THEORETICAL MODELING OF HYDRAULIC JUMPS IN RADIAL STILLING BASINS ENDED WITH SILLS

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ABSTRACT

The use of end sill is one of the common methods that have been used to optimize the hydraulic jump length. Extensive studies have been conducted to investigate the effect sills or end sills on the hydraulic jumps in rectangular basins but not yet in radial basins. The complete design of stilling basins needs the accurate estimation of the depth ratio of the formed hydraulic jump in the basin. In the present paper, a theoretical model is developed to predict the depth ratio of the radial hydraulic jump when a basin is ended with a sill. Both the momentum and continuity equation in one dimension are applied to the control volume where the basin begins and ends. Also, the energy equation is used to develop a theoretical model to evaluate the energy loss by the radial hydraulic jump when a sill exists at the end of the basin. An experimental program is conducted to collect experimental data to enable verification of the developed theoretical models. Good agreement between theoretical and experimental results is obtained. The developed models could be used safely to compute both the depth ratio of the jump and the energy loss by the jump formed in radial basin with an end sill.

Keywords: Hydraulic jump, Theoretical modeling, Stilling basin, Non-prismatic basins, Expanding channels, End sill

1. INTRODUCTION

Hydraulic jumps are advantageous for dissipating kinetic energy in stilling basins. It may be free or submerged depending on both the location and the initial depth of the jump relative to the gate. The different classifications of jumps were reported in Chow [1]. The hydraulic jump may be also formed in prismatic or in non-prismatic channels (diverging or sudden expanded), and may be forced or non-forced. Most of the studies on different types of hydraulic jump are presented in Hager [2].

Khalifa and McCorquodale [3], studied the radial hydraulic jump occurs in stilling basins with diverging side-walls. They concluded that the sequent depth ratio of the radial jump is less than that of the rectangular jump, and the length of the radial hydraulic jump is about 70% of that of rectangular jump with the same flow conditions. Also, the energy loss in a radial hydraulic jump is 15% higher than that of the rectangular jump. According to the various methods used in practice, stilling basins were arranged in a variety of geometrical configurations. On the other hand, sills or blocks were used in stilling basins to increase the rate of energy dissipation and to reduce the bed velocity in the region of the hydraulic jump. Many studies had been conducted to investigate the effect of sills in rectangular basins. The effect of the sill on the jump characteristics depends on factors such as the sill configuration, sill location and sill spacing when more than one sill was used. Several investigations dealt with the effect of sill on the hydraulic jump characteristics when the sill was constructed beneath hydraulic jump such as Shukry [4], Rajaratnam [5], Ohtsu and Yasuda [6], and Hager and Li [2]. Hager and Li gave one of these classifications of the forced hydraulic jump due to vertical sill. They classified the jump over vertical sill into A-jump, B-jump, minimum B-jump and C-jump. The A-jump was corresponding to the classical hydraulic jump, which was characterized by the maximum sequent depth ratio for the free jumps. They stated that, A-jump in which the jump characteristics are not influenced by the presence of sill (or weak effect are present) as the sill was found at the end of the surface roller and thus it is out side the effective zone for the sill to affect the jump flow. Other studies on the effect of vertical sill on the jump and different classification of jumps due to presence of sill could be reviewed in Hager [2]. Wafaie [8,9] investigated experimentally the free rectangular hydraulic jump phenomenon on roughened channel bed with dentated, solid, zigzagged bed sills, under different flow conditions, different bed sill heights, and different bed sill locations. Statistical analysis for the experimental results was made to obtain the best height and location of the bed sill. Recently, few studies were conducted to discuss the effects of negative step and/or end sill on the characteristics of the submerged hydraulic jump in radial basins, Negm et al. [10,11].

The above review indicated that the effect of end sill on free hydraulic jump in radial stilling basins was not investigated either theoretically or experimentally. This paper deals with the free hydraulic jump in radial basin with an end sill. The purpose of this research is to develop theoretical equations to predict the depth ratio and the energy loss ratio of the free radial hydraulic jump in the presence of an end sill at the end of the radial stilling basin.

2. DEVELOPMENT OF THEORETICAL MODELS

2.1 Depth Ratio (d_2/d_1)

The definition sketch of the radial hydraulic jump formed in smooth horizontal bed with end sill in a rectangular channel is shown in Figure 1. By applying the pressure-momentum relationship in the longitudinal direction yields:-

$$P_5 - P_1 - 2P_S \sin\frac{\theta}{2} + P_4 = \frac{\gamma Q}{g}(\beta_1 V_1 - \beta_5 V_5) \tag{1}$$

in which P_1: hydrostatic pressure before the jump, P_4: hydrostatic pressure just after the end sill, P_3: horizontal component of pressure on end sill, P_S: channel side pressure from the initial water depth to the end sill.

