ANALYSIS AND FORMULATION OF HYDRAULIC JUMPS IN SLOPING RECTANGULAR CLOSED CONDUITS

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ABSTRACT

Hydraulic jumps formed in closed rectangular sloping conduits aree investigated experimentally. Both positive and negative slopes are considered. The results are analyzed in terms of the inlet Froude number, the bottom slope and the inflow depth ratio. The experiments are conducted in a laboratory flume. Nineteen models are tested. Ten model for positive slope and another ten models nine negative slope. The analysis of results indicated that both the inlet Froude number and the bottom slope have major effects while the inflow depth ratio has a minor effect on the depth ratio of the jump at the outlet. Prediction model is formulated using multiple linear regressions. The model predictions are compared with the measurements as well as with the results of the previously developed prediction models using the same technique.

1. INTRODUCTION

Basic information on different types of hydraulic jumps in both open channels and closed conduits could be found in Rajaratnam [1]. The hydraulic jump in closed conduit occurs when the depth near the inlet of the conduit is less than the critical depth and the conduit outlet is submerged. The hydraulic jump formed in closed conduit below control gates has been frequently observed, Rajaratnam [2]. In closed conduits, the initial free surface of supercritical flow changes to a pressurized flow downstream from the jump and the conjugate depth is confined by the conduit height. The tailwater depth in that case provides the downstream subcritical free surface flow. The jump location in the conduit is very sensitive to any slight variation in the initial depth, conduit height, tailwater depth, or conduit slope. Thus it is necessary to investigate the relationships between the parameters affecting such interesting phenomenon.

Figure 1 shows a definition sketch for the hydraulic jump formed in sloping closed conduit. Using the momentum and continuity equations, a theoretical equation for the depth ratio of the hydraulic jump could be obtained. The direct solution of such theoretical equation of the hydraulic jumps in sloping closed conduits is not possible without an extensive experimental work to calibrate the equation as it contains many non-theoretical parameters. Ezzeldin et al. [3,4] showed that the dimensionless tailwater depth D_t/d_1 at the outlet of the closed conduit is a function of the initial Froude number F_1, the dimensionless initial depth d_1/D and the conduit slope S_o as:

$$\frac{D_t}{d_1} = f(S_o, F_1, \frac{d_1}{D}) \tag{1}$$

Where D_t is the depth of water just downstream the outlet of the conduit d_1 is the initial depth of jump, S_o is the slope of the conduit and F_1 is the initial Froude number($= Q/Bd_1\sqrt{gd_1}$) with Q being the discharge, B is the conduit width and g is the gravitational acceleration.

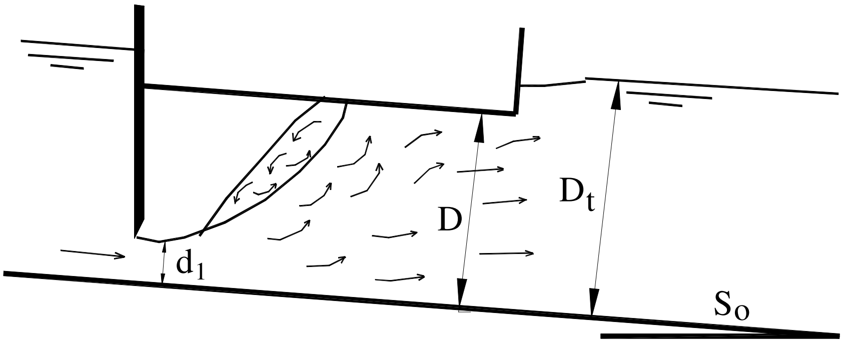

Figure 1. Definition sketch showing the jump formation inside a closed conduit

Equation (1) with S_o=0 is valid for hydraulic jumps formed in horizontal conduits. Such types of Hydraulic jumps were investigated by Lane and Kindsvater [5], Haindl [6] and Ezzeldin [7]. In case of sloping closed conduits, few studies on the hydraulic jump were carried out. Kalinske and Robertson [8] investigated the air pumping capacity of the jump for the case of sloping conduits. Smith and Chen [9] investigated the relative height of the hydraulic jump formed in steeply sloping square conduits without considering the tailwater depth conditions. They derived the basic theoretical equation for the relative height of the hydraulic jump formed in sloping square conduit using the 1-D momentum and continuity equations. It could not be solved properly because it contains many unknowns. Hence, they provided set of empirical equations of the form $H_j / D = aF_1^{1.4} + b$, (Hj being the height of jump, D is the conduit height, F_1 is the initial Froude number and a & b are two constants that depend upon the values of the relative height of conduit and the slope of the conduit).

