THEORETICAL MODELING OF HYDRAULIC JUMPS AT NEGATIVE STEP IN RADIAL STILLING BASINS WITH END SILL

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ABSTRACT

The hydraulic jump has been studied as an important method for energy dissipation downstream water structures. Several methods have been used to optimize the hydraulic jump length. One of the common methods was the use of end sill. Steps may be used to control the location of the hydraulic jump downstream of hydraulic structures. Extensive studies have been conducted to investigate the effect of steps in rectangular basins but not yet in radial basins. In the present paper, theoretical models are developed to predict the depth ratios of the radial hydraulic jumps at negative steps downstream of the control structures when the stilling basins is ended with a sill. Both the momentum and continuity equations in one dimension are applied to the control volume where the jump begins and ends. Both forms of the hydraulic jump (A and B) at negative step in the radial stilling basins are dealt. An experimental program is conducted to collect experimental data to enable verification of the developed theoretical models. Good agreement between theoretical and experimental results is obtained. The developed models are recommended for use in the design of radial stilling basin to compute the depth ratios which is needed to complete the dimensioning of the stilling basin.

Keywords: Hydraulic jumps, Theoretical modeling, Stilling basin, Non-prismatic stilling basins, Expanding channels, Negative steps, Sudden drop, End sill

INTRODUCTION

Hydraulic jumps are one of the most frequently used energy dissipators. It may be free or submerged depending on both the location and the initial depth of the jump relative to the gate. Most of the studies on different types of hydraulic jump are presented in Hager [1]. The hydraulic jump may be also formed in prismatic or in non-prismatic channels, and may be forced or non-forced.

Based on studies of Khalifa and McCorquodale [2] and Abdel-Aal [3], it was found that the relative depth of free radial jump as well as the length of the jump was shorter than those formed in rectangular channels, while the rate of energy loss increases through the the jump in radial basin compared to that in rectangular one.

A drop or negative step is used when the downstream depth is larger than the sequent depth for a classic jump to insure the jump occurrence and to provide more stability of the jump for a wide range of the downstream values. The available studies regarding the formation of hydraulic jumps at steps are for ones formed in rectangular basins. Hager [4] performed experimental and theoretical investigation on B-type jumps at abrupt drops. Hager and Bretz [5] discussed the characteristics of A and B jumps at negative steps. The ranges of relative depth and length representative of these types of jump were analyzed with particular attention to the design of stilling basins. Ohatsu and Yasuda [6] presented a systematic investigation on the characteristics of the hydraulic jump over a wide range of negative steps. All the cases were studied theoretically by the use of momentum equation with measurements of the pressure distribution over the face of the step. Other studies on negative and positive steps in sloping rectangular channels include those of Husain et al. [7], Quraishi. and Al-Brahim [8] and Negm [9]. Negm [9] reviewed the preevious and exteded their investigations by providing a simpliified theoretical solution for the depth ratio of hydraulic jumps at negative or positive steps in sloping rectangular channels. Armenio et al [10] investigated the pressure fluctuations beneath a hydraulic jump that developed over a negative step. The study was carried out experimentally using two different drops, an abrupt drop and a rounded one. The inflow and the outflow conditions were varied to obtain B-jump and a wave jump.

On the other hand, sills or blocks are used in stilling basins to increase the rate of energy dissipation and to reduce the bed velocity in the region of the hydraulic jump. Many studies have been conducted to investigate the effect of sills in rectangular basins. The effect of the sill on the jump characteristics depends on factors such as the sill configuration, sill location and sill spacing when more than one sill is used. Several investigations dealt with the effect of sill on the hydraulic jump characteristics when the sill is constructed beneath hydraulic jump such as Shukry [11], Rajaratnam [12], Ohtsu and Yasuda [13], and Hager and Li [14]. Hager and Li give one of these classifications of the forced hydraulic jump due to vertical sill. They classified the jump over vertical sill into A-jump, B-jump, minimum B-jump and C-jump. The A-jump is corresponding to the classical hydraulic jump, which is characterized by the maximum sequent depth ratio for the free jumps. They stated that, A-jump in which the jump characteristics are not influenced by the presence of sill (or weak effect are present) as the sill is found at the end of the surface roller and thus it is out side the effective zone for the sill to affect the jump flow. Other studies on the effect of vertical sill on the jump and different classification of jumps due to presence of sill could be reviewed in Hager [14]. Wafaie [15,16] investigated experimentally the free rectangular hydraulic jump phenomenon on roughened channel bed with dentated, solid, zigzagged bed sills, under different flow conditions, different bed sill heights, and different bed sill locations. Statistical analysis for the experimental results was made to obtain the best height and location of the bed sill. Recently, few studies were conducted to discuss the effects of negative step and/or end sill on the characteristics of the submerged hydraulic jump in radial basins, Negm et al. [17,18,19].

This paper concentrates on the development of a theoretical prediction model to predict the depth ratios of the free hydraulic jump at negative step in radial stilling basin with end sill. Both B and A jumps are considered. The one dimensional momentum and continuity equations are used to develop the required model. The models are verified with a collected experimental data.

DEVELOPMENT OF THE THEORETICAL MODEL

B-JUMP

Figure (1a) presents a definition sketch for negative B-jump that could be formed in radial stilling basin provided with a vertical negative step and end sill. This type of jump is formed such that it begins upstream of the step and ends downstream of the step. The assumed pressure distributions are also shown.

Both the 1-D momentum and continuity equations are used to develop a theoretical design model for computing the sequent depth ratio for the free radial hydraulic jump formed in radial stilling basin provided with a drop. The present development is based on the following assumptions: (a) the flow is radial and steady (b) the liquid is incompressible (c) the channel is horizontal and has smooth boundaries, (d) hydrostatic pressure distribution at the beginning of the jump, at the end of the jump, at the step and the sill (e) uniform velocity distribution, i.e. the values of the kinetic energy correction factor and the momentum correction factor, α and β are considered unity, and (f) the effects of air entrainment and turbulence are neglected.