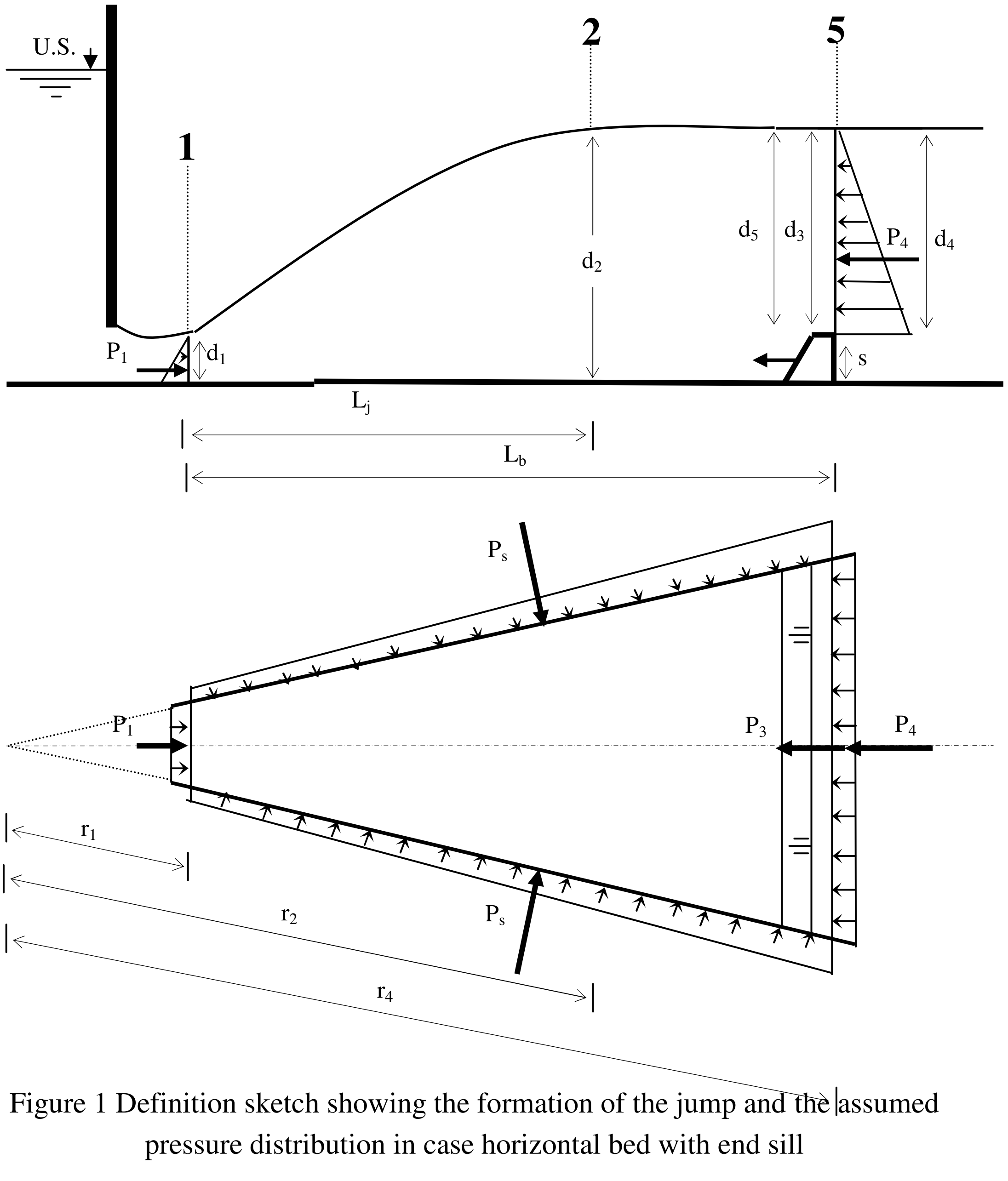

Figure 1 Definition sketch showing the formation of the jump and the assumed pressure distribution in case horizontal bed with end sill

These forces may be expressed as follows

$$P_1 = \frac{\gamma d_1^2 b_1}{2} \ , \ P_4 = \frac{\gamma d_4^2 b_4}{2} \ , \ P_3 = \frac{1}{2}\gamma b_3 (s\, d_5 + s\, d_3 + s^2) \tag{2}$$

$$P_s = \frac{\gamma}{6}\Big[(r_2 - r_1)(d_1^2 + d_2^2 + d_1 d_2) + d_2^2 + (r_4 - r_2)(d_2^2 + (d_4+s)^2 + d_2\,(d_4+s))\Big] \tag{3}$$

Substituting for P_1 ,P_4, P_3, and P_S from equations (2), and (3) respectively in equation (1).

$$\frac{\gamma d_4^2 b_4}{2} - \frac{\gamma d_1^2 b_1}{2} - \frac{\gamma}{3}\Big[(r_3 - r_1)(d_1^2 + d_2^2 + d_1 d_2) + (r_4 - r_2)(d_2^2 + (d_4+s)^2 + d_2\,(d_4+s))\Big]\sin\frac{\theta}{2}$$
$$+ \frac{1}{2}\gamma b_3 (s d_5 + s d_3 + s^2) = \frac{\gamma Q}{g}(\beta_1 V_1 - \beta_4 V_4) \tag{4}$$

Applying the continuity equation:

$$Q = b_1\, d_1\, V_1 = b_4\, (d_4+s)\, V_4 \tag{5}$$

Where b_1, and b_4 are the channel width at the beginning, and the end of the basin.

$$b_1 = 2 r_1 \sin \theta/2, \ b_4 = 2 r_4 \sin \theta/2 \tag{6}$$

Substituting from equation (6), in equation (5), and solving for V_4 then:

$$V_4 = V_1\, r_1\, d_1 / r_4\, (d_4+s) = V_1 / r_s\, (d_s+S) \tag{7}$$

Where $d_4/d_1 = d_s$, $s/d_1 = S$, and $r_4/r_1 = r_s$

Substituting from equations (6) and (7) in equation (4), and assuming $\beta_1 = \beta_4 = 1.0$ then.

$$\gamma r_4 d_4^2 \sin\frac{\theta}{2} - \gamma r_1 d_1^2 \sin\frac{\theta}{2} - \frac{\gamma}{3}\begin{bmatrix}(r_2 - r_1)(d_1^2 + d_2^2 + d_1 d_2) \\ + (r_4 - r_2)(d_2^2 + (d_4+s)^2 + d_2 (d_4+s)\end{bmatrix}\sin\frac{\theta}{2}.$$
$$+ \gamma r_4 (s d_5 + s d_4 + s^2)\sin\frac{\theta}{2} = \frac{2\gamma}{g} v_1 r_1 d_1 \sin\frac{\theta}{2}\Big(V_1 - \frac{V_1}{r_s\,(d_s + S)}\Big) \tag{8}$$