Recently, Ezzeldin, Negm and Attia [3,4] investigated the hydraulic jumps in rectangular sloping closed conduit with small slopes. The effect of positive slope Ezzeldin et al. [3] and the effect of negative slope Ezzeldin et al. [4] were considered. Empirical equations (2) for positive slope and (3) for negative slope were developd in terms of the initial Froude number, the conduit bottom slope, S_o, and the relative initial depth of jump, d_1/D.

$$\frac{D_t}{d_1} = 7.018 - 3.782F_1 + 1.5732F_1^{1.5} + 121.169S_o - 2119.53S_o^2 + 0.5554\frac{d_1}{D} \tag{2}$$

Eq.(2) has $R^2 = 0.9876$ and SEE = 0.1248. It is valid for the following ranges of the input parameters $4 \le F_1 \le 6$, $0.0 < S_o \le 0.02$ and $0.21 \le d_1/D \le 0.35$

$$\frac{D_t}{d_1} = 7.743 - 4.1022F_1 + 1.6805F_1^{1.5} - 128.058S_o + 3501.5S_o^2 + 1.0492\frac{d_1}{D} \tag{3}$$

Eq.(3) has $R^2 = 0.9848$ and SEE = 0.13753. It is valid for the following ranges of the input parameters $4 \le F_1 \le 6$, $-0.02 \le S_o < 0.0$ and $0.21 \le d_1/D \le 0.35$

2. EXPERIMENTAL SETUP AND PROCEDURE

The experiments were conducted in a recirculating self contained tilting glass sided flume in the Hydraulics Laboratory of Faculty of Engineering, Zagazig University. The flume is 3.0 m long, 10 cm wide and 31 cm deep. A discharge control valve was used to regulate the flow rate. The bottom slope was adjusted using a screw jack located at the upstream end of the flume while at the downstream end; the flume was allowed to rotate freely about a hinged pivot. The slope was directly determined using a slope indicator. A downstream adjustable gate was used to regulate the tailwater surface elevation.

A typical test model consisted of a clear prespex sheet of 50 cm long and 10 cm wide fixed horizontally near the middle of the flume working section above the flume bed by an amount equals the conduit height, D. At the downstream end of this horizontal sheet (at the outlet), another sheet was fixed above it vertically to retain the flow at the outlet of the closed conduit. The sides of the prespex sheets were fixed to each other and to the flume sides by colorless silicon rubber to ensure good fixation and to prevent leakage from the sides.

The experiments were carried out mainly by Ezzeldin et al. [3,4] using five different conduit heights, D, of 6, 7, 8, 9 and 10 cm. Ten positive bottom slopes and another nine negative bottom slopes, S_o, were used. The used slopes were 0.002, 0.004, 0.005, 0.0067, 0.008, 0.01, 0.013, 0.016, 0.018 and 0.02. The slopes were selected based on the flume facility. Five different flow rates ranged from 342 lit/min to 234 lit/min were used for each particular conduit height and bottom slope. The initial Froude number ranged from 4 to 6. For each conduit height, the upstream control gate was adjusted to produce an initial supercritical depth, d_1, and the downstream adjustable gate was adjusted to control the tailwater outlet depth, $D_{t,}$ and which enabled the jump to be formed at a certain fixed location in the conduit. The discharge was measured using a pre-calibrated orifice meter. Depth measurements were taken using a point gauge with an accuracy of ± 0.1 mm. For each run, the initial depth of jump, the flow rate and the depth of water just downstream the conduit outlet were measured.