In the present study, the control volume where the momentum equation is applied starts from the vena contracta (if a free flow occurs) and ends just down stream the end sill in the direction of the flow Figure (1a). The momentum equation for the case of B-jump at negative step when a sill is existed at the end of the basin written as follows:

$$P_5 - P_1 - 2\,P_S \sin\frac{\theta}{2} - P_3 + P_4 = \frac{\gamma Q}{g}(\beta_1 V_1 - \beta_5 V_5) \qquad (1)$$

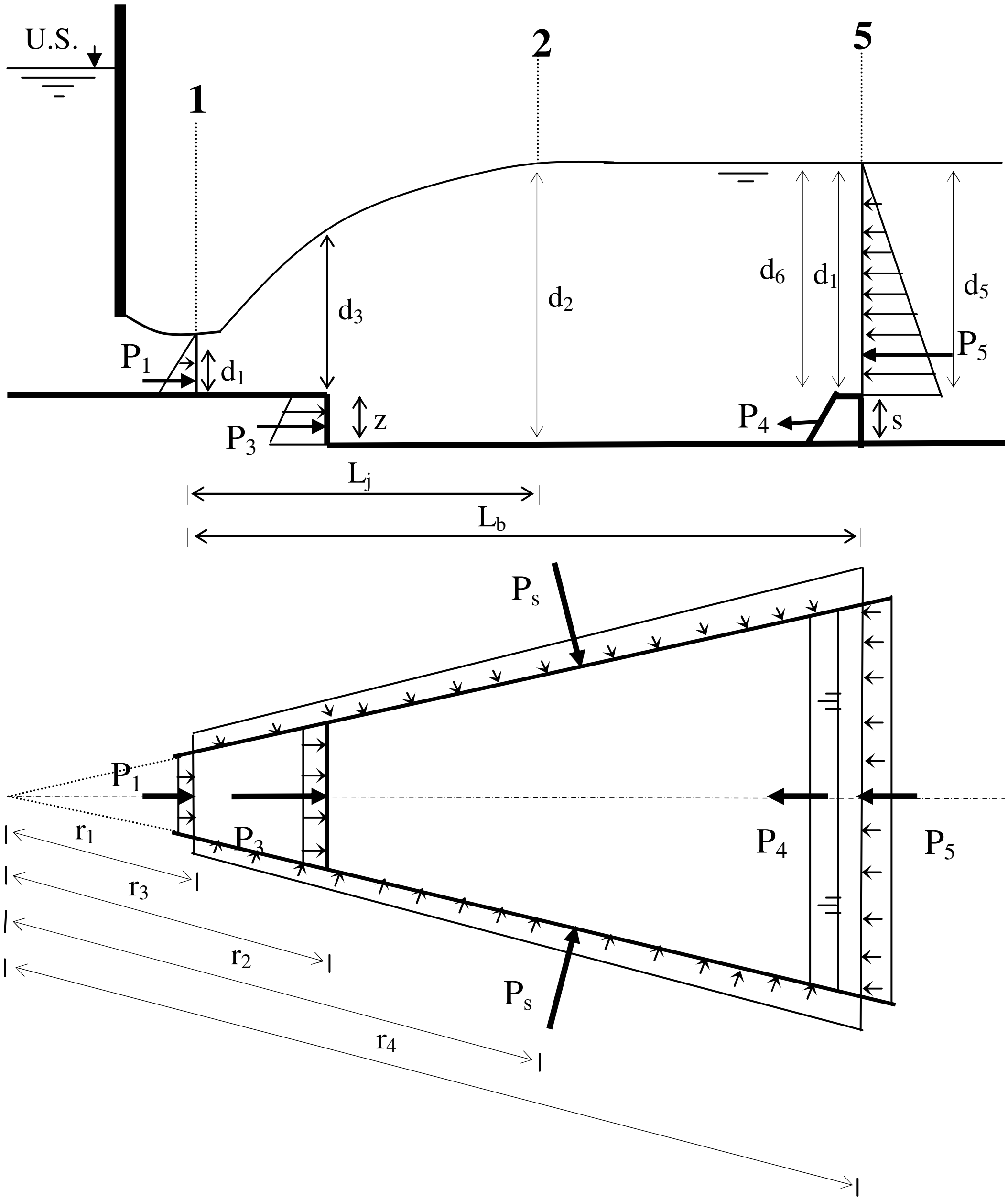

Figure 1a. Definition sketch showing the B-jump at negative step in radial basin with end sill. Also, the control volume, the forces and the pressure distribution are indicated

in which P_1 is the hydrostatic pressure at the beginning of the jump the jump, P_5 is the hydrostatic pressure just downstream the end sill, P_3 is hydrostatic pressure due to the step, P_4 is horizontal component of pressure due to end sill and P_S is channel side pressure from the initial water depth to the end sill. These forces may be exppressed as follows:

$$P_1 = \frac{\gamma d_1^2 b_1}{2},\ P_5 = \frac{\gamma d_4^2 b_4}{2},\ P_3 = \frac{\gamma z(2d_3 + z)b_3}{2},\ P_4 = \frac{1}{2}\gamma b_4(s\, d_6 + s\, d_4 + s^2) \qquad (2)$$

$$P_s = \frac{\gamma}{6}\begin{bmatrix}(r_3-r_1)(d_1^2+d_3^2+d_1d_3)+(r_2-r_3)[d_2^2+(d_3+z)^2+d_2(d_3+z)]+ \\ (r_4-r_2)(d_2^2+(d_4+s)^2+d_2(d_4+s))\end{bmatrix} \quad (3)$$

Substituting for P_1, P_5, P_3, P_4, and P_S from equations (2), and (3) in equation (1), one obtains

$$\begin{aligned}&\frac{\gamma d_4^2 b_4}{2}-\frac{\gamma d_1^2 b_1}{2} \\ &-\frac{\gamma}{3}\begin{bmatrix}(r_3-r_1)(d_1^2+d_3^2+d_1d_3)+(r_2-r_3)(d_2^2+d_3^2+2zd_3+z^2+d_2d_3+d_2z)+ \\ (r_4-r_2)(d_2^2+(d_4+s)^2+d_2(d_4+s))\end{bmatrix}\sin\frac{\theta}{2} \\ &-\frac{\gamma}{2}z(2d_3+z)b_3+\frac{1}{2}\gamma b_4(sd_6+sd_4+s^2)=\frac{\gamma Q}{g}(\beta_1 V_1-\beta_5 V_5)\end{aligned} \quad (4)$$

Applying the continuity equation:

$$Q= b_1\, d_1\, V_1= b_4\,(d_4+s)\, V_5 \quad (5)$$

Where b_1, and b_4 are the channel width at the beginning, and the end of the basin. We have

$$b_1=2r_1\sin\theta/2,\; b_2=2r_2\sin\theta/2,\; b_3=2r_3\sin\theta/2,\; b_4=2r_4\sin\theta/2 \quad (6)$$

