

## **THERMAL - ECONOMICAL ANALYSIS AND COMPARISON BETWEEN PYRAMID CONFIGURATION AND SIGNAL SLOPE SOLAR STILL**

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### **ABSTRACT**

This paper presents an analytical study as well as thermal and economical comparison between two solar stills configurations; the Pyramidal and single slope. A mathematical model has been developed to simulate the two configurations and study their thermal performance. The weather meteorological data of Aswan City (south of Egypt) was used, and the daily total energy received by each still basin was calculated. The main performance parameters such as still productivity and efficiency have been presented for the whole year. In addition, the economical assessment of the distilled water production cost has been carried out. On the basis of the yearly performance results, the single slope still was found to be slightly more efficient and economical than the pyramidal one.

**Keywords:** Solar, Desalination, Still, Pyramid, Economics

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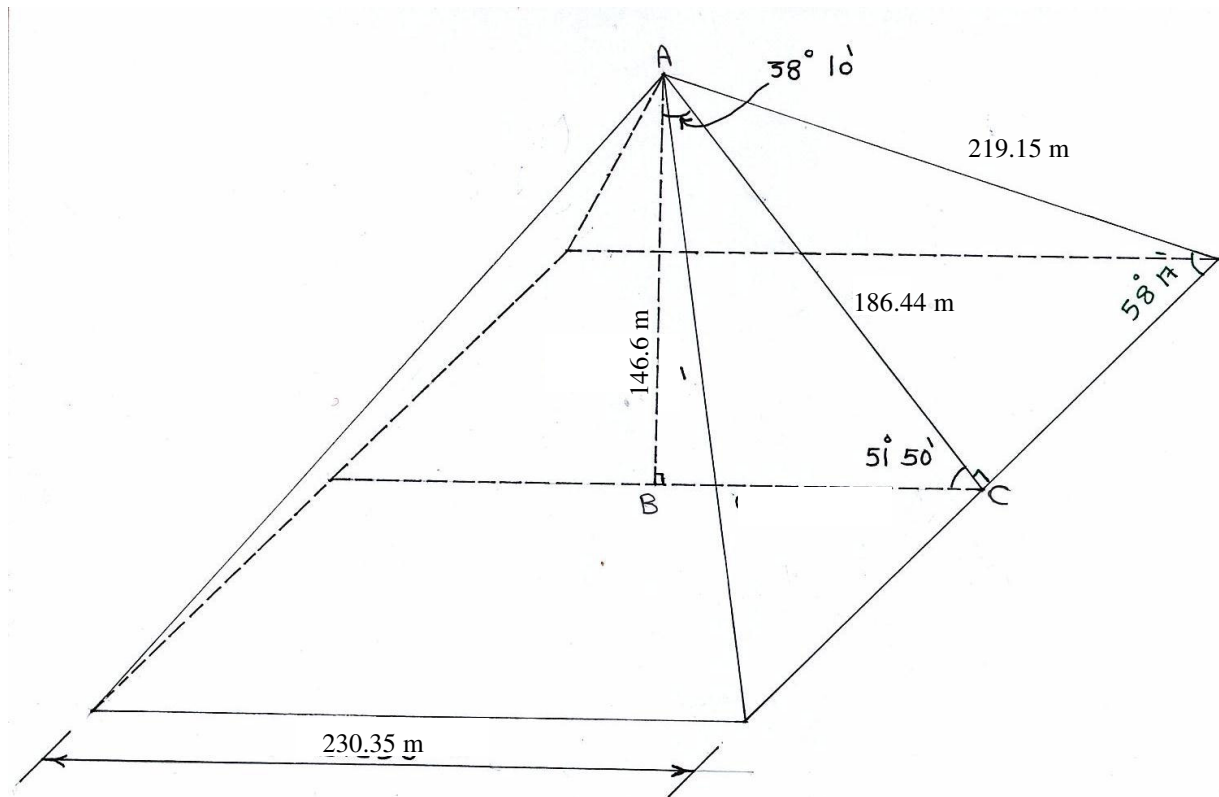
## **INTRODUCTION**

Desalination has become one of the important “ non-conventional” water resource that plays an important role in solving fresh water scarcity in different region in the world as the Middle East. The cost of desalinated water is continuously decreasing such that desalination, of both brackish and seawater, became technically and economically feasible and competitive to other water resources particularly in remote areas. Energy cost represents a major contribution to desalinated water unit cost. Semiat [1] optimized a relative cost of energy stand alone desalinated plant in the range of 30 % to 50 % of the product water cost.