Dividing equation (8) by γ, $\sin\theta/2$, and $r_1 d_1^2$:

$$r_s d_s^2 - 1 - \frac{1}{3}\left[\Big(\frac{r_2}{r_1} - 1\Big)\Big(1 + \frac{d_2^2}{d_1^2} + \frac{d_2}{d_1^2} + \frac{d_1 d_2}{d_1^2}\Big) + + \Big(\frac{r_4}{r_1} - \frac{r_2}{r_1}\Big)\left\{\Big(\frac{d_2^2}{d_1^2} + \frac{(d_4+s)^2}{d_1^2} + \frac{d_2 (d_4+s)}{d_1^2}\right\}\right]$$
$$+ \frac{r_4}{r_1}\Big(\frac{s}{d_1}\frac{d_5}{d_1} + \frac{s}{d_1}\frac{d_4}{d_1} + \frac{s^2}{d_1^2}\Big) = \frac{2V_1^2}{g d_1}\Big(\frac{r_s(d_s + S) - 1}{r_s(d_s + S)}\Big) \tag{9}$$

Let $d_2/d_1=d_o$, $r_4/r_1=r_s$, $s/d_1=S$, and $d_4/d_1=d_s$, $d_5/d_1=d_{s*}$
Multiplying equation (9) by $3r_s(d_s+S)$, with simplifying.

$$\begin{aligned}&d_o^2 r_s (d_s+S)(1-r_s)-d_o r_s (d_s+S)\left[r_o-1+(d_s+S)(r_s-r_o)\right]+r_s (d_s+S)\\&\left[3r_s d_s^2-2+3r_s (Sd_{s*}+Sd_s+S^2)-r_o-(d_s+S)^2(r_s-r_o)\right]\\&-6F_1^2\left[r_s(d_s+S)-1\right]=0.0\end{aligned} \tag{10}$$

Alternatively, equation (10) could be rearranged to take the easily solved form

$$F_1=\sqrt{\frac{d_o^2 r_s (d_s+S)(1-r_s)-d_o r_s (d_s+S)\left[r_o-1+(d_s+S)(r_s-r_o)\right]+r_s (d_s+S)\left[3r_s d_s^2-2+3r_s (Sd_{s*}+Sd_s+S^2)-r_o-(d_s+S)^2(r_s-r_o)\right]}{6\left[r_s(d_s+S)-1\right]}} \tag{11}$$

Equation (11) tends to the previously developed one by Abdel-Aal et al. [12] for hydraulic jump in smooth radial stilling basin (case of no sill, i.e. $S=0$, $r_o=r_s$, $d_o=d_s$)

2.2 Energy Loss Ratio E_L/E_1

The energy loss can be obtained by applying the energy equation as follow:

$$E_L= E_1-E_2 \tag{12}$$

From Figure (1) the specific energy at the beginning of the jump (E_1), and at the end of the jump E_2 can be written as:

$$E_1=d_1+\frac{\alpha_1 V_1^2}{2g} \tag{13}$$

$$E_2=d_2+\frac{\alpha_2 V_2^2}{2g} \tag{14}$$

Keeping in mind that $F_1=\dfrac{V_1}{\sqrt{gd_1}}$, and $V_2=\dfrac{V_1}{r_o d_o}$, substituting in equation (14) to get.

$$E_2=d_2+\frac{V_1^2}{2gr_o^2d_o^2} \tag{15}$$

$$E_2=d_2+\frac{d_1F_1^2}{2r_o^2d_o^2} \tag{16}$$

$$E_2=d_1\left(\frac{2r_o^2d_o^3+F_1^2}{2r_o^2d_o^2}\right) \tag{17}$$

From equation (13)

$$E_1=d_1\left(1+\frac{1}{2}F_1^2\right) \tag{18}$$

From equations (17), and (18) then:

$$\frac{E_2}{E_1}=\frac{2\,r_o^2 d_o^3+F_1^2}{r_o^2 d_o^2(2+F_1^2)} \tag{19}$$

$$\frac{E_L}{E_1}=1-\frac{E_2}{E_1} \tag{20}$$

Substituting from equation (19) in equation (20) then:

$$\frac{E_L}{E_1}=\frac{r_o^2 d_o^2(2+F_1^2-2\,d_o)-F_1^2}{r_o^2 d_o^2(2+F_1^2)} \tag{21}$$

3. EXPERIMENTAL WORK

The experimental work of this study was conducted using a re-circulating adjustable flume of 15.0 m long, 45 cm deep and 30 cm wide, Habib [13]. The discharges were measured using pre-calibrated orifice meter fixed on the feeding pipeline. The tailgate fixed at the end of the flume was used to control the depth of flow for each run. The radial basin was made from a clear prespex to enable visual inspection of the phenomenon being under investigation. The model length was kept constant at 130 cm and a constant angle of divergence of 5.28^o was used. The model was fixed in the middle third of the flume between its two side-walls as shown in Figure 2. A smooth well painted baffle block of wood was formed to fit well inside the basin model extending from one side of the model to the other side at the end of the basin to simulate the end sill. The end sill has an upstream slope of 1:1 and vertical face from the downstream side. The wood was well painted by a waterproof material (plastic) to prevent wood from changing its volume by absorbing water. Three different heights of the end sill (viz 3, 4 and 5 cm) were tested under the same flow conditions. The range of the experimental data were as follows: Froude numbers (2.0-7.0), r_o (1.2-1.4), and relative height of the end sill, s/d_1 (0.0 – 3.4).