Where b_2, and b_3 are the channel width at the end of the jump, and at the step. Substituting from equations (6) in equation (5), and solving for V_5 then:

$$V_5= V_1\, r_1\, d_1/\, r_4\,(d_4+s) =V_1/\, r_s\,(d_s+S) \quad (7)$$

Where $d_4/d_1 = d_s$, $s/d_1 = S$, and $r_4/r_1 = r_s$
Substituting from (6) and (7) in (4), and assuming $\beta_1=\beta_5=1.0$ to get

$$\begin{aligned}&\gamma r_4 d_4^2\sin\frac{\theta}{2}-\gamma r_1 d_1^2\sin\frac{\theta}{2} \\ &-\frac{\gamma}{3}\begin{bmatrix}(r_3-r_1)(d_1^2+d_3^2+d_1d_3)+(r_2-r_3)(d_2^2+d_3^2+2zd_3+z^2+d_2d_3+d_2z) \\ +(r_4-r_2)(d_2^2+(d_4+s)^2+d_2(d_4+s)\end{bmatrix}\sin\frac{\theta}{2} \\ &-\gamma r_3 z(2d_3+z)\sin\frac{\theta}{2}+\gamma r_4(sd_6+sd_4+s^2)\sin\frac{\theta}{2}=\frac{2\gamma}{g}v_1 r_1 d_1\sin\frac{\theta}{2}(V_1-\frac{V_1}{r_s(d_s+S)})\end{aligned} \quad (8)$$

Dividing Eqn. (8) by γ, then by $\sin\theta/2$, and then by $r_1 d_1^2$, Eqn. (8) tends to:

$$r_s d_s^2 - 1 - \frac{1}{3}\left[\begin{array}{l}(\frac{r_3}{r_1}-1)(1+\frac{d_3^2}{d_1^2}+\frac{d_3}{d_1^2}+\frac{d_1 d_3}{d_1^2})+(r_o-\frac{r_3}{r_1}) \\ (\frac{d_2^2}{d_1^2}+\frac{d_3^2}{d_1^2}+\frac{2zd_3}{d_1^2}+\frac{z^2}{d_1^2}+\frac{d_2 d_3}{d_1^2}+\frac{d_2 z}{d_1^2})+(\frac{r_4}{r_1}-\frac{r_2}{r_1}) \\ \left((\frac{d_2^2}{d_1^2}+\frac{(d4+s)^2}{d_1^2}+\frac{d2(d4+s)}{d_1^2}\right)\end{array}\right] - \frac{r_3 z}{r_1 d_1^2}(2d_3+z) \tag{9}$$

$$+\frac{r_4}{r_1}(\frac{s}{d_1}\frac{d_6}{d_1}+\frac{s}{d_1}\frac{d_4}{d_1}+\frac{s^2}{d_1^2}) = \frac{2V_1^2}{gd_1}(\frac{r_s(d_s+S)-1}{r_s(d_s+S)})$$

Assume that $r_3/r_1=r$, $d_2/d_1=d_o$, $z/d_1=Z$, $r_4/r_1=r_s$, $s/d_1=S$, $d_3/d_1=d$ and $d_4/d_1=d_s$, $d_6/d_1=d_{s*}$ Multiplying equation (9) by $3r_s\,(d_s+S)$, and simplify to obtain

$$d_o^2 r_s (d_s+S)(r-r_s) - d_o r_s (d_s+S)\left[(d+Z)(r_o-r)+(d_s+S)(r_s-r_o)\right] + r_s(d_s+S)$$
$$\left[\begin{array}{l}3r_s d_s^2 - 3 - 3rZ(2d+Z) + 3r_s(Sd_{s*}+Sd_s+S^2) - (1+d+d^2) \\ (r-1)-(d+Z)^2(r_o-r)-(d_s+S)^2(r_s-r_o)\end{array}\right] - 6F_1^2\left[r_s(d_s+S)-1\right] = 0.0 \tag{10}$$

Equation (10) may be rearranged to take to the following explicit form

$$F_1 = \sqrt{\frac{\begin{array}{l}d_o^2 r_s (d_s+S)(r-r_s) - d_o r_s (d_s+S)\left[(d+Z)(r_o-r)+(d_s+S)(r_s-r_o)\right] \\ +r_s(d_s+S)\left[\begin{array}{l}3r_s d_s^2 - 3 - 3rZ(2d+Z) + 3r_s(Sd_{s*}+Sd_s+S^2) - (1+d+d^2) \\ (r-1)-(d+Z)^2(r_o-r)-(d_s+S)^2(r_s-r_o)\end{array}\right]\end{array}}{6\left[r_s(d_s+S)-1\right]}} \tag{11}$$

Equation (10) tends to the previously developed equation by Abdel-Aal et al. [3] for smooth radial basin i.e. no step and no end sill (i.e. Z=0.0, S=0.0, $r=r_o$, $r_s=r_o$ and $d=d_s=d_o$). Also, it tends to the equation of Negm et al. [20] for radial basin provided with a negative step (no end sill). For end sill only and no negative step, Eqn. (10) tends to that of Habib et al. [21]. Furthermore, if the basin is rectangular and contains negative step only (case of B-jump $r=r_o=1.0$, and d=1.0), Eqn. (10) tends to the previously developed equations by Hager [4] and Hager and Bretz [5] respectively.

A-JUMP

For the case of A-jump formed at a negative step in radial stilling basin with end sill that is formed entirly upstream of the step as shown in Figure (1b), the side pressure force could be expressed as follows:

$$P_S = \frac{\gamma}{6}\left[\begin{matrix}(r_2 - r_1)(d_1^2 + d_2^2 + d_1 d_2) + (r_3 - r_2)(d_3^2 + d_2^2 + d_2 d_3) + (r_4 - r_3) \\ [(d_3 + z)^2 + (d_4 + s)^2 + (d_3 + z)(d_4 + s)]\end{matrix}\right] \quad (12)$$

The momentum equation in the direction of the flow may be written in the following form:

$$P_5 - P_1 - P_3 + P_4 - 2P_S \sin\frac{\theta}{2} = \frac{\gamma Q}{g}(\beta_1 V_1 - \beta_5 V_5) \quad (13)$$