3. RESULTS AND DISCUSSIONS

Figures 2a to 2e present the variations of D_t/d_1 with the bottom slope (from –0.02 to 0.02) for different inlet Froude number at fixed value of d_1/D. It is clear that the depth ratio increases with the increase of the slope at particular inlet Froude number. The lowest value is due to the maximum negative slope (-0.02) and the maximum value is due to the maximum positive slope (0.02). This could be explained by the fact that, in case of the positive slope, the weight component acts in the direction of the flow while in case of the negative slope, it acts in a direction opposite to the flow direction imposing more resistance to the flow. This in turn results in a reduction of the depth ratio and the rate of reduction increase as the negative slope increases. On the other hand, the jump depth ratio at the outlet of the conduit increases by the increase of the inlet Froude number at fixed value of d_1/D and particular value of the slope.

Figures 3a present the variation of D_t/d_1 with S_o for different values of d_1/D at particular inlet Froude number of $F_1=4.093$. Clearly, the inlet flow depth ratio, d_1/D has a minor effect on the variation of D_t/d at fixed F_1. Figures 3b present the variation of D_t/d_1 with d_1/D for different values of So at particular F_1 of 4.617. The figure indicates that variation of D_t/d_1 is mostly minor with the increase of d_1/D. Figure 3c shows the variations of D_t/d_1 with F_1 for different S_o at particular d_1/D of 0.263. The figure confirmed what has been indicated by Figure 2.

3.1. Prediction Model

Eighteen regression models were tested based on the following criteria:

- The correlation coefficient, r.
- The mean relative error, mre.
- The root mean square error, rmse.

Table 1 shows the parameters involved in each of the tested models as well as the corresponding test criteria. The models were presented in table 1 in an ascending order according to the value of the correlation coefficient. From Table 1 it could be stated that model no. 17 has a good predictive power for the jump depth ratio. This model takes the form:

$$\frac{D_t}{d_1} = 7.2286 - 3.8401F_1 + 1.59595F_1^{1.5} + 63.5819S_o + 489.9137S_o^2 + 0.76652\frac{d_1}{D} \quad (4)$$

Eq.(4) has $R^2 = 0.988$ and SEE = 0.13996. It is valid for the following ranges of the input parameters $4 \le F_1 \le 6$, $-0.02 \le S_o \le 0.02$ and $0.21 \le d_1/D \le 0.35$.