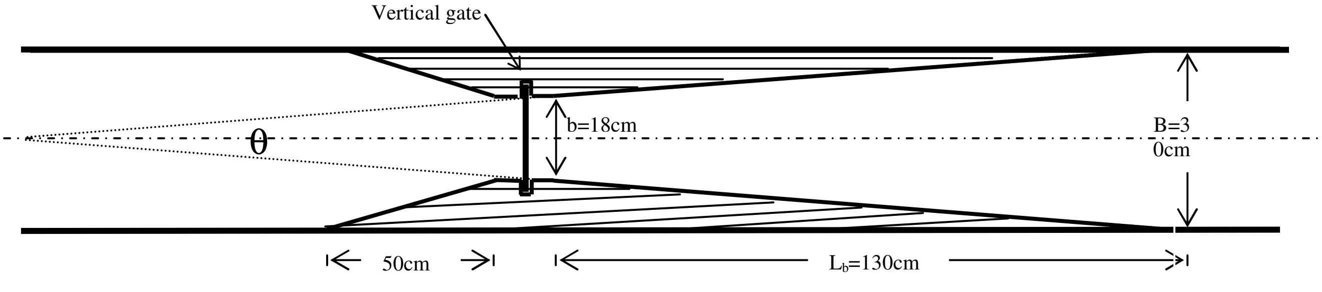

Figure 2 General sketch for smooth radial stilling basin model in plan

Each model was tested using five different gate openings and five discharges for each gate opening. The measurements were recorded for each discharge. The total number of runs was 100. A typical test procedure consisted of (a) a gate opening was fixed and a selected discharge was allowed to pass. (b) the tailgate was adjusted until a free hydraulic jump is formed. (c) once the stability conditions were reached, the flow rate, length of the jump, water depths upstream and at the vena contracta downstream of the gate in addition to the tail water depth and the depth of water above the step were recorded. The length of jump was taken to be the section at which the flow depth becomes almost fixed. These steps were repeated for different discharges and different gate openings and so on till the required ranges of the parameters being under investigation were covered.

4. VERIFICATION OF THE DEVELOPED MODELS

Figure 3 presents the comparison between theoretical values of Froude number ($F_{1\ The}$) as computed from Eqn. (11) for no sill case and the its values as computed from the measurements ($F_{1\ Exp}$) for free hydraulic jump formed on smooth radial basin. The coefficient of determination (R^2) between theoretical and measured values of Froude number is 0.992 while the relative mean absolute error (MRE) is 0.024. Similarly, Figure 4 presents the same comparisons for the case when the basin ends with a sill of 1:1 sloping US face and vertical DS face based on equation (11) as compared with those from measurements. The value of the coefficient of determination (R^2) in this case is 0.973 while the value of mean relative absolute error is 0.035.

Figure 5 shows the comparison between theoretical values and experimental ones (the relationship between d_2/d_1 and F_1) for typical values of s/d_1 of (0, 3.4, 2.4, 2, 1.5 and 1.3) when ro=1.3. All these figures from 3 to 5 indicate good agreement between theoretical and measured values.

Figure 6 presents the comparison between theoretical values of energy loss ratio as computed from Eqn. (21) for no sill case and the measured values for free hydraulic jump formed on smooth radial basin. The coefficient of determination (R^2) between theoretical and measured values of Froude number is 0.968 while the relative mean absolute error (MRE) is 0.04.

Similarly, Figure 7 presents the same comparisons for the case when the basin ends with a sill of 1:1 sloping US face and vertical DS face based on equation (21) as compared with those from measurements. The value of the coefficient of determination in this case is 0.931 while the value of mean relative absolute error is 0.037.

Figure 8 shows the comparison between theoretical values and experimental ones (the relationship between E_L/E_1 and F_1) for typical values of s/d_1 of (0, 3.4, 2.4, 2, 1.5 and 1.3) when r_o=1.3. All these figures from 6 to 8 indicate good agreement between theoretical and measured values.

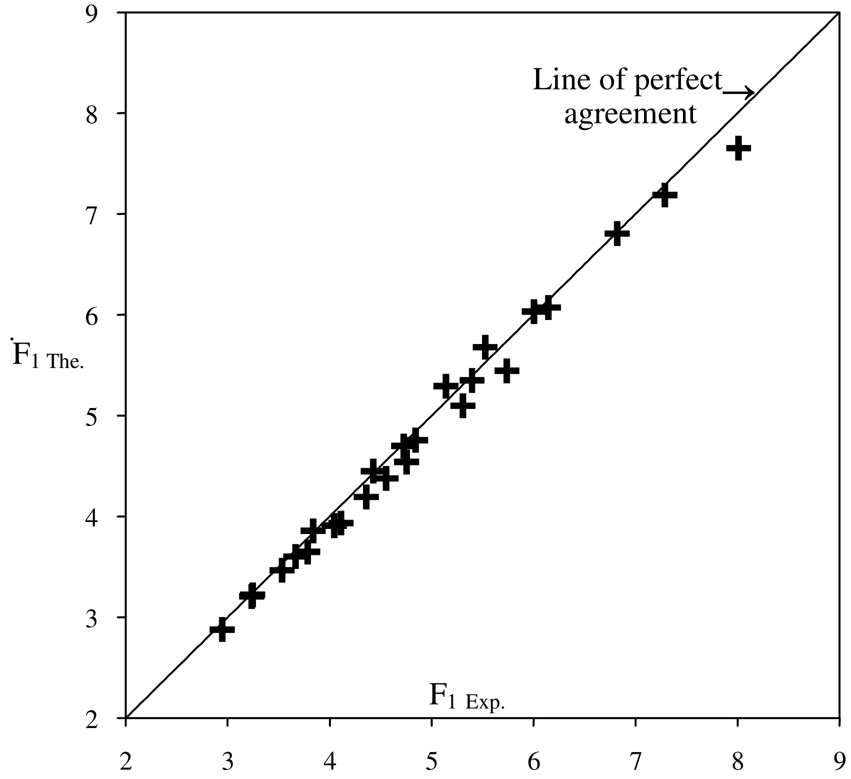

Figure 3. Verification of Eq. (11) for free radial jump on smooth bed without end sill