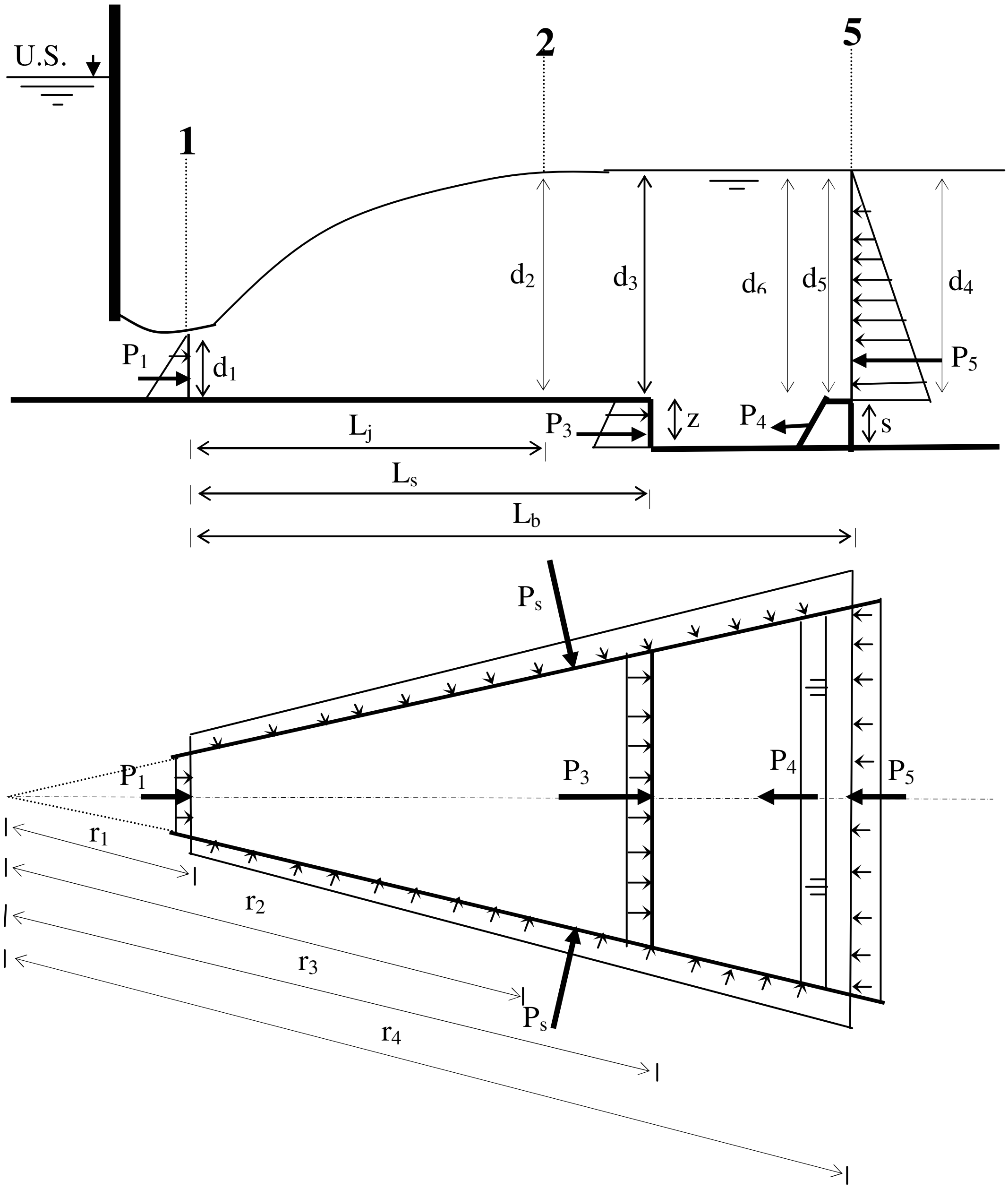

Figure 1b. Definition sketch showing the A-jump at negative step in radial basin with end sill. Also, the control volume, the forces and the pressure distribution are indicated

Substituting from equations (2), and (12) in equation (13):

$$\frac{\gamma d_4^2 b_4}{2}-\frac{\gamma d_1^2 b_1}{2}-\frac{\gamma}{2}z(2d_3+z)b_3+\frac{1}{2}\gamma b_4(sd_6+sd_4+s^2)-\frac{\gamma}{3}\sin\frac{\theta}{2}*$$
$$\begin{bmatrix}[(r_2-r_1)(d_1^2+d_2^2+d_1d_2)+(r_3-r_2)(d_3^2+d_2^2+d_2d_3)\\ +(r_4-r_3)[(d_3+z)^2+(d_4+s)^2+(d_3+z)(d_4+s)]\end{bmatrix}=\frac{\gamma Q}{g}(\beta_1 V_1-\beta_5 V_5) \quad (14)$$

Similar to the case of B-jump formed at negative step in radial basin, one could obtain

$$d_o^2 r_s (d_s+S)(1-r\ )-d_o r_s (d_s+S)\left[r_o-1+d\ (r\ -r_o)\right]+r_s (d_s+S)$$
$$\begin{bmatrix}3r_s d_s^2-3-3rZ(2d+Z)+3r_s(Sd_{s*}+Sd_s+S^2)-(r_o-1)-d^2(r-r_o)\\ -[(d+Z)^2+(d_s+S)^2+(d+Z)(d_s+S)]\ \ (r_s-r\ )\end{bmatrix} \quad (15)$$
$$-6F_1^2\left[r_s(d_s+S)-1\right]=0.0$$

Equation (15) may be rearranged to take to the following explicit form

$$F_1=\sqrt{\frac{d_o^2 r_s (d_s+S)(1-r\ )-d_o r_s (d_s+S)\left[r_o-1+d\ (r\ -r_o)\right]+r_s (d_s+S)\begin{bmatrix}3r_s d_s^2-3-3rZ(2d+Z)+3r_s(Sd_{s*}+Sd_s+S^2)-(r_o-1)-d^2(r-r_o)\\ -[(d+Z)^2+(d_s+S)^2+(d+Z)(d_s+S)]\ \ (r_s-r\ )\end{bmatrix}}{6\left[r_s(d_s+S)-1\right]}} \quad (16)$$

Equation (15) reduces to the equation developed by Negm et al.[20] for A-jump at negative step (no end sill). For end sill only and no negative step, Eqn. (15) tends to that of Habib et al. [21]. Also, for smooth radial stilling basin (i.e. Z=0.0, S=0.0, $r=r_o$, $r_s=r_o$ and $d=d_s=d_o$), it reduces to the previously developed equation by Abdel-Aal et al. [3]. Furthermore the present Eqn. (15) tends to to the previously developed equation by Hager and Bretz [5] if the basin is rectangular and provided with only negative step (case of A-jump where $r=r_o=1.0$, and $d=d_o$-Z)