The problem of conventional energy source availability, possible depletion and cost as well as the environmental impacts (trend to reduce CO<sub>2</sub> emission in order to minimize globe greenhouse thermal effect) necessities the reconsideration of renewable resources as solar. Direct solar distillation (using stills) is one of the promising renewable energy alternative especially in areas of relatively high solar intensity as in the Middle East. The obvious advantage of solar desalination is that energy required for conversion is free and its technology is very simple. However the disadvantages are equally obvious; the conversion efficiency is low and dependent on the solar intensity, which varies with geographical location, time of the year, clock time and to the extent of cloudiness, fog or haze. Fath [2] stated that even under the ideal solar conditions, only about four to five liter of fresh water could be obtained each day for each square meter of solar collection area. Many solar stills configurations have been investigated and published in literature, see Fath [2], however non has considered the Pyramidal still. The motivation for this configuration arises from the fact that pyramid configuration and dimension

(particularly the Great pyramid of Giza, Egypt) presents a mystery that motivates many to investigate its secrets. Two striking numerical “coincidence” quickly emerge from its data; i) the slant height (611.5 Ft) when divided by one half the length of its side (377.9 Ft) results in the enigmatic ratio known as the “golden ratio” or Phi (1.618) as was known later during the Renaissance Era, and ii) twice the perimeter (6064 Ft) is the length that turns out to be precisely equal to the length of a degree of latitude at the equator. Figure (1) shows the general dimensions of the Great Pyramid of Giza, Egypt.

This study presents, see also Hassabou [3], a numerical analysis of the great pyramid configuration solar still. Thermal and economical comparison with the conventional single sloped basin still will be investigated.



**Fig. (1) diagrammatic sketch and general dimensions of the Great Pyramid.**

## THERMAL THEORETICAL STUDY

The basic elements of analyzing a conventional solar still are the transparent cover and the insulated basin. Both singled slope and the pyramid stills will be analyzed similarly except that the pyramid transparent cover takes the shape of four sides facing the geographical directions of East, West, North and South, while the single slope glass cover is facing south. Figure (2) illustrates the pyramid and single slope still configurations and the stills overall heat balance. The selected scaled down base area of both stills is 1.5274 m<sup>2</sup> (1.235 m x 1.235 m). The governing transient energy equations for the system components can be written as follows, see also Hassabou [3] and Darwish et. Al. [4]:

### Solar Energy

The total horizontal insolation is given by: -

$$I_{t,h} = I_{b,n} \cos \theta_z + I_{d,h} \quad (1)$$

Where  $I_{d,h}$  represents the diffused radiation on a horizontal surface

And the total insolation on inclined surface from horizon by angle  $\beta$  is given by:

$$I_{t,\beta} = I_{b,n} \cos \theta_i + \{ I_{d,h} (1 + \cos\beta) / 2 + \rho I_{t,h} (1 - \cos\beta) / 2 \} \quad (2)$$

Where  $\theta_i$  is the angle of incidence calculated as:

$$\cos \theta_i = \sin \alpha \cos \beta + \cos \alpha \sin \beta \cos (\gamma - A)$$

### Still Heat Balance

$$(\alpha_g A_g + \tau_g \alpha_w A_b) H_s = A_g q_{ga} + A_b q_b + C_s dT_w/dt \quad (3)$$

### Glass Cover Heat Balance

$$A_g q_{ga} = A_b (q_c + q_r + q_e) + \alpha_g A_g H_s \quad (4)$$

Where:

$$q_c = 0.8831 (T_w - T_g) [(T_w - T_g) + (P_w - P_g) * (T_w + 273) / (0.265 - P_w)]^{0.33}$$

$$q_e = 1.92 \times 10^{-3} (p_w - p_g) L_w [(T_w - T_g) + (P_w - P_{wg}) * (T_w + 273) / (0.265 - P_w)]^{0.33}$$

$$q_r = 0.9 \sigma [(T_w + 273)^4 - (T_g + 273)^4] \quad , \quad q_b = h_b A_b (T_w - T_b)$$

$$q_{ga} = q_{air} + q_{sky} \quad , \quad q_{air} = h_{ga} (T_w - T_a) \quad , \quad h_{ga} = a + b (v)^n$$

$$T_{sky} = (T_{air} - 10) \quad q_{sky} = 0.9 \sigma [(T_g + 273)^4 + (T_a + 273)^4]$$

