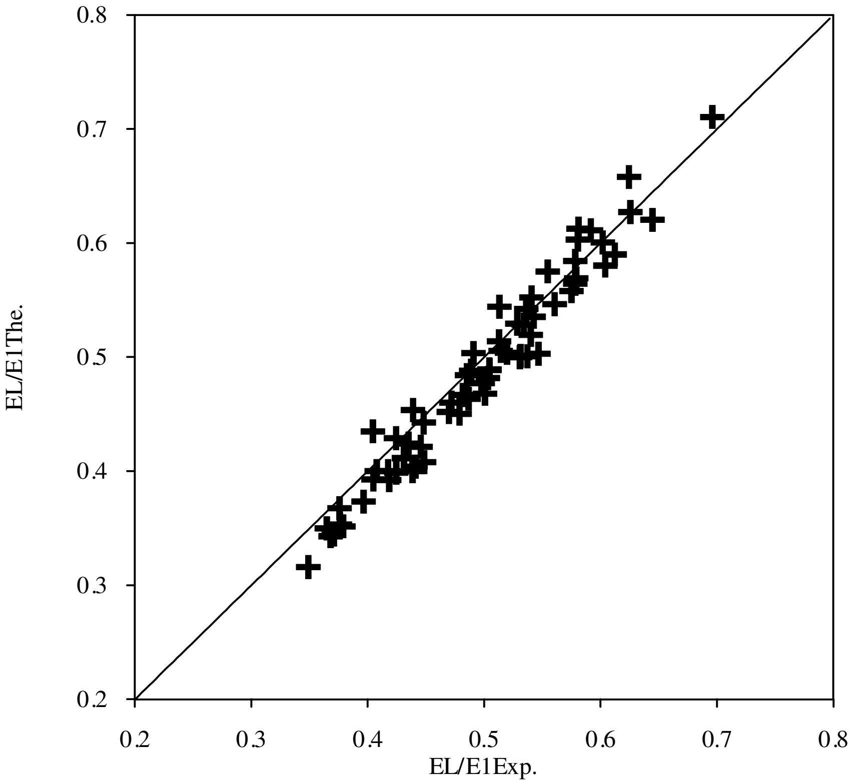

Figure 3a. Equation (8) versus experimental results for B-jump at negative step in radial stilling basins

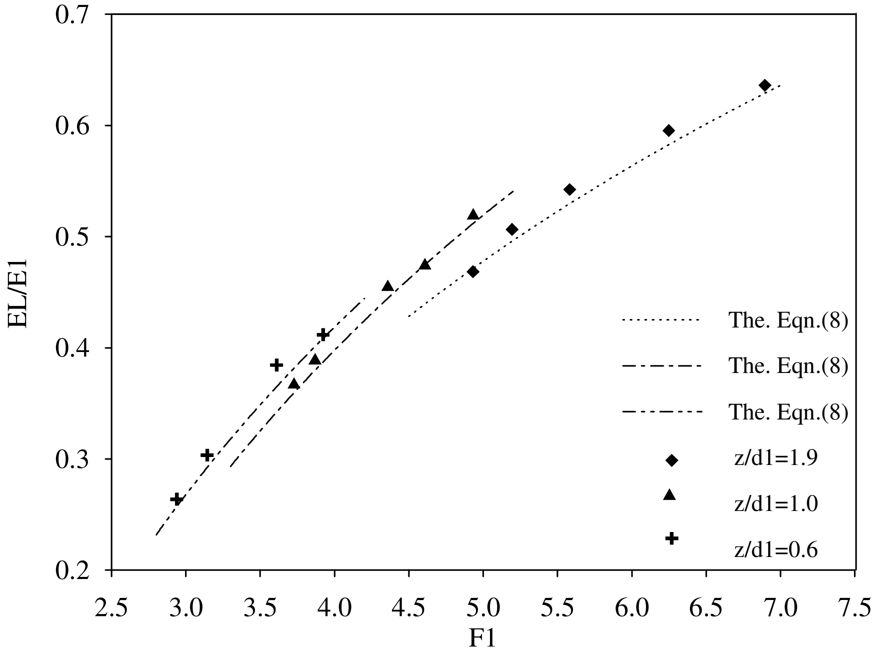

Figure 3b. Typical relationship between E_L/E_1 and F_1 for different z/d_1 at r=1.0, Showing both experimental results and theoretical ones based on Eqn.(8).