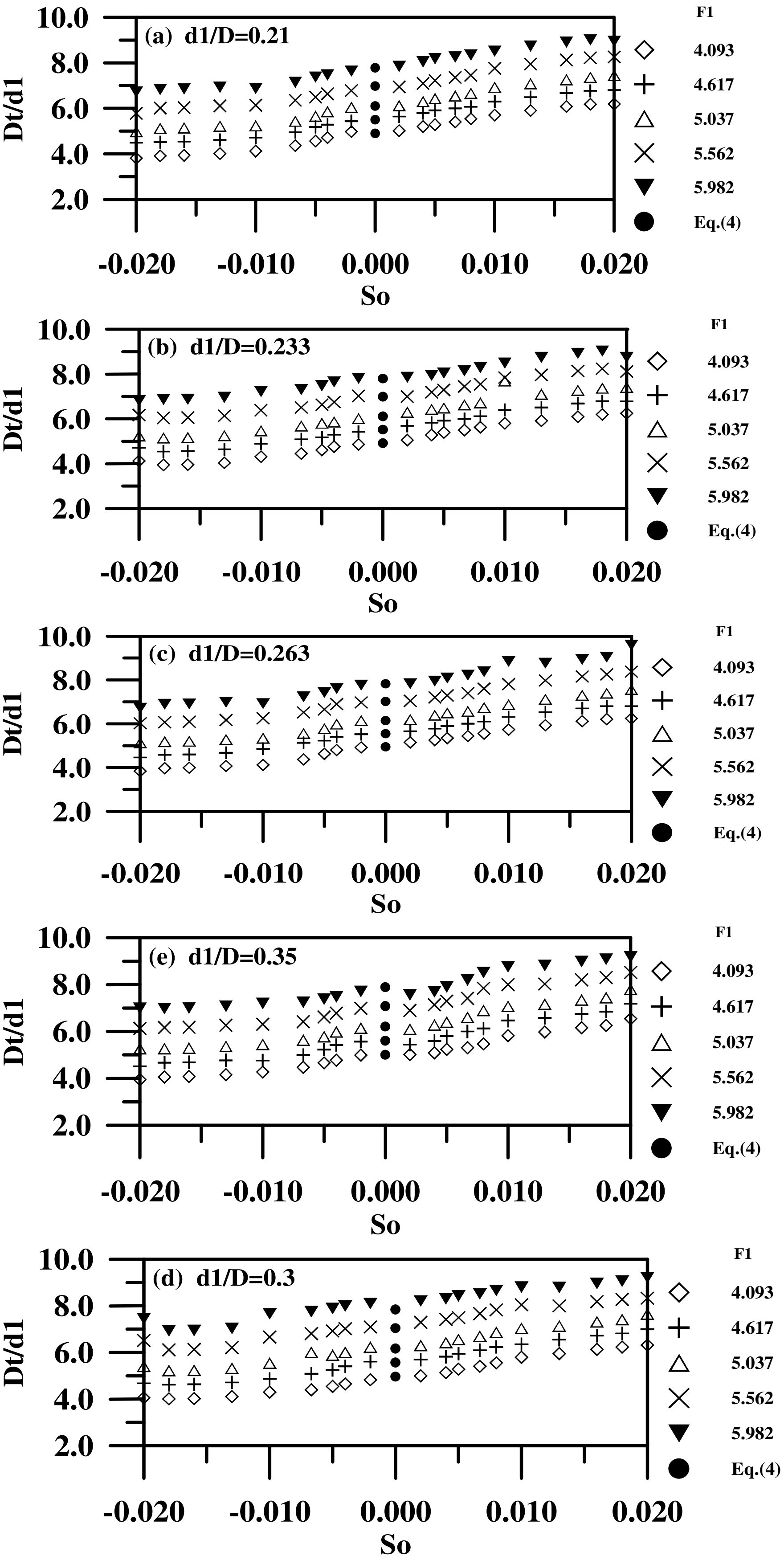

Figure 2. Variations of D_t/d_1 with slope for different values of F_1 and particular value of d_1/D, (a) $d_1/D=0.21$, (b) $d_1/D=0.233$, (c) $d_1/D=0.263$, (d) $d_1/D=0.30$ and (e) $d_1/D=0.35$

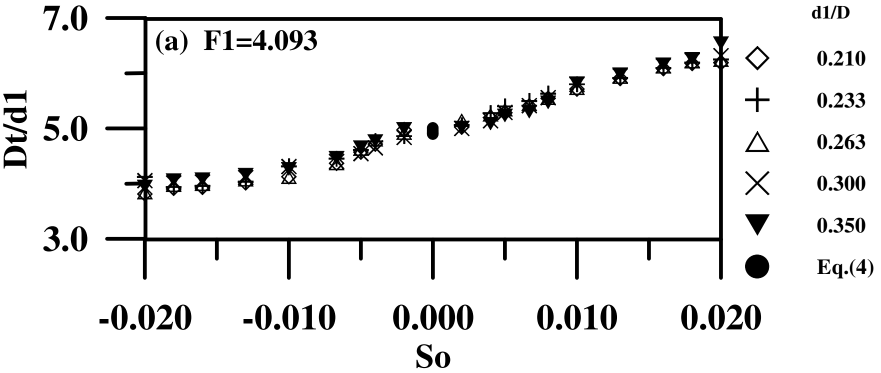

Figure 3a. Typical variations of D_t/d_1 with slope for different values of d_1/D and particular value of F_1.