The still instantaneous evaporation rate  $D_i(t)$ , the still daily productivity  $D_d$ , daily efficiency  $\eta_d$  and annual average efficiency  $\eta_y$  are:

$$\text{Evaporation Rate } D_i(t) = q_e(t) / L_{wg} \quad (5)$$

$$\text{Productivity } D_d = \sum D_i(t) \Delta t \quad (6)$$

$$\eta_d = \sum q_e(t) \Delta t / \sum H_s(t) \cdot \Delta t \quad (7)$$

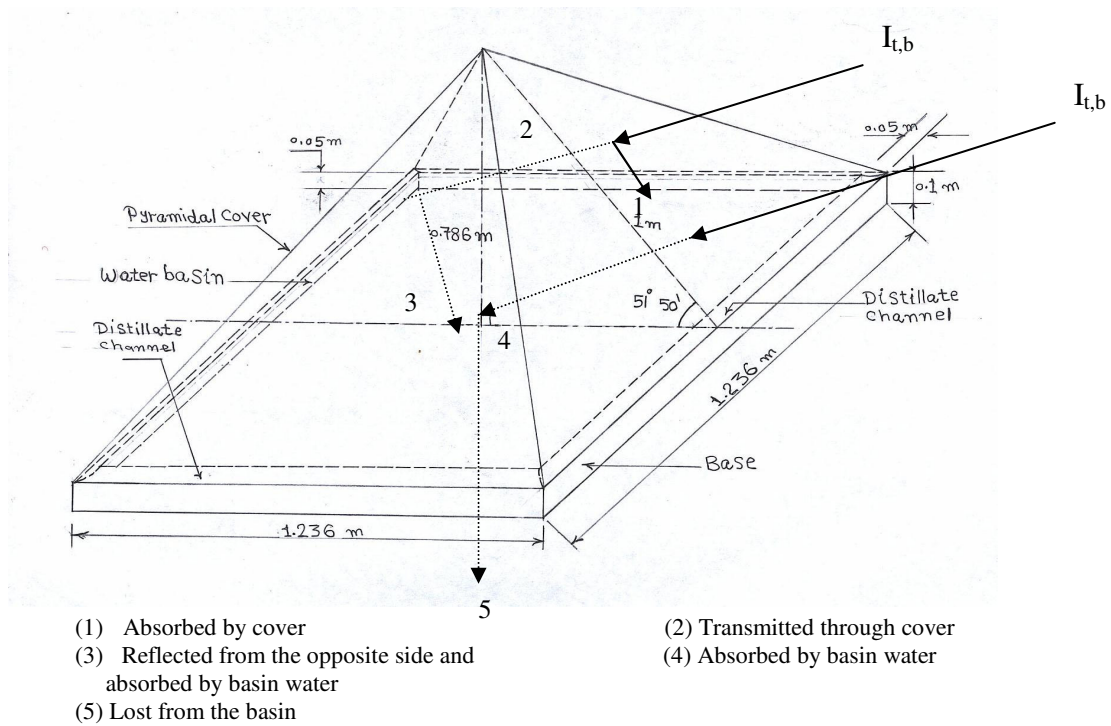
$$\eta_y = \text{annual yield} * L_{wg} / \text{annual solar energy} \quad (8)$$

Performance predictions of the stills are obtained by solving equations (1) to (8) for the temperatures, heat rates, productivities and efficiencies. The method of solving these simultaneous equations is given in both Hassabou [3] and Darwish [4]. The general assumptions for the solution of these equations can be summarized as follows:

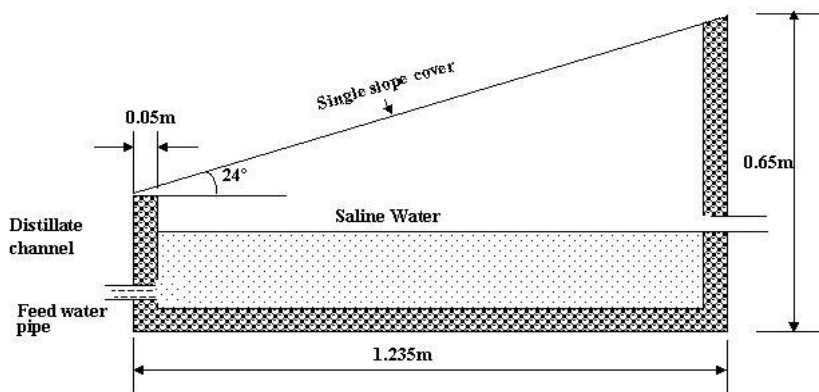
1. The physical properties of the saline water are constant over the operating temperature range,
2. No temperature variation in basin water in all direction,
3. The glass cover of the single sloped still is tilted up towards the south horizon by an angle of 24 degrees (the latitude of the city of Aswan, Egypt). This orientation is often selected since it averages the noontime insolation peaks over the year,

4. There is no leakage of vapor in or out the stills,
5. The wind speed is constant along the day and represents the measured daily average,
6. The ground heat capacity is 10 % of the still total heat capacity.

Configuration parameters for both stills are presented in table (1).