5. EXPERIMENTAL RESULTS AND DISCUSSIONS

Figures 4a and 4b present typical variations of the relationship between d_2/d_1 and F_1 for B-jump formed at negative step positioned at r=1.0 (L_s/L_b=0.0) and r=1.17 (L_s/L_b=0.25). Also, Figures 5a and 5b present the typical variations of L_j/d_1 with F_1 for the same two positions while Figure 6a and 6b show the typical variations of E_L/E_1 with F_1. These figures indicate similar trend of variation for the same relationship, i.e. linear variation for d_2/d_1 and nonlinear variations for both L_j/d_1 and E_L/E_1. In all these figures, the jump property (d_2/d_1, L_j/d_1 or E_L/E_1) increases with the increase of F_1 at particular z/d_1 due to the increase in the corresponding flow rate. At particular F_1, both d_2/d_1 and L_j/d_1 increase with the increase of z/d_1 with a higher rate of increase at the position r=1.17 compared to rate of increase at the position r=1.0. Regarding the energy loss ratio, it decreases with the increase of z/d_1 at particular F_1 as shown from Fgures 6a and 6b. The rate of reduction in E_L/E_1 increases with the increase of r from r=1.0 to r=1.17. Table 1 shows the percentage increase or dcrease in the jump property for unit increase in z/d_1 at the different relative positions of the negative step.

Table 1. Percentages of increase or decrease in value of hydraulic jump characteristics for each unit increase in z/d_1

r or L_s/L_b	% increase in d_2/d_1	% increase in L_j/d_1	% decrease in E_2/E_1
1.0 or 0.0	7.0	5.0	5.7
1.17 or 0.25	11.0	6.8	15.0

The increase in the depth ratio is due to the increase in the depth of flow which is turn is due to the increase in the specific energy at the section where the jump ends while the incoming flow is subcritical leading to a rise in the water surface. This rise in the water surface increases the weight of the jump which makes the supercritical jet of flow takes long distance to decay and in turn longer length of jump. This increase in the depth of flow and in turn in the weight of jump decreases the rate of the formed eddies and reverse motion that are created within the jump leading to a reduction in the rate of the energy dissipation compared to the case of no step (free bed).

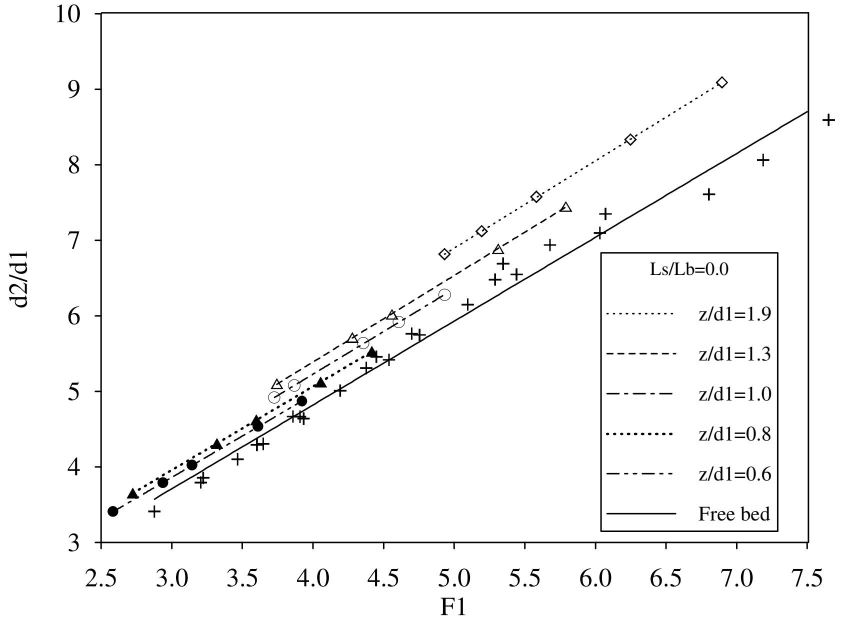

Figure 4a. Typical experimental relationship between d2/d1 and F1 for different z/d1 at r=1.0