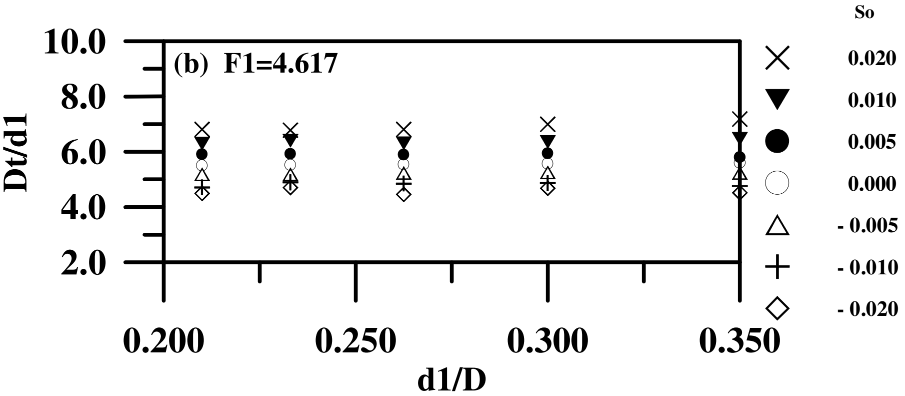

Figure 3b. Typical variations of D_t/d_1 with d_1/D for different values of S_o and particular value of F_1.

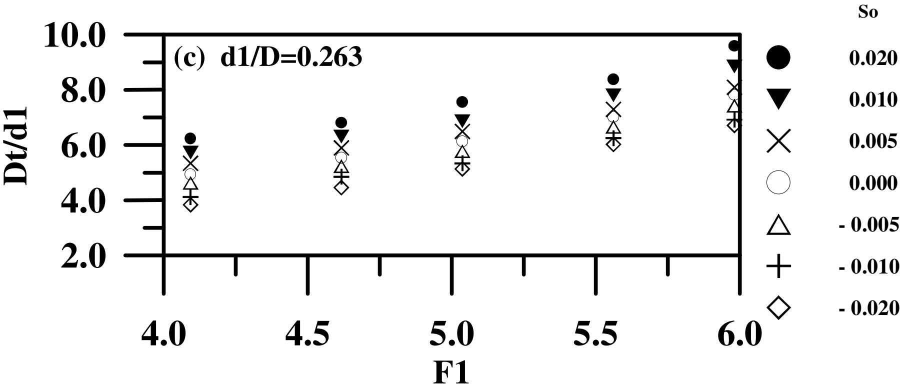

Figure 3c. Typical variations of D_t/d_1 with F_1 for different values of S_o and particular value of d_1/D.

Table 1 Evaluation of tested regression models in this study

Model	Involved Parameters	r^2	mre	rmse	max % error
5	d_1/D	0.001	0.178	1.288	66.5
4	S_o^2	0.002	0.178	1.288	70.3
14	$F_1^{1.5}+S_o^2+d_1/D$	0.345	0.111	0.779	31.3
3	S_o	0.350	0.148	1.039	39.2
7	$S_o+S_o^2$	0.352	0.148	1.037	38.6
1	F_1	0.629	0.111	0.785	29.0
9	$F_1+S_o^2$	0.631	0.111	0.783	31.5
13	$F_1+S_o^2+d_1/D$	0.632	0.112	0.782	30.5
2	$F_1^{1.5}$	0.632	0.111	0.782	29.9
6	$F_1+F_1^{1.5}$	0.635	0.110	0.778	32.3
8	F_1+S_o	0.979	0.024	0.188	12.6
11	$F_1+S_o+d_1/D$	0.980	0.023	0.184	12.9
10	$F_1^{1.5}+S_o$	0.982	0.022	0.174	12.1
16	$F_1+S_o+S_o^2+d_1/D$	0.982	0.021	0.171	10.6
12	$F_1^{1.5}+S_o+d_1/D$	0.983	0.021	0.170	12.1
15	$F_1+F_1^{1.5}+S_o+d_1/D$	0.986	0.018	0.154	11.4
17	$F_1+F_1^{1.5}+S_o+S_o^2+d_1/D$	0.988	0.016	0.139	11.7

3.2. Comparisons

Figure 4a shows the comparison between Eq.(4) and Eq.(2) for positive slope while Figure 4b compare the results of Eq.(4) with those of Eq.(3) for negative slope. Clearly good agreement was achieved. Since, Figures 4a and 4b are crowded with the observations, it may be better to indicate the predictive power of Eq.(4) in comparison with the experimental results as well as those of Eqs.(2) and (3) for a selected set of data as could be depicted from Figures 6c for $d_1/D=0.350$. Figures 4c confirmed that Eq.(4) is capable of predictive the depth ratio of the jump at the outlet of the rectangular closed conduit for both positive and negative slopes using the same parameters as previously developed by Ezzeldin et al. [3,4] for each slope separately..