(a) Pyramidal still



(b) Single Slope Still

Fig.(2) Pyramid and Single Slope stills configurations



**Table (1) Some important properties for performance analysis of pyramidal and single slope solar stills.**

<b>Property classification</b>	<b>Property</b>	<b>Value</b>	<b>Units</b>	<b>Notes</b>
<b>Physical properties</b>	$\tau_g$	0.9		
	$\alpha_g$	0.1		
	$\alpha_w$	0.89		
	$C_w$	4186	J/kg °C	
<b>Configuration values</b>	$A_b$	1.5274	m <sup>2</sup>	
	$h_b$	0.812	W/°C	
	$A_{g(p)}$	2.472	m <sup>2</sup>	for $\beta=51^\circ 50'$
	$A_{g(s)}$	1.67	m <sup>2</sup>	for $\beta=24^\circ$
	$C_{s(p)}$	0.10052X10 <sup>6</sup>	J/°C	water depth = 0.01 m
	$C_{s(s)}$	0.10239X10 <sup>6</sup>	J/°C	water depth = 0.01 m.
<b>Others</b>	$\phi$	24	<b>Degrees</b>	<b>For Aswan City</b>
	L	33	Degree s	For Aswan City
	Longitude of Standard zone meridian	30	Degree s	For Aswan City
	$\rho$	0.26		<b>For desert</b>

## RESULTS & DISCUSSION

Table (2) illustrates typical measured data global solar radiation for the city of Aswan (1994). Aswan city data is used since it is the most reliable measured data for both direct and diffused radiation (another station is in Cairo, where dust and pollution make the data not as reliable). The table shows also the calculated values of the solar energy input to for both pyramid and single slope still's. The variation in the daily global radiation, the daily incident and absorbed radiation on both stills are shown in Figure (3), for the twelve months. From both Table(2) and Figure (3), it is clear that; i) the incident and absorbed radiation for the pyramid is higher than that of the single slope still (with February and October exceptions) ii) the daily average yearly incident radiation and absorbed radiation for the pyramidal still are 4% higher than that of the single slope still, (iii) the daily average yearly solar radiation losses for the pyramidal still is 1% higher than that for the single slope one, and (iv) the ratio of solar radiation losses to the incident radiation for pyramidal still is the same as that of the single slope still. The maximum global solar energy occurs in July (25 MJ/m<sup>2</sup>.day) and the minimum occurs in January (7.6 MJ/m<sup>2</sup>.day). Temperature, energies distribution indicates that the peak values occur almost near noon time in winter and one hour after noon time in summer.

Figures (4) and (5) show typical performance comparison between pyramidal and single slope stills for both winter (February) and summer (July) months. The Figures show that the incident resultant solar energies received by the single slope still is 8 % higher than that received by the pyramid in winter while it is 5 % lower in summer. Due to the larger radiation losses from the cover surface of the pyramid, the daily yield of the single slope still is 30 % higher than that of the pyramid in winter and 3 % higher in summer.

Figure (6) shows the monthly variation and the yearly average daily productivity and thermal efficiency. As expected, and due to the higher energy received and lower losses, the single sloped still is relatively more productive and efficient than the pyramid still configuration. The maximum and minimum productivities follows the solar energy trend and occur in July (4.25 Lit/m<sup>2</sup>.day) and January (0.65 Lit/m<sup>2</sup>.day) respectively. Similarly, the maximum and minimum efficiencies follows the solar energy and productivity trends and occur in August (40 %) and January (17 %) respectively. The annual average daily productivity is very close for both pyramid and single slope stills and is around 2.6 Lit/m<sup>2</sup>.day and the annual average daily efficiency is 30 % for pyramid and 33 % for single slope stills.