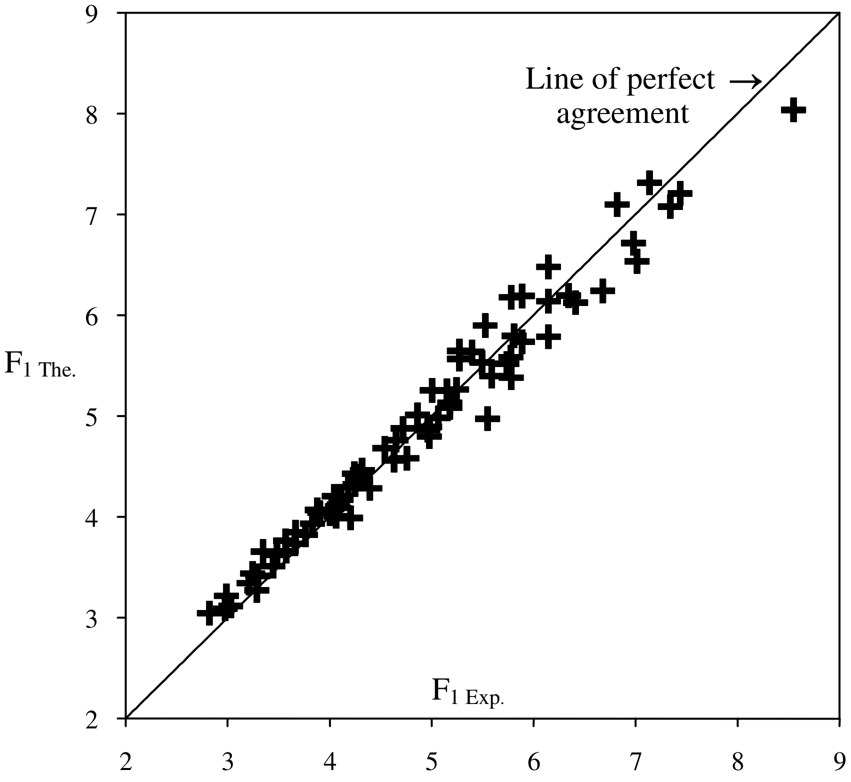

Figure 4. Verification of Eq. (11) for free radial jump on smooth bed with end sill

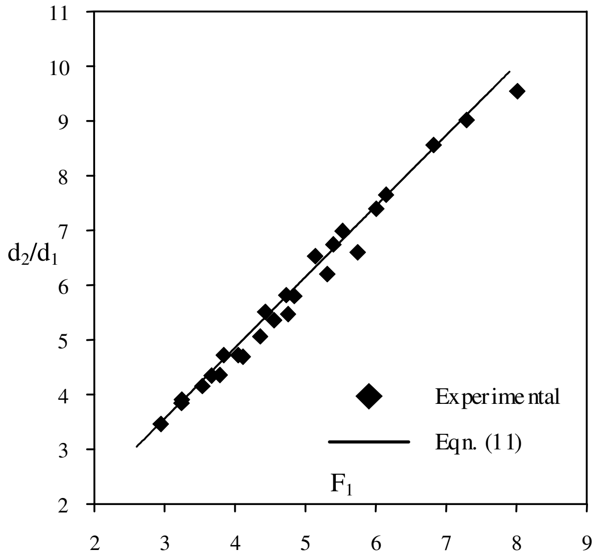

Figure 5a. Relationship between d_2/d_1 and $F_1$for smooth bed without end sill, s/d_1=0.0, and r_o=1.3

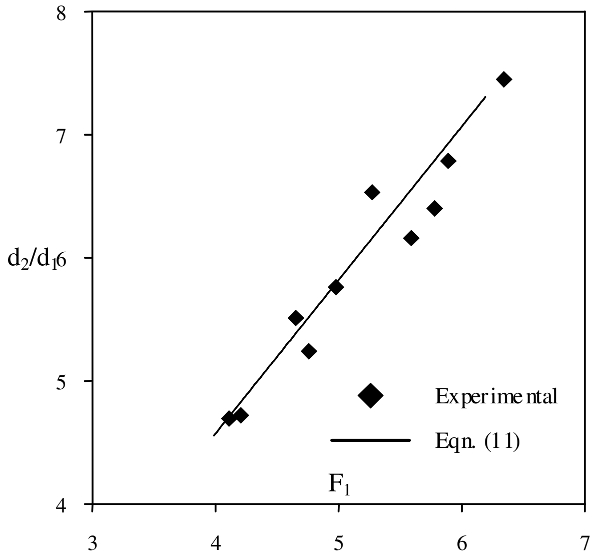

Figure 5d. Relationship between d_2/d_1 and $F_1$for smooth bed with end sill, s/d_1=2.0, and r_o=1.3

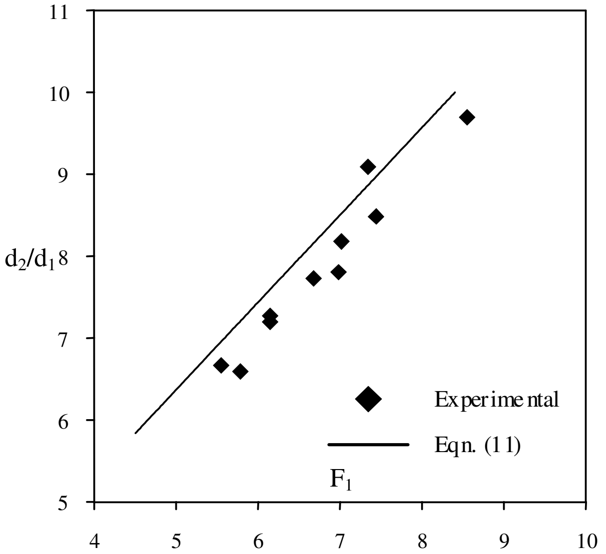

Figure 5b. Relationship between d_2/d_1 and $F_1$for smooth bed with end sill, s/d_1=3.4, and r_o=1.3