3.3 Further Considerations

All models of table 1 were based on the full data set of size 475 records. The coefficient of the last model (no. 17) were re-evaluated using 75% of the observations while the remaining 25% of observations were left for testing the validity of the model. The regression toolbox of the Neural Connection [10] was used to evaluate the regression coefficients of the model. This model (no.18) will have the following form:

$$\frac{D_t}{d_1} = 6.98984 - 3.682F_1 + 1.54697F_1^{1.5} + 63.61585S_o + 443.7936S_o^2 + 0.8137\frac{d_1}{D} \qquad (5)$$

Eq.(5) has $R^2 = 0.99$ and SEE = 0.131. It is valid for the same ranges as for Eq.(4). The R^2 for validation data is 0.986 and that of test is 0.99. Table 2 summarize the same criteria as those in table 1 for Eq.(5).

Table 2. Evaluation of last model using 75% of the data

Model	Involved Parameters	r^2	mre	rmse	max % er	data set
18tr	$F_1 + F_1^{1.5} + S_o + S_o^2 + d_1/D$	0.990	0.015	0.130	8.70	training
18V	$F_1 + F_1^{1.5} + S_o + S_o^2 + d_1/D$	0.989	0.017	0.153	9.40	validation
18t	$F_1 + F_1^{1.5} + S_o + S_o^2 + d_1/D$	0.990	0.013	0.126	4.30	test

The training data set (75%) was used to evaluate the coefficients of the model. The other 25% was used to verify or testing the validity of the by generating values of the target variable based on the input without prior information on the actual output. The data sets were randomly selected to ensure proper calibration and correct verification of the model.

Comparing the performance criteria for models no.17 and no.18, some slight improvement in the performance of model no. 18 was obtained. For example, the maximum error was reduced from 11.7% to 9.4% while the mre was increased by 0.1% to reach 1.7% instead of 1.6%. Figure 5a shows the comparison between predicted D_t/d_1 and the measured ones for verification data set.

Figure 5b shows the comparison between Eq.(4) and Eq.(5). Clearly both equations could be used safely to predict the depth ratio of the hydraulic jump at the outlet of the rectangular closed conduit.

Figure 5c shows the variations of the residuals with the estimated values of D_t/d_1 for both models no.17 and no.18. The values of the residuals are very small and are symmetrically distributed around the line of zero error with a negligible correlation coefficient indicating the validity of both models in the prediction. The values of the mean, standard deviation and correlation coefficient are 0.00422, 0.1393 and 0.0268 for Eq.(5) and are 0.0000026, 0.1391 and 0.0000057 for Eq.(4).