**Table (2) Daily solar radiation track for pyramidal and single slope solar stills.**

Month No.	1	2	3	4	5	6	7	8	9	10	11	12	Daily average yearly
$I_{t,h}$ (MJ/m <sup>2</sup> )	7.62	17.3	14.4	21.5	24.2	23.1	25.0	21.1	17.4	15.4	10.4	10.05	<b>17.3009</b>
$I_{t,\beta,n}$ (MJ)	3.29	2.09	4.41	6.17	7.29	7.41	7.76	7.11	4.59	2.94	3.05	2.75	4.90466
$I_{t,\beta,e}$ (MJ)	3.31	8.52	6.28	9.43	9.75	9.16	11.8	8.58	6.37	6.46	5.31	4.715	7.47732
$I_{t,\beta,s}$ (MJ)	4.48	12.8	8.47	12.1	12	12.2	13.3	10.9	10.2	10.4	6.92	7.27	10.079
$I_{t,\beta,w}$ (MJ)	4.65	7.82	7.48	10.5	10.9	12.1	11.4	10.7	10.2	8.23	4.63	5.123	8.6377
$I_{t,\beta 4}$ (p) (MJ)	15.7	31.2	26.6	38.1	40.0	40.8	44.4	37.3	31.3	28.0	19.9	19.85	<b>31.105</b>
$I_{t,\beta}$ (s) (MJ)	13.1	33.9	25.1	37.1	36.8	38.3	42.1	35.1	30.4	28.7	19.1	19.32	<b>29.938</b>
$I_{abs}$ (p) (MJ)	11.2	22.2	18.9	27.1	29.6	29	30.7	31.6	26.5	22.3	19.9	14.2	<b>22.236</b>
$I_{abs}$ (s) (MJ)	9.36	24.2	17.9	26.4	26.2	27.3	29.9	25	21.6	20.5	13.6	13.76	<b>21.3181</b>
$I_{loss}$ (p) (MJ)	4.5	8.9	7.7	10.9	10.4	11.7	12.7	10.7	9.02	8.07	5.73	5.718	8.86865
$I_{loss}$ (s) (MJ)	3.78	9.79	7.23	10.7	10.6	11.0	12.1	10.1	8.76	8.28	5.52	5.564	8.61997
$I_{loss}$ (p) %	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.8
$I_{loss}$ (s) %	28.8	28.8	28.8	28.7	28.8	28.8	28.8	28.8	28.8	28.8	28.8	28.80	28.8

Different design, operational and environmental parameters can influence the productivity and efficiency of the solar stills . Most of these parameters have been investigated previously by Fath & Hosny [5] and others. Figure (7) shows the effect of one important parameter affect the pyramid configuration; i.e. inclination angle of pyramid sides. This parameter affect the pyramid glass cover area, incident and absorbed solar energy, and system heat capacity, and therefore influence the still productivity. The Figure shows that the angle 50 degree (very near to the Great Pyramid of Giza angle of 52 degree) gives the best still productivity. Significant reduction takes place when the tilt angle increases above 60 degree.

## **ECONOMICAL STUDY & RESULTS**

Many factors affect the product water unit cost of desalination technologies and similarly the solar distillation. Both capital and running (and so the total) costs are influenced by the unit size, site location, feed water properties, product water required quality, qualified staff availability, .....etc., see Fath [6]. The main economical advantages of solar distillation are; i) it does not require much infra-structure, and ii) its simplicity to locally design, install, operate and maintain. On the other hand its main economical disadvantage is the unit size limitation due to the large area required (about 250 m<sup>2</sup> per one m<sup>3</sup> of product water).

### Economical Analysis of water Unit Cost

After Govind and Tiwari [7], the Capital Recovery Factor (CRF), the First Annual Cost (FAC), the Sinking Fund Factor (SFF), the Annual Salvage Value

(ASV), Annual Cost (AC), and finally the Annual Cost per Liter (AC/L) can be expressed as:

$$\text{CRF} = i (1+i)^n / [ (1+i)^n - 1 ] \quad (9)$$

$$\text{FAC} = P (\text{CRF}) \quad (10)$$

$$\text{SFF} = (i) / [ (1+i)^n - 1 ] \quad (11)$$

$$\text{ASV} = (\text{SFF}) S \quad (12)$$

$$\text{AC} = \text{FAC} + \text{AMC} - \text{ASV} \quad (13)$$

$$\text{AC/L} = \text{AC} / M \quad (14)$$

Where: AMC is annual Maintenance Cost, and has been taken as 15 % FAC.

Tables (3) and (4) shows the details of cost analysis which shows that the pyramid unit cost is little more expensive than the single slope still and averages to 0.13 L.E. / Liter (about 0.03 \$/Liter).

**Table (3) Cost Breakdown of Solar Stills In L.E. ( L.E. = 0.25 \$)**

Still Component	Pyramidal Solar Still (PSS)	Single Slope Solar Still (SSSS)
Galvanized steel 0.4 mm thick. at LE 3.1/kg	70	70
Upper glazing (Tephlon) at LE15/m <sup>2</sup>	50	25
Rubber material	15	15
Black paint	25	25
Pump	100	100
Galvanized iron Pipes and brass nipples	100	100
Labor	175	175
Insulation (sawdust)	-	-
Internal coating	50	25
<b>Total cost</b>	<b>585</b>	<b>535</b>
Life (years)(n)	10	10
Salvage value(s) (50% cost of reusable material (LE)	135	135