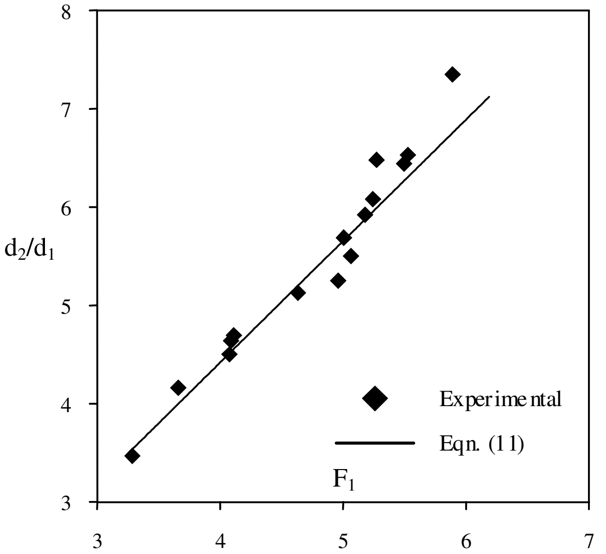

Figure 5e. Relationship between d_2/d_1 and $F_1$for smooth bed with end sill, s/d_1=1.5, and r_o=1.3

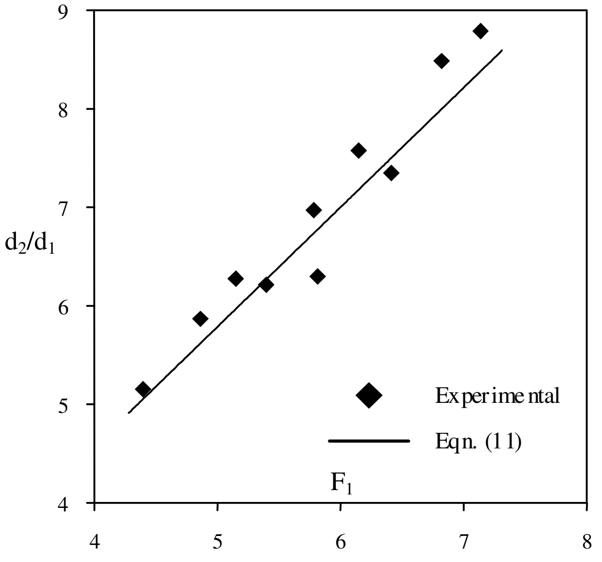

Figure 5c. Relationship between d_2/d_1 and $F_1$for smooth bed with end sill, s/d_1=2.4, and r_o=1.3

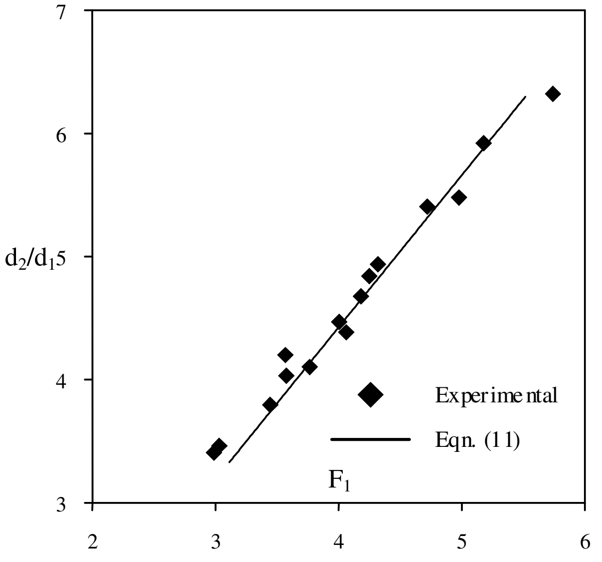

Figure 5f. Relationship between d_2/d_1 and $F_1$for smooth bed with end sill, s/d_1=1.3, and r_o=1.3

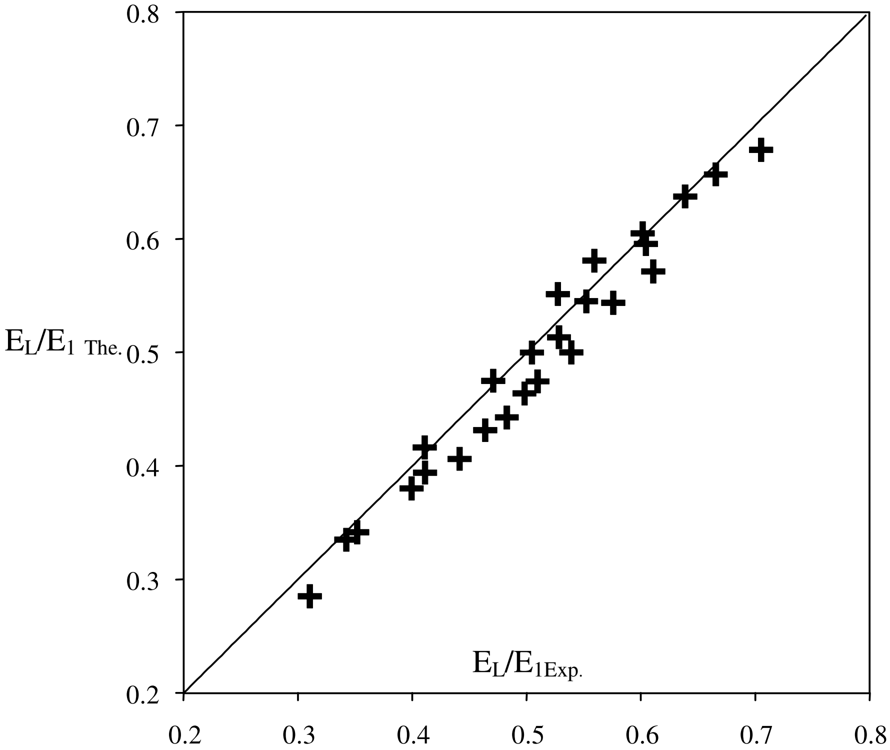

Figure 6. Verification of Eqn. (21) for free radial jump on smooth bed without end sill