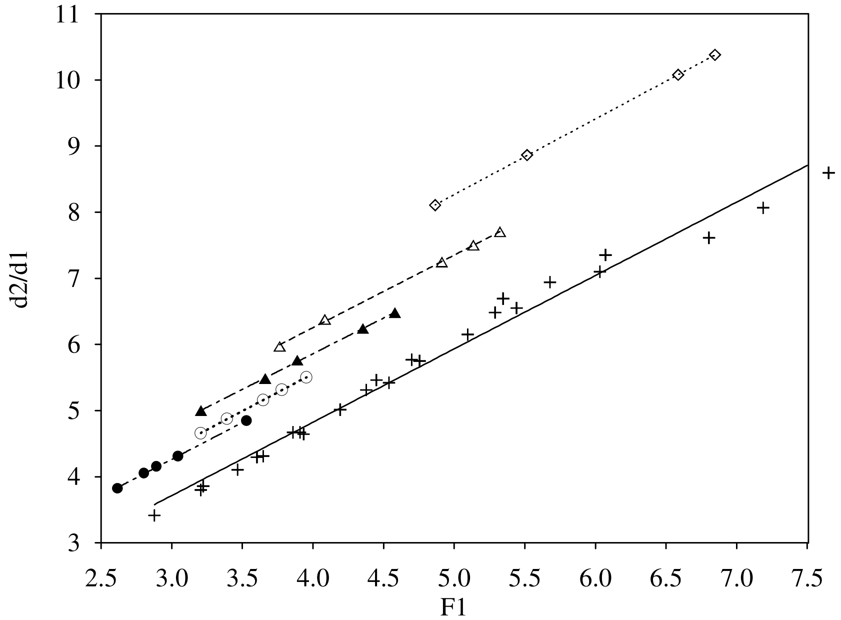

Figure 4b. typical experimental relationship between d2/d1 and F1 for different z/d1 at r=1.17

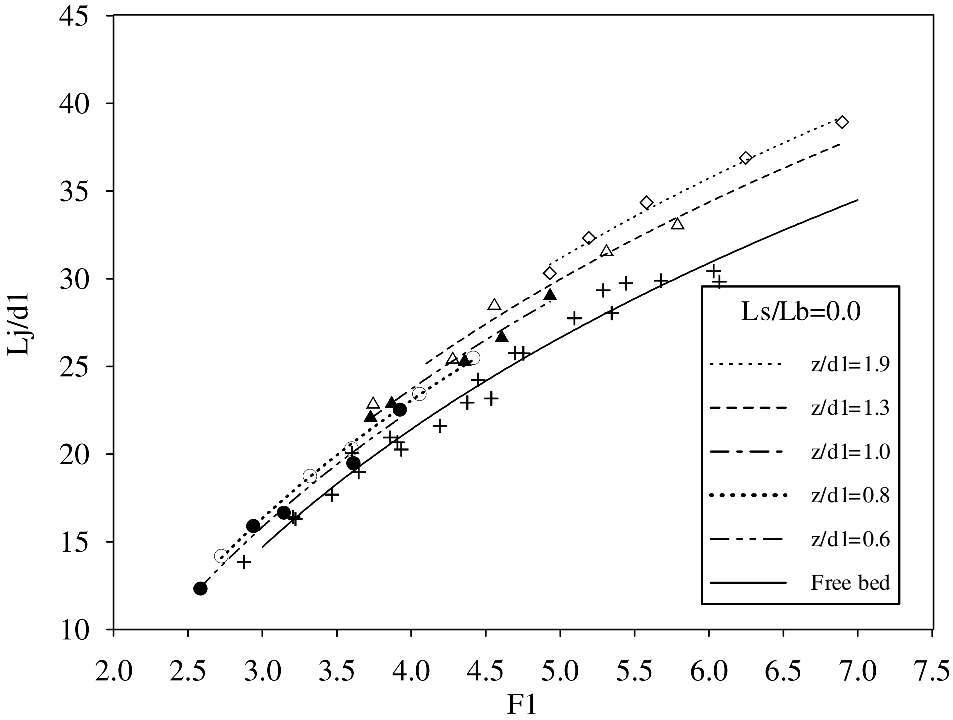

Figure 5a. Typical experimental relationship between L_j/d_1 and F_1 for different z/d_1 at r=1.0 (L_s/L_b=0.0)