**Table (4) Product Water Unit Cost, In L.E. ( L.E. = 0.25 \$)**

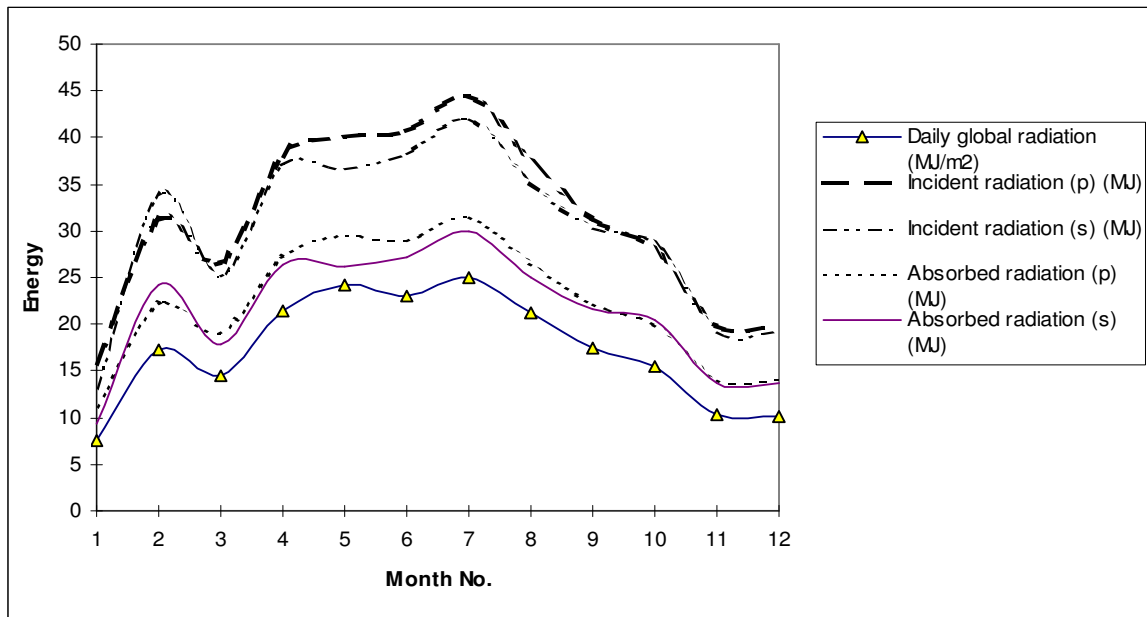
	<b>Pyramidal still</b>	<b>Single slope still</b>
<b>Life(Years)</b>	<b>10</b>	<b>10</b>
<b>Interest rate</b>	<b>12%</b>	<b>12%</b>
<b>Initial investment</b>	<b>585</b>	<b>535</b>
<b>Salvage value</b>	<b>135</b>	<b>135</b>
<b>CRF</b>	<b>0.295</b>	<b>0.295</b>
<b>SFF</b>	<b>0.0569</b>	<b>0.0569</b>
<b>FAC</b>	<b>172.58</b>	<b>157.83</b>
<b>ASV</b>	<b>7.68</b>	<b>7.68</b>
<b>Maintenance cost</b>	<b>25.88</b>	<b>23.7</b>
<b>Annual cost</b>	<b>190.78</b>	<b>173.85</b>
<b>Annual yield (liters)</b>	<b>1510.5</b>	<b>1532.7</b>
<b>Annual cost per liter (LE/Liter)</b>	<b>0.126</b>	<b>0.113</b>