EXPERIMENTAL WORK

The experimental work of this study is conducted using a re-circulating adjustable flume of 15.0 m long, 45 cm deep and 30 cm wide, Habib [22]. The discharges were measured using pre-calibrated orifice meter fixed in the feeding pipeline. The tailgate fixed at the end of the flume was used to control the tail-water-depth of flow. The radial basin was made from a clear prespex to enable visual inspection of the phenomenon being under investigation. The model length was kept constant at 130 cm and the angle of the divergence was kept constant to 5.28^o. The model was fixed in the middle third of the flume between its two side-walls as shwon in Figure (2). A smooth block of wood was formed to fit well inside the basin model extending from upstream the gate by 5.0 cm to the position where the drop was desired. Also a smooth baffle block of wood was formed to fit well inside the basin model extending from one side of the model to the other side at the end of the basin to simulate the end sill. The end sill has an upstream slope of 1:1 and vertical face from the downstream side. The wood was painted very well by a waterproof material (plastic) to prevent wood from changing its volume by absorbing water. A fixed height of the drop of 2.5 cm was used at different positions of the drop ($r_3=r_1$, $r_3=1.16r_1$, $r_3=1.33r_1$, $r_3=1.5r_1$, and $r_3=1.67r_1$) downstream from the gate opening were tested under the same flow conditions. The end sill was tested using three different heights (3 cm, 4cm and 5 cm) when the step was present under smilar flow conditions. The range of the experimental data were as follows: Froude numbers (2.0-7.0), r_o (1.2-1.4), relative position of the drop, r (1-1.67), relative height of the drop, z/d_1 (0.6 – 4.2), and relative height of end sill, s/d_1 (0.0-3.4).

Each model was tested using five different gate openings and five discharges for each gate opening. The measurements were recorded for each discharge. The total number of runs was 275. A typical test procedure consisted of: (a) a gate opening was fixed and a selected discharge was allowed to pass, (b) the tailgate was adjusted until a free hydraulic jump is formed, (c) once the stability conditions were reached, the flow rate, length of the jump, water depths upstream and at the vena contracta downstream of the gate in addition to the tail water depth and the depth of water above the step were recorded. The length of jump was taken to be the section at which the flow depth becomes almost level. These steps were repeated for different discharges and different gate openings and so on till the required ranges of the parameters being under investigation were covered.

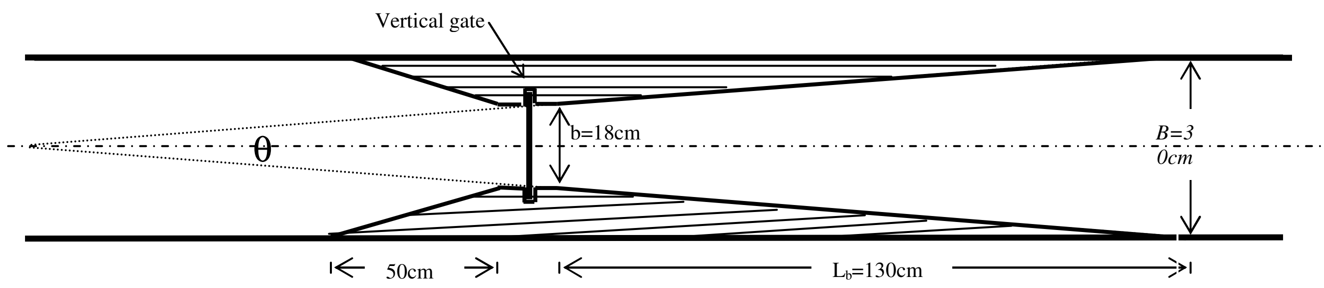

Figure 2. General sketch of the smooth radial stilling basin model

VERIFICATION OF THE DEVELOPED MODELS

Figure 3 presents the comparison between theoretical values of Froude number ($F_{1\ The}$) as computed from equation (11) and the its values as computed from the measurements ($F_{1\ Exp}$) for B-jump formed at negative step in radial basin ended with a sill. The coefficient of determination (R^2) between theoretical and measured values of Froude number is 0.966 while the relative mean absolute error (MRE) is 0.032 indicating good a greement between theoretical and measured values.

Similarly, Figure 4 presents the same comparisons for the A-jump where the values from equation (16) are compared with those from measurements and the values from measurements The values of the coefficient of determination in this case is 0.977 while the values of mean relative absolute error is 0.026.

Also, Figures 5a to 5f and 6a to 6f present the comparison between the theoretical values and the experimental ones for B-jump (r=1.17) and A–jump (r=1.33) respectively, (in terms of the relationship between d_2/d_1 and F_1) for typical values of s/d_1 of (3.4, 2.4, 2, 1.5, 1.3 and 0.9) and typical values of z/d_1 of (1.9, 1.6, 1.2, 1.0, 0.8 and 0.7) respectively when r_o=1.3. Inspection of all these figures indicates that good agreements between theoretical and measured values are achieved.