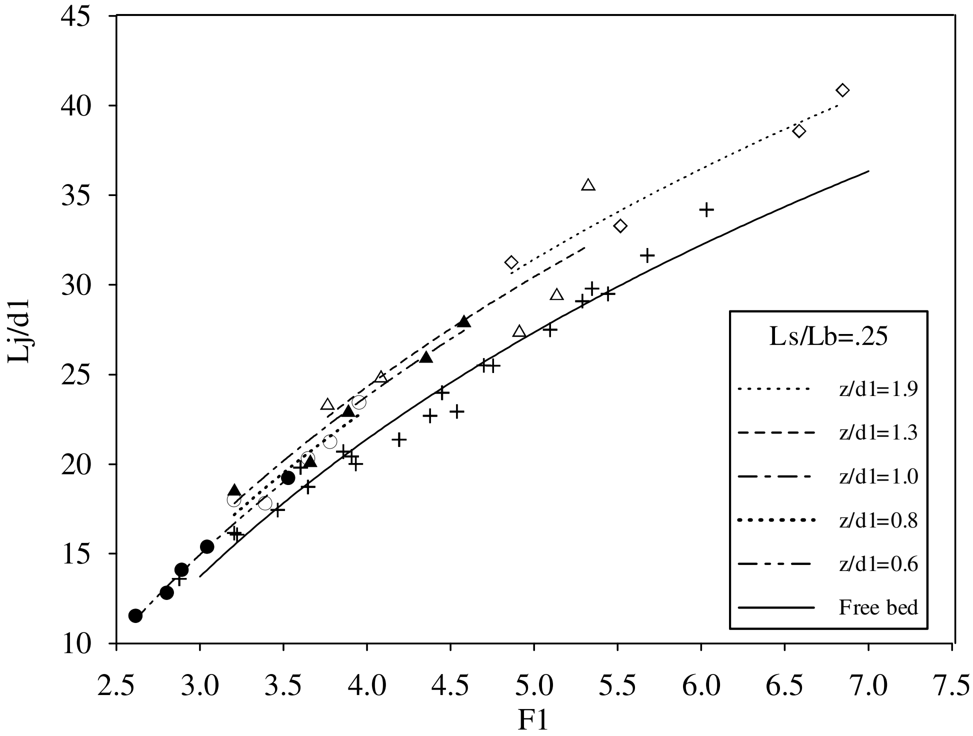

Figure 5b. Typical experimental relationship between L_j/d_1 and F_1 for different z/d_1 at r=1.17 (L_s/L_b=0.25)