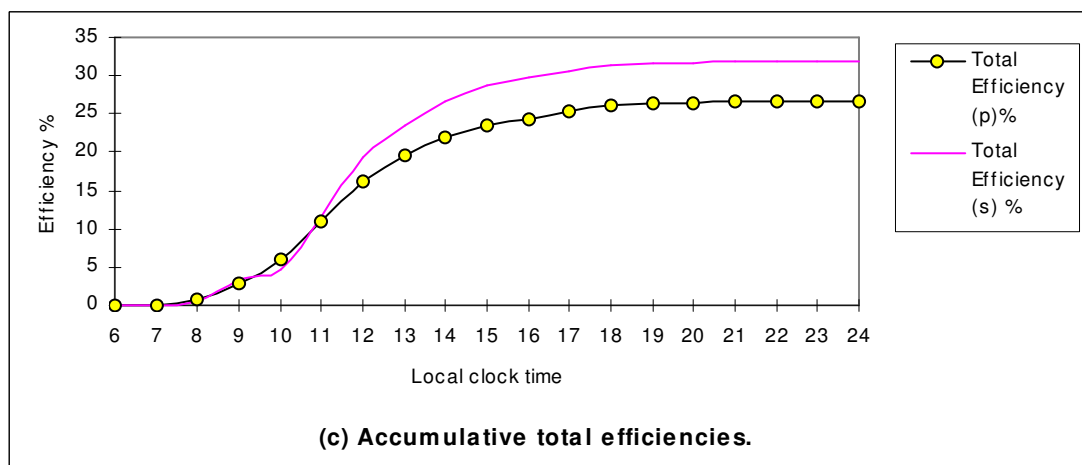
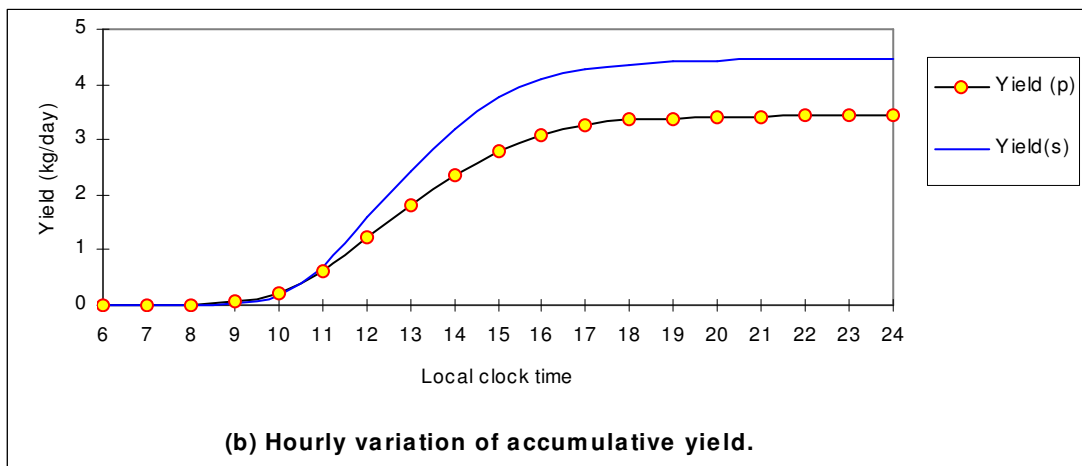
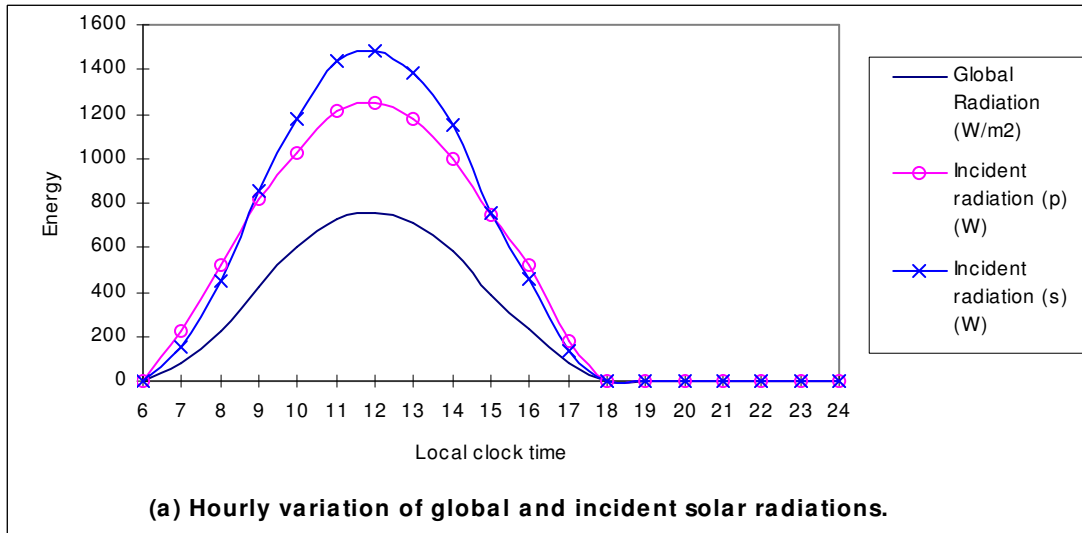
## CONCLUSION

- 1- Analytical study and comparison between two solar stills configurations; the Pyramidal and single slope, have been presented. A mathematical model has been developed to simulate the thermal analysis of these configurations and study their performance. The weather meteorological data of Aswan City (south of Egypt) was used, since it is the most reliable measured data for both direct and diffused radiation.
- 2- The daily total energy received by each still basin, still productivity and efficiency have been presented for the whole year. On the basis of the yearly performance results, the single slope still was found to be slightly more efficient than the pyramidal one. The solar energies received by the single slope still is 8 % higher than that received by the pyramid in winter while it is 5 % lower in summer. Due to the larger radiation losses from the cover surface of the pyramid, the daily yield of the single slope still is 30 % higher than that of the pyramid in winter and 3 % higher in summer. The annual average daily productivity is very close for both pyramid and single slope stills and is around 2.6 Lit/m<sup>2</sup>.day and the annual average daily efficiency is 30 % for pyramid and 33 % for single slope stills.
- 3- The pyramid tilt angle of 50 degree (very near to the Great Pyramid of Giza angle of 52 degree) gives the best still productivity. Significant reduction takes place when the tilt angle increases above 60 degree.
- 4- The product water cost is around 120 L.E./m<sup>3</sup> (30 \$/m<sup>3</sup>) is very high as compared to 1-2 \$/m<sup>3</sup> for conventional desalination