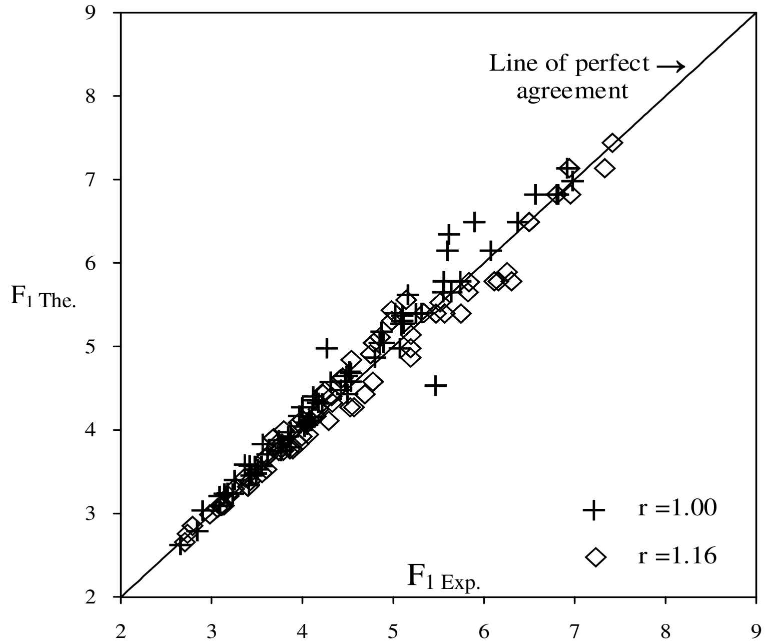

Figure 3. Verification of Eqn. (11) for –ve B-jump

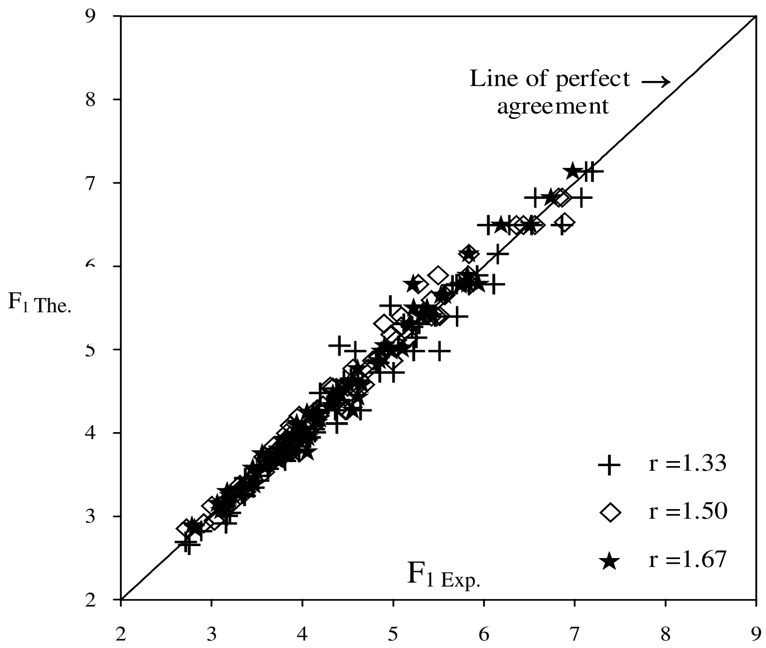

Figure 4. Verification of Eqn. (16) for –ve A-jump

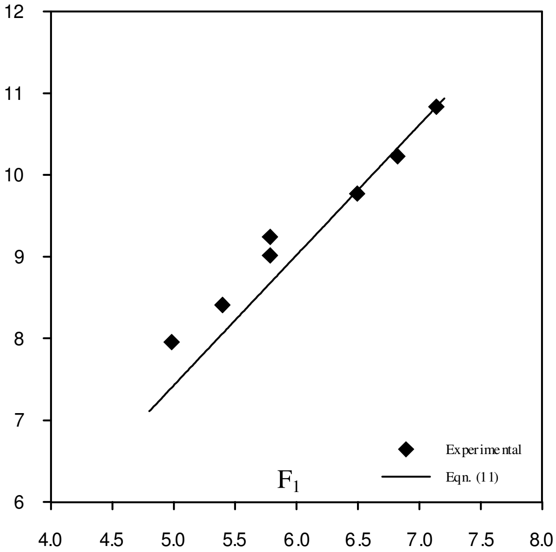

Figure 5a. Eqn. (11) versus measurements for z/d_1=1.9 at r=1.17, s/d_1=3.4, and r_o=1.3 (-ve B-J)

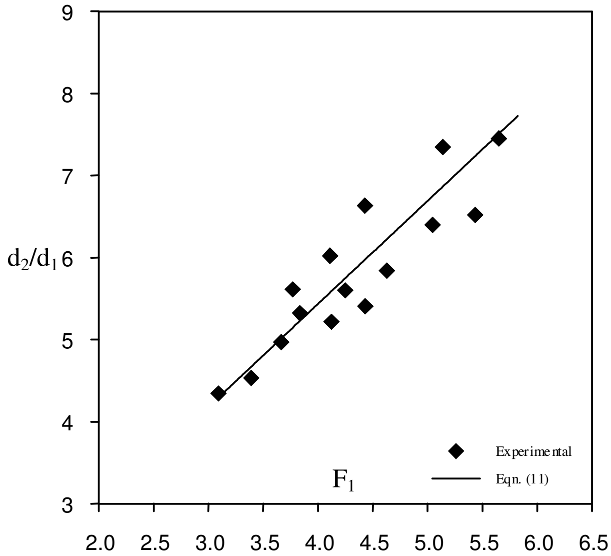

Figure 5d. Eqn. (11) versus measurements for z/d_1=1.0 at r=1.17, s/d_1=1.5, and r_o=1.3 (-ve B-J)

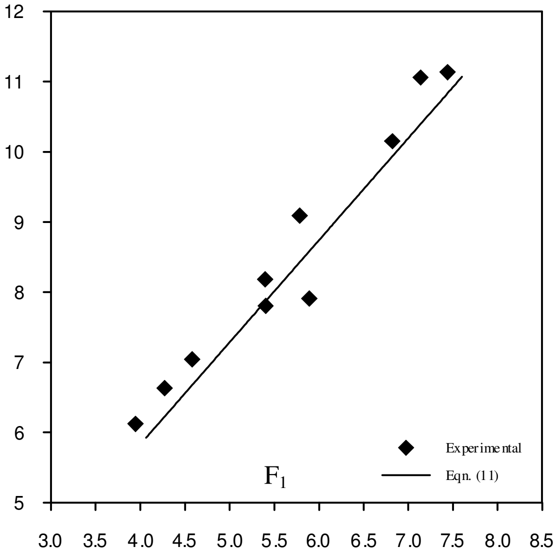

Figure 5b. Eqn. (11) versus measurements for z/d_1=1.6 at r=1.17, s/d_1=2.4, and r_o=1.3 (-ve B-J)

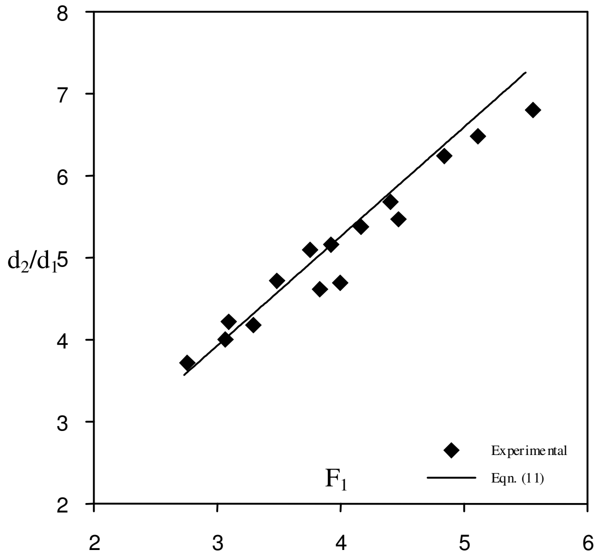

Figure 5e. Eqn. (11) versus measurements for z/d_1=0.8 at r=1.17, s/d_1=1.3, and r_o=1.3 (-ve B-J)