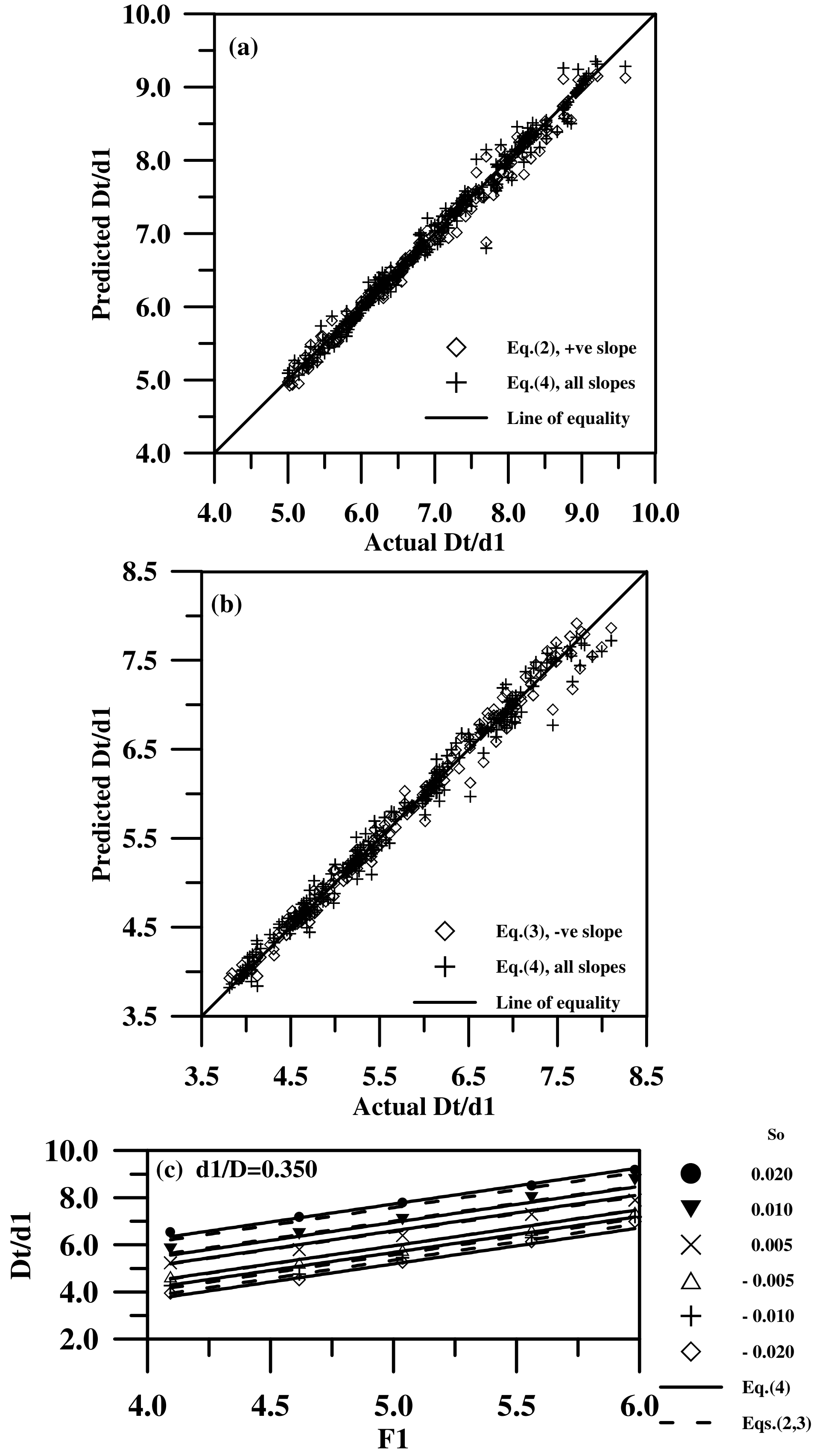

Figure 4. Comparison between actual, present model for +ve and –ve slopes and previous models for (a) +ve slope (b) -ve slope, (c) Present model versus previous ones