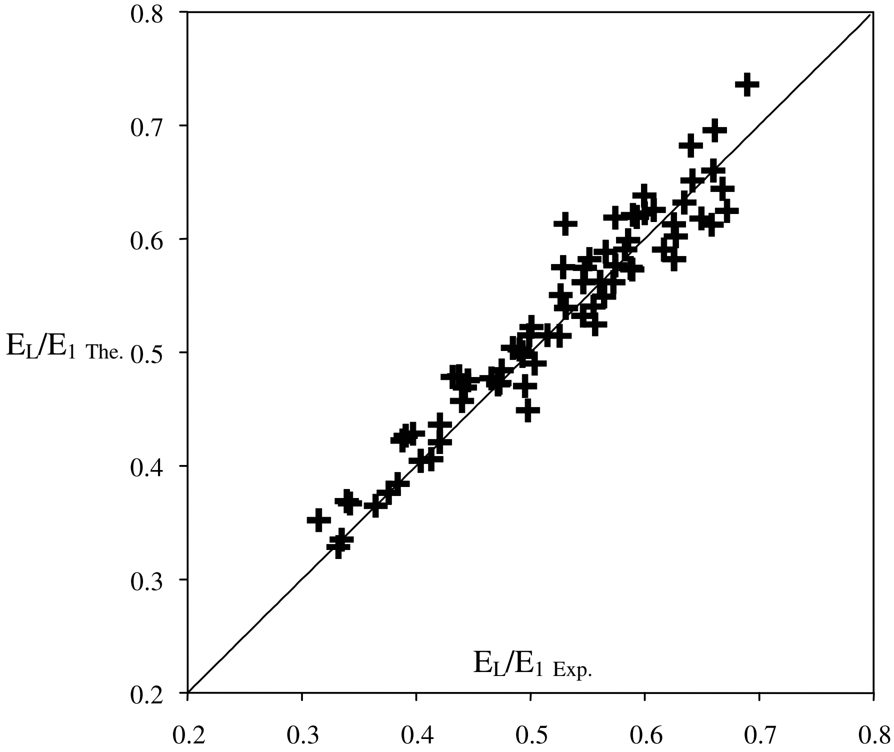

Figure 7. Verification of Eqn. (21) for free radial jump on smooth bed with end sill

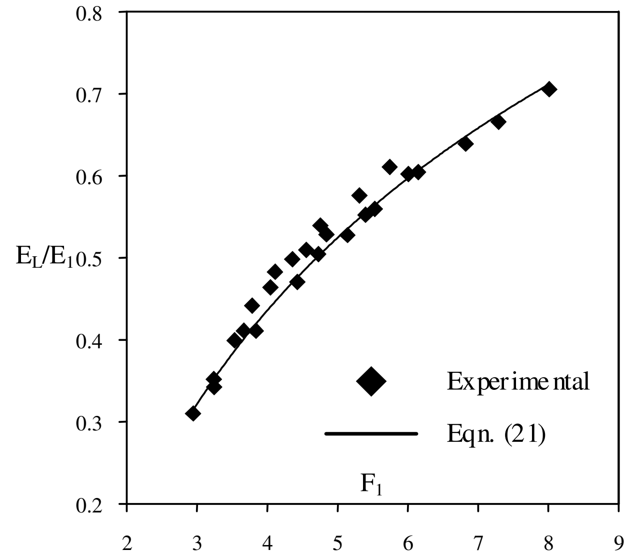

Figure 8a. Relationship between E_L/E_1 and F_1 for smooth bed without end sill, $s/d_1=0.0$, and $r_o=1.3$

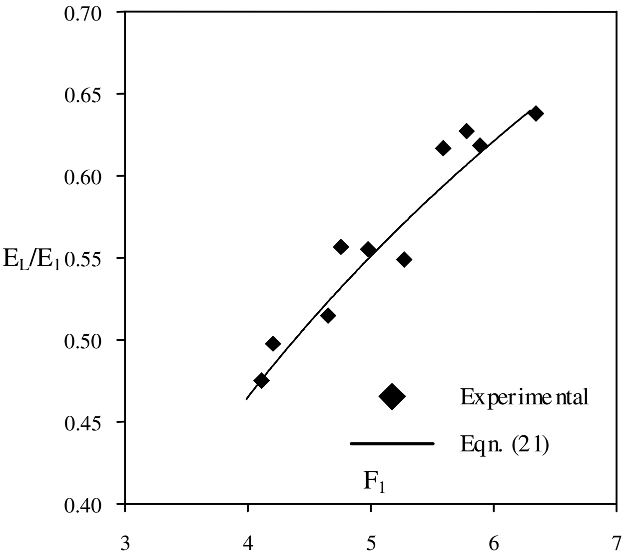

Figure 8d. Relationship between E_L/E_1 and F_1 for smooth bed without end sill, $s/d_1=2.0$, and $r_o=1.3$

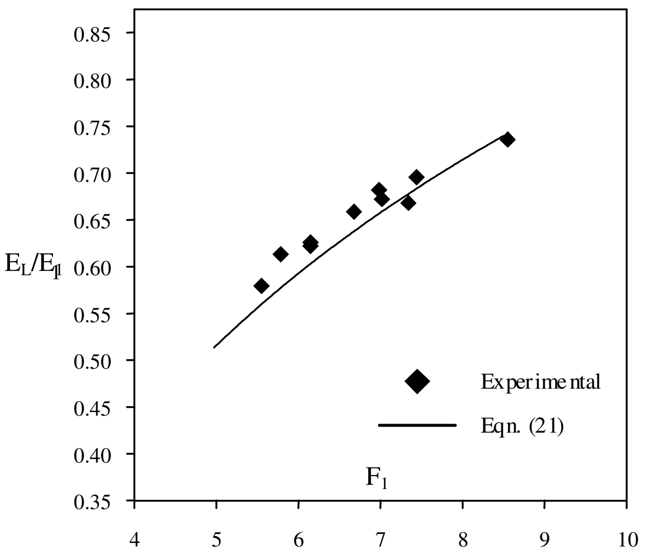

Figure 8b. Relationship between E_L/E_1 and F_1 for smooth bed without end sill, $s/d_1=3.4$, and $r_o=1.3$