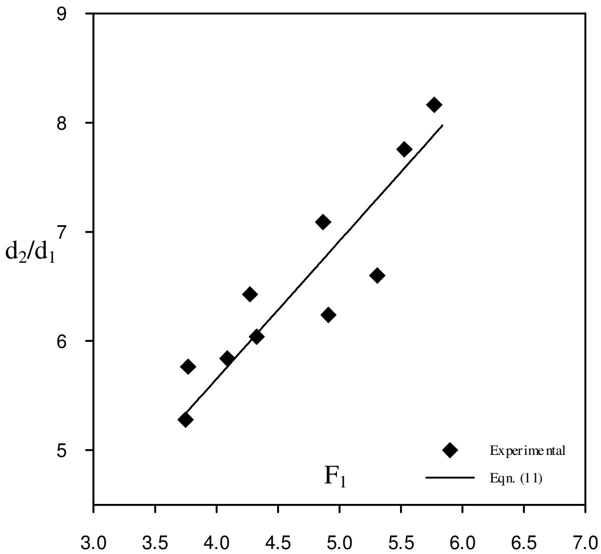

Figure 5c. Eqn. (11) versus measurements for z/d_1=1.2 at r=1.17, s/d_1=2.0, and r_o=1.3 (-ve B-J)

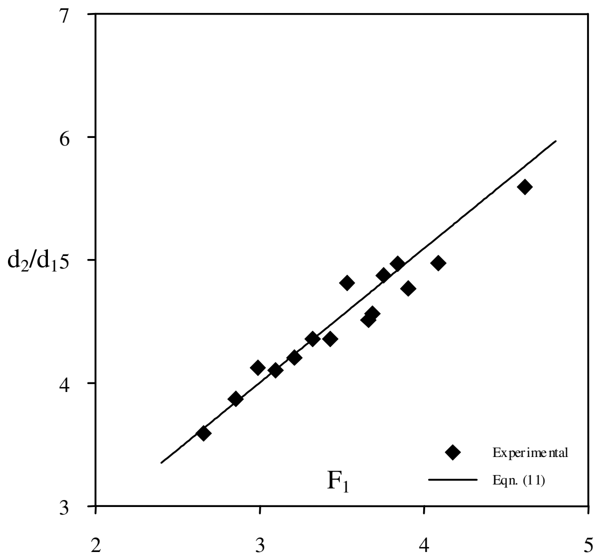

Figure 5f. Eqn. (11) versus measurements for z/d_1=0.7 at r=1.17, s/d_1=0.9, and r_o=1.3 (-ve B-J)

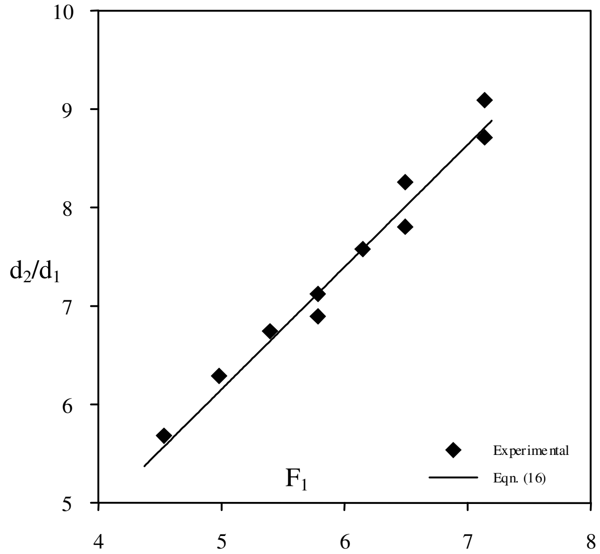

Figure 6a. Eqn.(16) versus measurements for z/d_1=1.9 at r=1.33, s/d_1=3.4, and r_o=1.3 (-ve A-J)

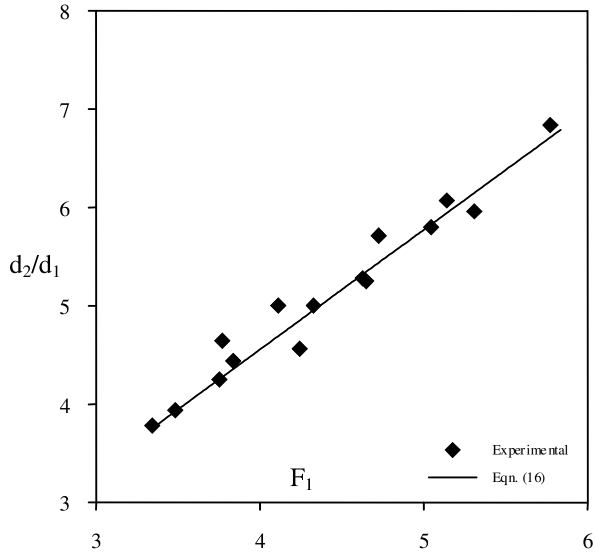

Figure 6d. Eqn.(16) versus measurements for z/d_1=1.0 at r=1.33, s/d_1=1.5, and r_o=1.3 (-ve A-J)

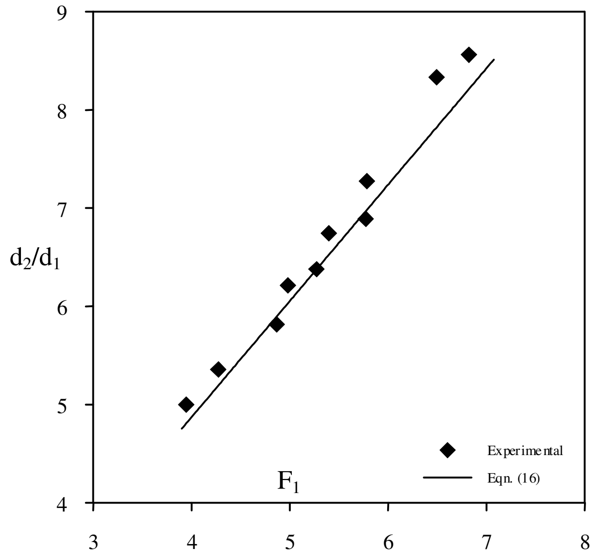

Figure 6b Eqn.(16) versus measurements for z/d_1=1.6 at r=1.33, s/d_1=2.4, and r_o=1.3 (-ve A-J)