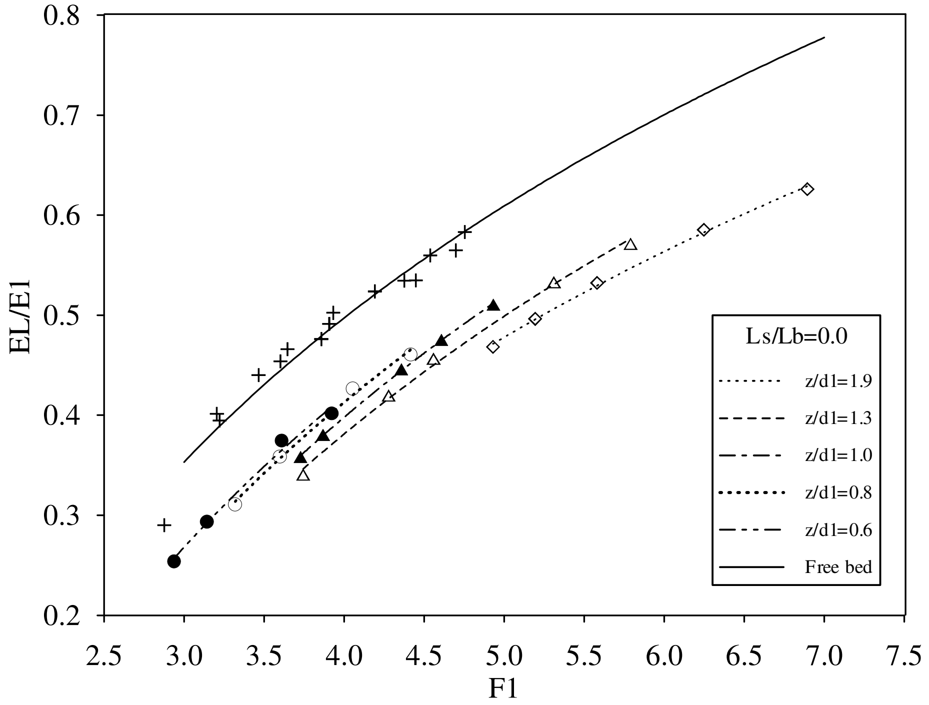

Figure 6a. Typical experimental relationship between EL/E1 and F1 for different z/d1 at r=1.0

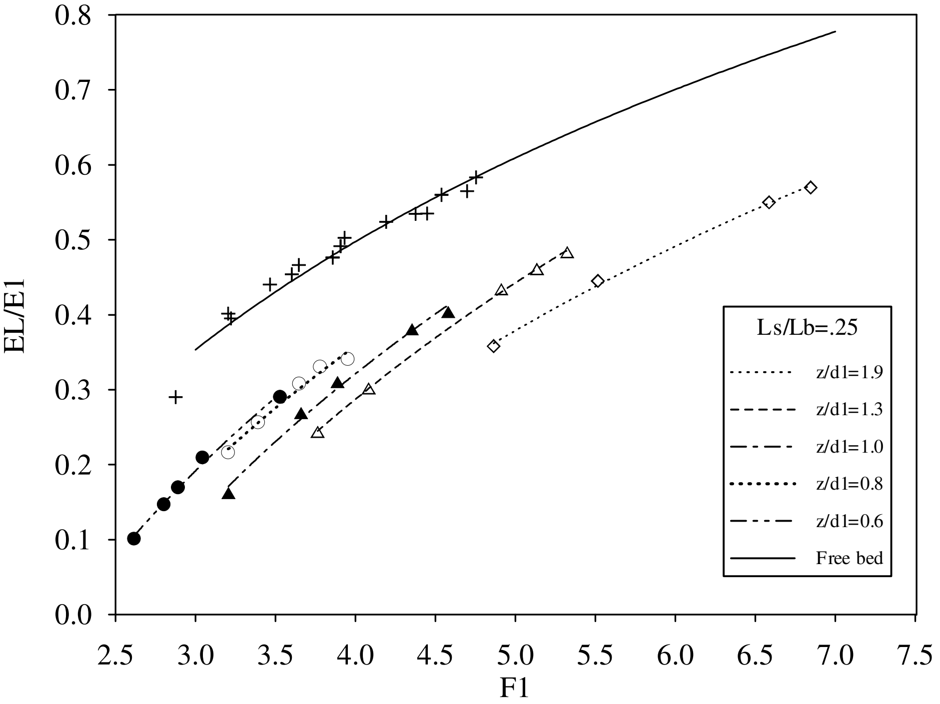

Figure 6b. Typical experimental relationship between EL/E1 and F1 for different z/d1 at r=1.17