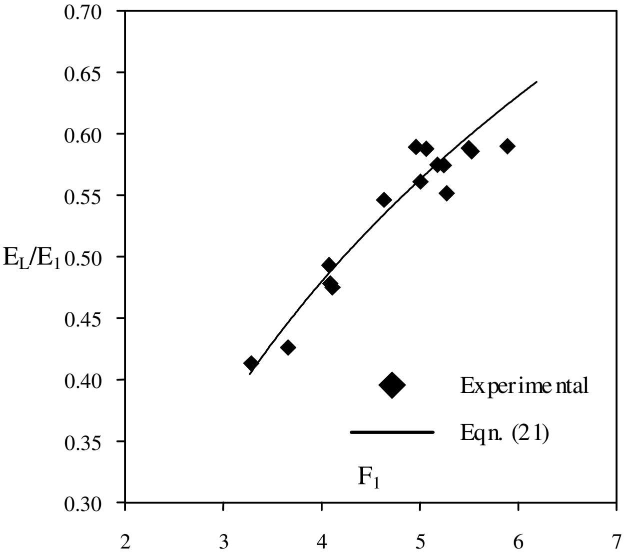

Figure 8e. Relationship between E_L/E_1 and F_1 for smooth bed without end sill, $s/d_1=1.5$, and $r_o=1.3$

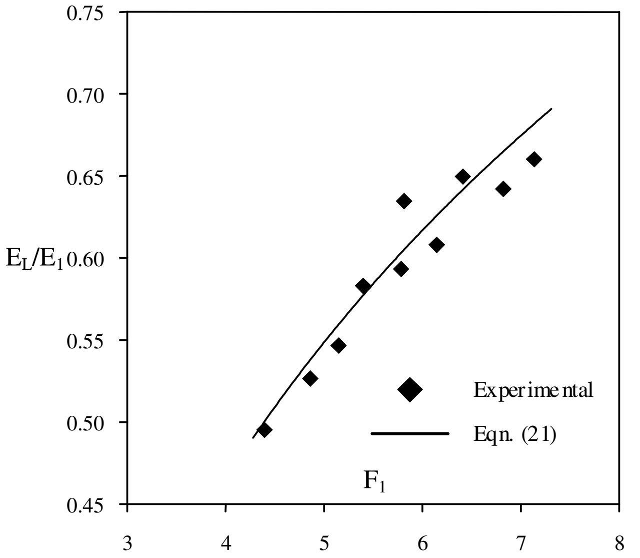

Figure 8c. Relationship between E_L/E_1 and F_1 for smooth bed without end sill, $s/d_1=2.4$, and $r_o=1.3$

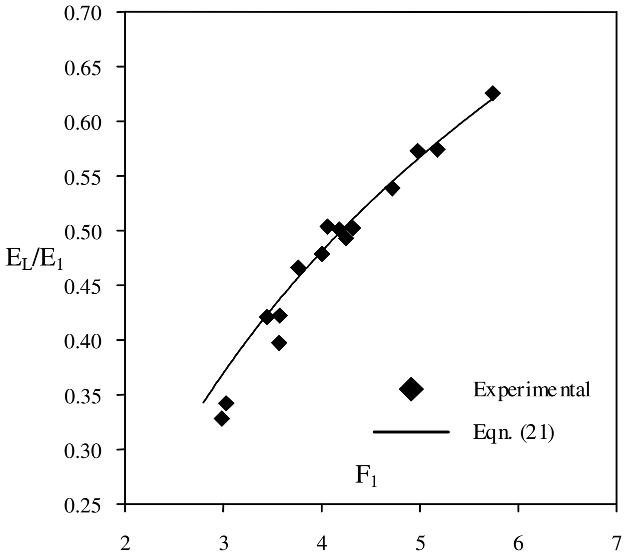

Figure 8f. Relationship between E_L/E_1 and F_1 for smooth bed without end sill, $s/d_1=1.3$, and $r_o=1.3$

5. CONCLUSIONS

Theoretical models, equations (11) and (21) were developed for the prediction of the depth and energy loss ratios of the hydraulic jump that could be formed in radial stilling basins when the basin ends with a sill of 1:1 sloping US face and vertical DS face. An experimental program was conducted to collect experimental data on free hydraulic jumps formed in both smooth radial-stilling basins and in those ends with a sill. The developed theoretical models were verified using the collected experimental data. Good agreement was obtained with mean relative absolute error (MRE) of 0.035 and 0.037 for depth ratio and the energy loss ratio respectively in case of end sill. The correlation coefficients between theoretical and experimental values for these cases were 0.973 and 0.931 for depth and energy loss ratios respectively. The obtained results indicated that the developed equations could be used safely in the design of radial stilling basins provided with end sills.

NOTATIONS

b = contracted width of the channel;
B = width of the channel;
d_1 = water depth at vena contracta downstream the gate where the channel width is b_1;
d_2 = sequent water depth where the channel width is b_2;
d_3 = water depth over end sill where the channel width is b_3;
d_4 = water depth just downstream the end sill over end sill (channel width = B);
d_5 = water depth just upstream the end sill over end sill where the channel width is b_5;
d_o = the relative water depth, d_2/d_1;
d_s = the ratio of $d_3$to d_1;
d_{s*} =the ratio of d_5 to d_1;
F_1 = Froude's number at the initial depth;
L_j = the length of the hydraulic jump;
L_b= the length of the stilling basin;
P_1 = the hydrostatic pressure at the beginning of the jump;
P_2 = the hydrostatic pressure just at the end of the jump;
P_s = channel side pressure force;
P_3= horizontal component of pressure on end sill;
P_4= pressure force just downstream the end sill;
Q = rate of flow;
r_1 = radius at the beginning of the jump;
r_2 = radius at the end of the jump;
r_o = the ratio of r_2 to r_1;
r_3 = radius at the end sill;
r_s = the ratio of r_3 to r_1;
R^2= the coefficient of determination:
s = the sill height;
S = the ratio of s to d_1;
V_1= average velocity at the initial depth;

V_2= average velocity at the sequent depth;
V_4= average velocity just downstream the end sill;
γ = the specific weight, and
θ = the angle of divergence.

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