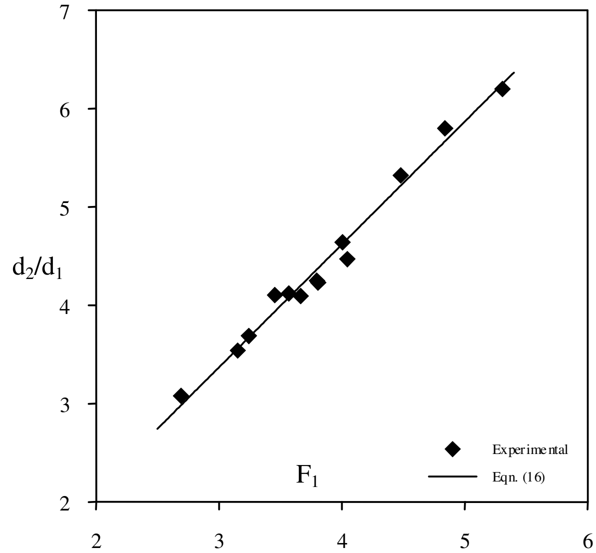

Figure 6e. Eqn.(16) versus measurements for z/d_1=0.8 at r=1.33, s/d_1=1.3, and r_o=1.3 (-ve A-J)

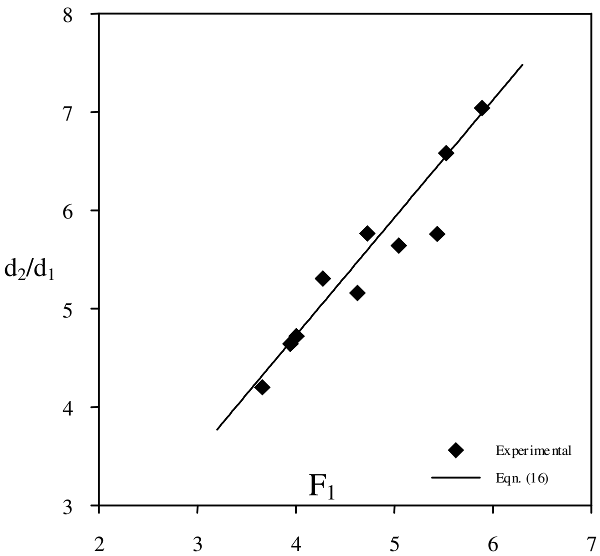

Figure 6c. Eqn.(16) versus measurements for z/d_1=1.2 at r=1.33, s/d_1=2.0, and r_o=1.3 (-ve A-J)

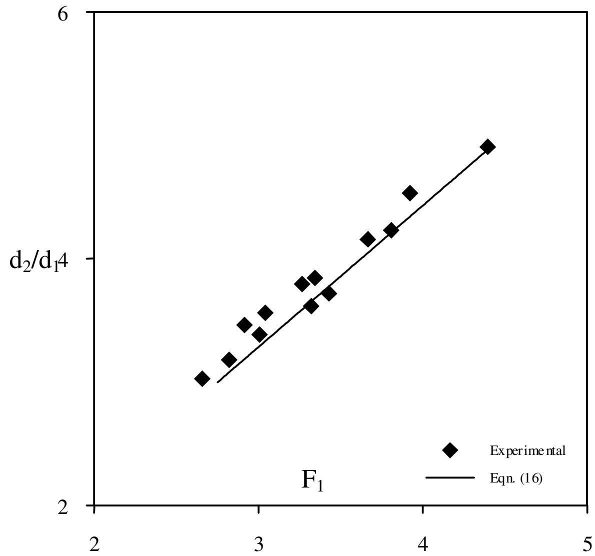

Figure 6f. Eqn.(16) versus measurements for z/d_1=0.7 at r=1.33, s/d_1=0.9, and r_o=1.3 (-ve A-J)

CONCLUSIONS

Theoretical models, equations (11) and (16) are developed for the prediction of the depth ratios of the different forms of hydraulic jumps that could be formed in radial stilling basins when a negative step (sudden drop) is existed in the basin which ends with sill. The jumps may be negative B- jump or negative A- jump at a sudden drop. An experimental program is conducted to collect experimental data on the two types of jumps using radial basin of constant divergence angle and fixed length but variable positions of the step in the basin and different heights of the end sill. The developed theoretical models are verified using the collected experimental data. Good agreement is obtained with mean relative absolute error (MRE) of 0.032 and 0.026 for B and A-jumps respectively. The coefficients of determination R^2 between theoretical and experimental values are 0.982 and 0.988 for the case of B and A- jumps respecively. The obtained results indicated that the developed equation could be used safely in the design of radial stilling basins provided with a negative step and ended with a sill.

NOTATIONS

b = contracted width of the channel;
B = width of the channel;
d_1 = water depth at vena contracta downstream the gate;
d_2 = sequent water depth;
d_4 = water depth over end sill;
d_5 = water depth just downstream the end sill over end sill;
d_6 = water depth just upstream the end sill over end sill;
d_o = the relative water depth, d_2/d_1;
d_3 = depth of water above the step;
d = the ratio of d_3 to d_1;
d_s = the ratio of $d_4$to d_1;
d_{s*} =the ratio of d_6 to d_1;
F_1 = Froude's number at the initial depth;
L_j = the length of the hydraulic jump;
L_b= the length of the stilling basin;
P_1 = the hydrostatic pressure at the beginning of the jump the jump;
P_2 = the hydrostatic pressure just downstream the end of the jump;
P_s = channel side pressure force;
P_3 = pressure force due to step;
P_5= pressure force just downstream the end sill;
Q = rate of flow;
r_1 = radius at the beginning of the jump;
r_2 = radius at the end of the jump;
r_o = the ratio of r_2 to r_1;
r_3 = radius at the end of the step;
r_4 = radius at the end sill;
r = the ratio of r_3 to r_1;
R^2= the coefficient of determination:

s = the sill height;
S = the ratio of s to d_1;
V_1 = average velocity at the initial depth;
V_2 = average velocity at the sequent depth;
V_5 = average velocity just after the end sill;
z = the drop height;
Z = the ratio of z to d_1;
γ = the specific weight, and
θ = the angle of divergence.

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