6. STATISTICAL MODELING OF JUMP CHCRACTERISTICS

Since it is difficult to derive a pure theoretical equation for the length of jump, the nature of function of Eqn. (1) should be determined to enable the prediction of the length of jump. This could be done by proposing many different forms and trying to fit each form to the experimental data using the statistical techniques. The multiple linear regression is used for this purpose. Among the different trials, the following Eqn. (9) is found to be suitable. It has a determination coefficient of R^2=0.941 and mean relative absolute error, MRE, of 0.056. The residuals are uncorrelated with the predicted values as the correlation coefficient between them approches zero (R=7.16E-08). Figure 7a presents the comparison between the predicted and the measured values of L_j/d_1 for all measured data. The values are very close to the line of perfect agreement indicating good agreement between predicted and measured values of L_j/d_1. The variations of the residuals with the predicted values of L_j/d_1 are shown in Figure 7b. Clearly, the reiduals are small, symmetrically distributed around the line of zero error and uncorrelated indicating the validity of the developed model.

$$L_j/d_1 = -0.135+5.525\ F_1-4.407\ r+3.274\ z/d_1 \qquad (9)$$

Figure 8 presents a typical comparison between measured and predicted L_j/d_1 as a function of F_1 for different z/d_1 of 0.6, 1.6 and 1.9 at r=1.0. Clearly, the trends of the predicted values are the same as those of the measured one indicating the merits of Eqn. (9) in the prediction of L_j/d_1.

Similar models could be developed for both depth ratio d_2/d_1 and for the energy loss ratio E_L/E_1 as given by equations (10) and (11) respectively.

$$d_2/d_1= -3.231+1.115\ F_1+2.529\ r+1.143\ z/d_1 \qquad (10)$$

$$E_L/E_1= -0.818-0.175\ F_1+1.122\ F_1^{0.5}\ -0.195\ r-0.037\ z/d_1 \qquad (11)$$

Equations (10) and (11) have R^2 of 0.970 and 0.977 and MRE of 0.032 and 0.021 respectively. The correlation coefficients of the residuals with the predicted values are 9.59E-07 and 4.47E-02 respectively. Similar figures as those of 7a, 7b and 8 are prepared for the depth ratio and the energy loss ratio but not presented here to reserve space and to avoid repetition.

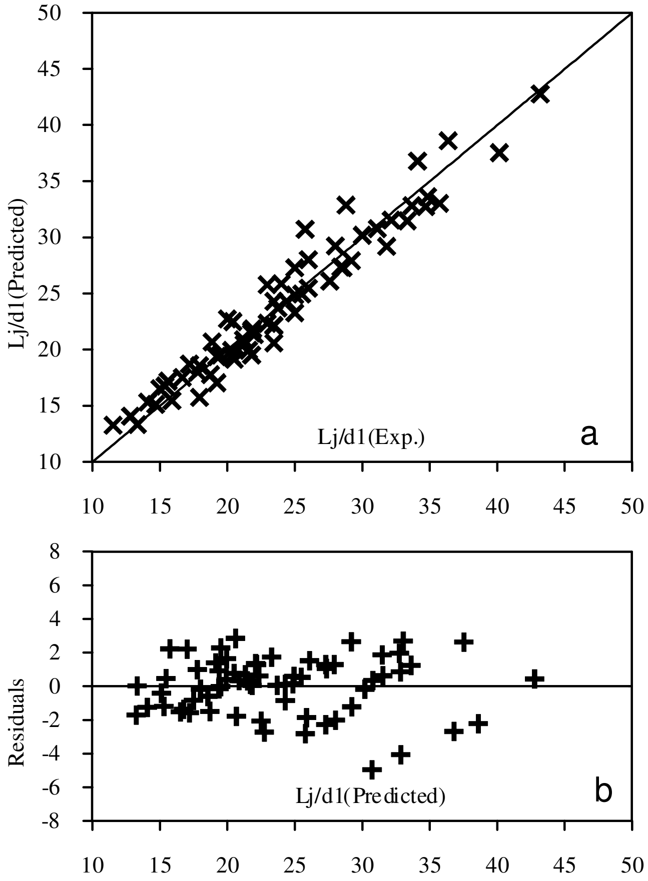

Figure 7. Results of the prediction model Eq. (9), (a) predicted versus measured and (b) residuals versus predicted values