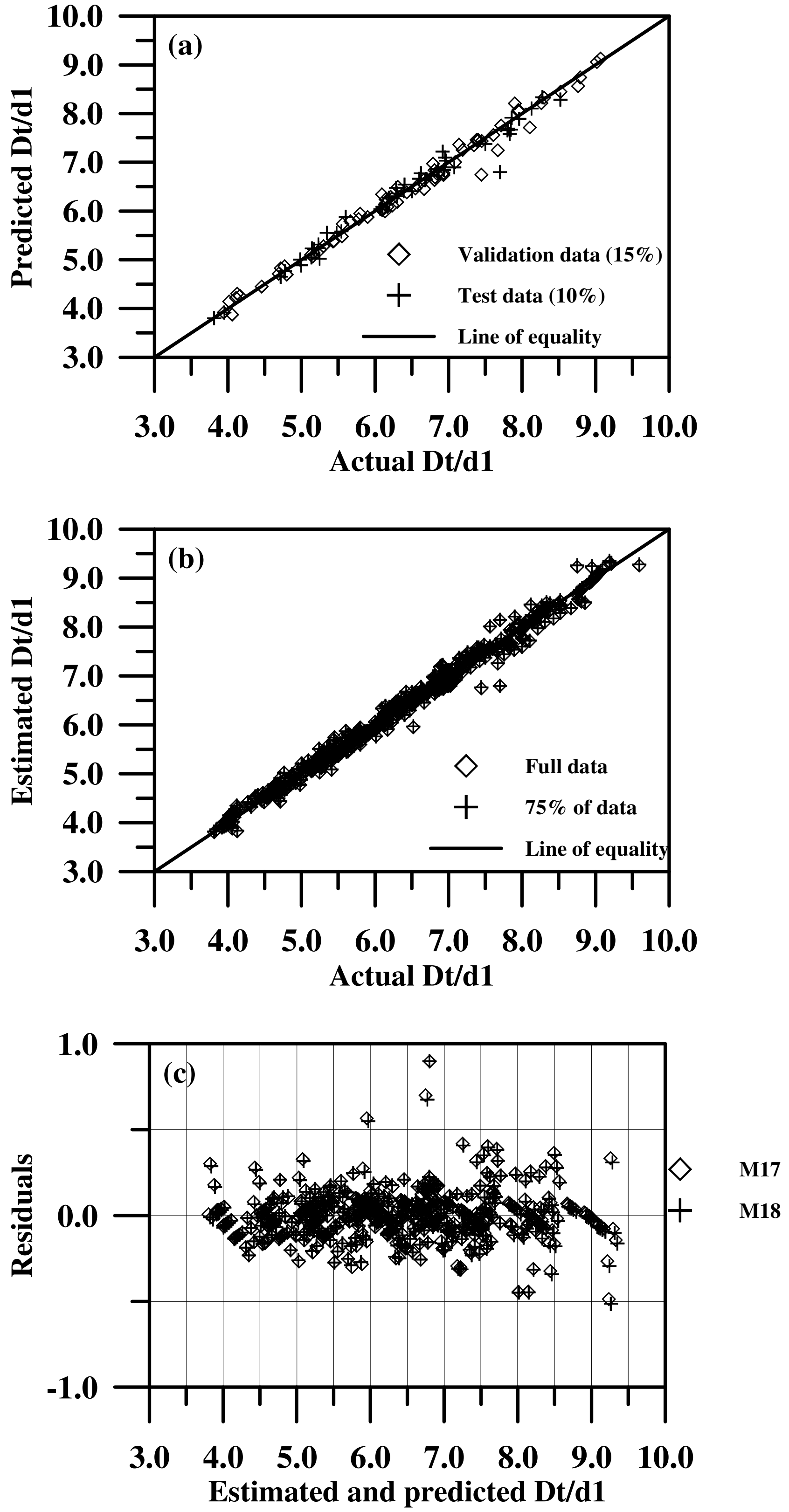

Figure 5. Performance of Eq.(5), (a) comparison of measurements with prediction of Eq.(5) for validation data, (b) comparison between Eq.(4) and Eq.(5) with the measurements and (c) distribution of the residuals of Eq.(5) and Eq.(4).

4. CONCLUSIONS

Hydraulic jumps formed in rectangular closed conduit with both positive and negative bottom slopes were analyzed based on an extensive experimental investigation. It was found that both the bottom slope S_o, and the inlet Froude number F_1, have major effect on the variations of the jump outlet depth ratio while the inflow depth ratio d_1/D, is of minor importance. The jump depth ratio increases with the increase of F_1 and increases with the increase of the bottom slope of the conduit. The negative bottom slope produces values of the jump outlet depth ratio which are lower than those produced by the positive slope. The values of the horizontal bottom slope are between the values of the negative and those of the positive slopes. A general prediction model was proposed in the same form of the previously developed models by Ezzeldin et al. [3,4] for jumps formed in conduits with either positive or negative slope. The model was calibrated using the full set of data by assigning positive sign for positive slope and negative sign for negative slope. The prediction of the proposed model was compared to the previously developed models for both positive and negative slopes. The predicted results agreed well with the experimental observations as well as with those of the previously developed models using the same technique. Also, the model was calibrated using 75% of the data and verified with the remaining 25% based on random selection. The calibrated model with 75% of the observations showed better performance than the model that calibrated based on all the data. However, the differences are practically negligible.

NOMENCLATURE

The following symbols are used in this paper
d_1 = initial depth of supercritical flow = d1 on the figures,
D = conduit height;
D_t = depth of flow at the outlet, =Dt on the figures,
S_o = conduit slope;
F_1 = initial Froude number;
D_t/d_1 = dimensionless outlet tailwater depth=Dt/d1 on the figures; and
d_1/D = dimensionless initial depth or conduit height ratio, =d1/D.

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