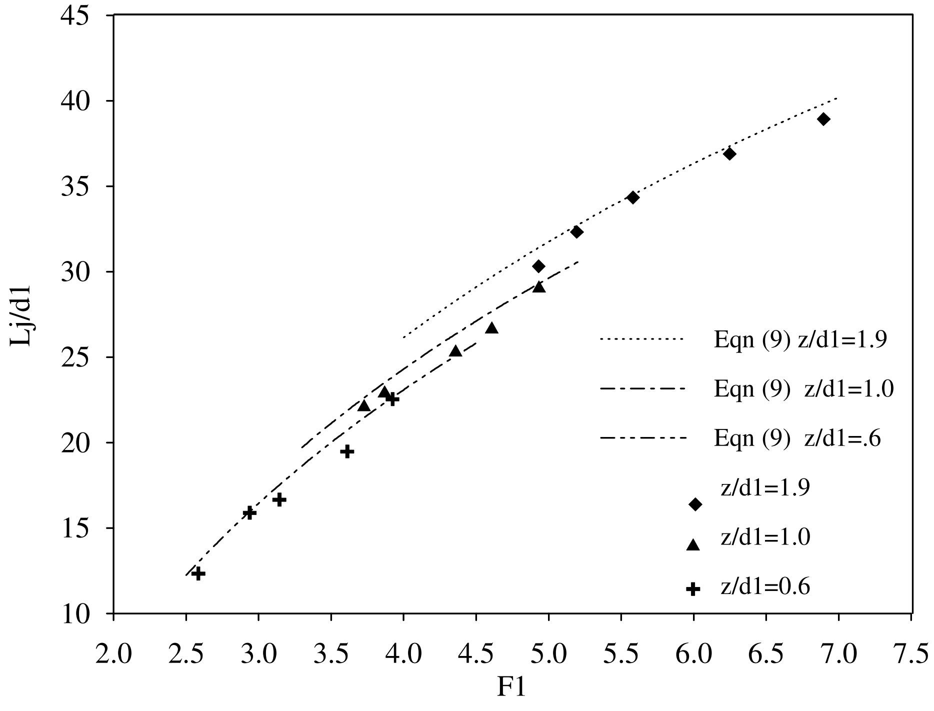

Figure 8. Comparison between prediction of equation (9) and measured Lj/d1 for typical values of z/d1 at r=1.0

7. CONCLUSIONS

An experimental program was conducted to investigate the effect of presence of negative step in radial stilling basin on the hydraulic characteristics of the free radial hydraulic B-jump. The effects of the initial Froude number, the relative position of the step in the basin and the relative height of the step on the chracteristics of the hydraulic B-jump were addressed. The experimental results were compared the developed theoretical and statistical models for the energy loss ratio and the length of jump ratio respectively. The presence of negative step in the first half of the radial stilling basin affects the depth ratio and the length of jump ratio positively while the energy loss is affected negatively. Among the investigated positions of the negative step, the one at the position r=1.17 (1/4 of the basin length) has the maximum effects on the jump chcracteristics than the other positions. At this position a unit increase in z/d_1 produces about 7% increase in L_j/d_1.

NOTATIONS

b = contracted width of the channel;
B = width of the channel;
d_1 = water depth at vena contracta downstream the gate (initial depth);
d_2 = sequent water depth;
d_o = the relative water depth of the jump, d_2/d_1;
d_3 = depth of water above the step;
d = the ratio of d_3 to d_1;
F_1 = Froude's number at the initial depth;
L_b= length of stilling basins;
L_j = the length of the hydraulic jump;
L_s=length from the gate to the end of the step in the basin;
Q = rate of flow;
r_1 = radius at the beginning of the jump;
r_2 = radius at the end of the jump;
r_o = the ratio of r_2 to r_1;
r_3 = radius at the end of the step in the basin;
r = the ratio of r_3 to r_1;
R^2= the coefficient of determination;
R = correlation coefficient;
P_1 = the hydrostatic pressure at the beginning of the jump;
P_2 = the hydrostatic pressure just at the end of the jump;
P_s = channel side pressure force;
P_3 = = the hydrostatic pressure on the face of the step;
V_1 = average velocity at the initial depth;
V_2 = average velocity at the sequent depth;
z = the drop height;
Z = the ratio of z to d_1; and
θ = the angle of divergence.

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