technologies. For drinking water and direct man use in remote areas (islands, mountains, ...etc.) the cost of solar distillation product water of 0.12 L.E./liter (0.03 \$/liter) could be acceptable as compared to water transportation.



**Figure (3) Monthly average variation of daily solar thermal energies for pyramidal and single slope stills.**



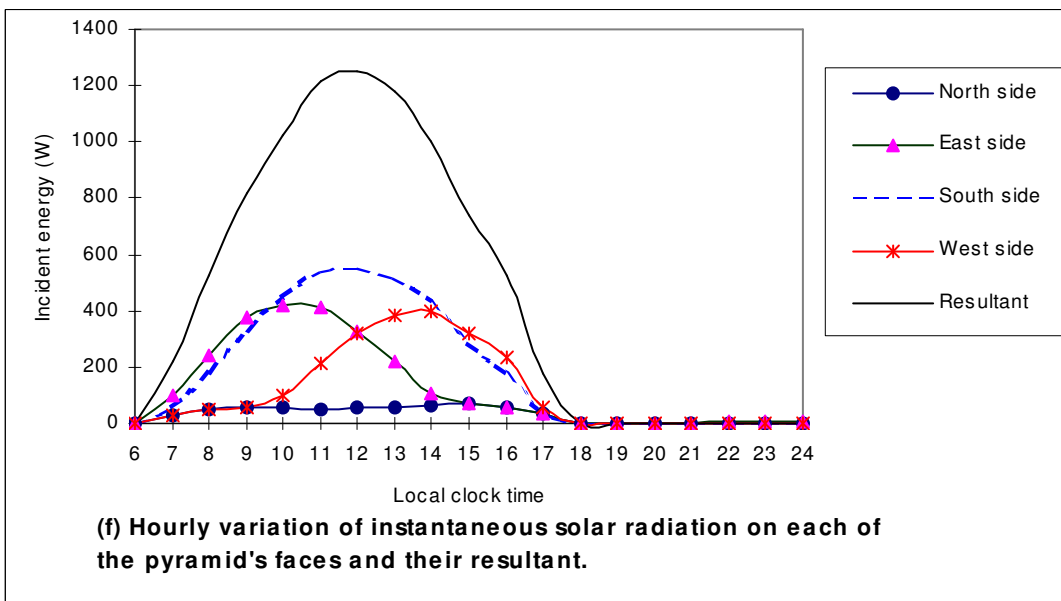
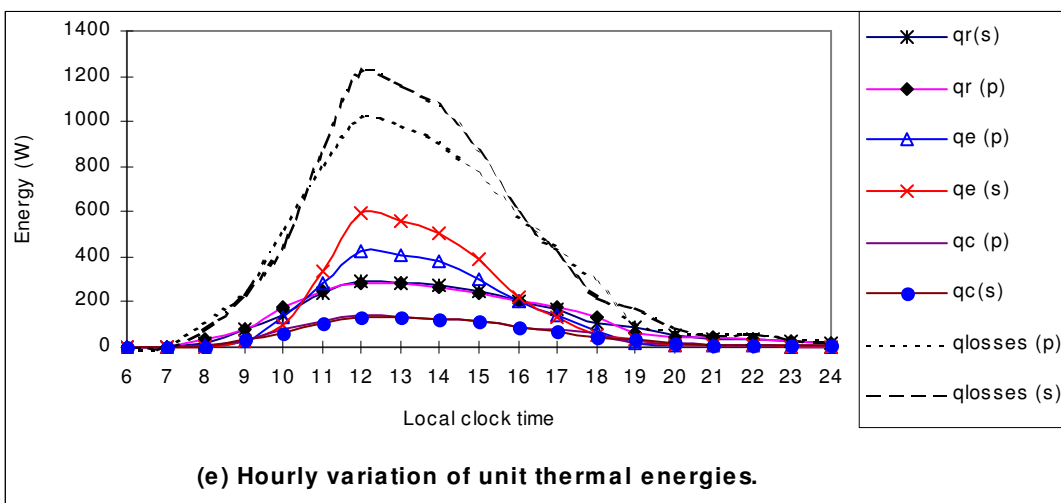
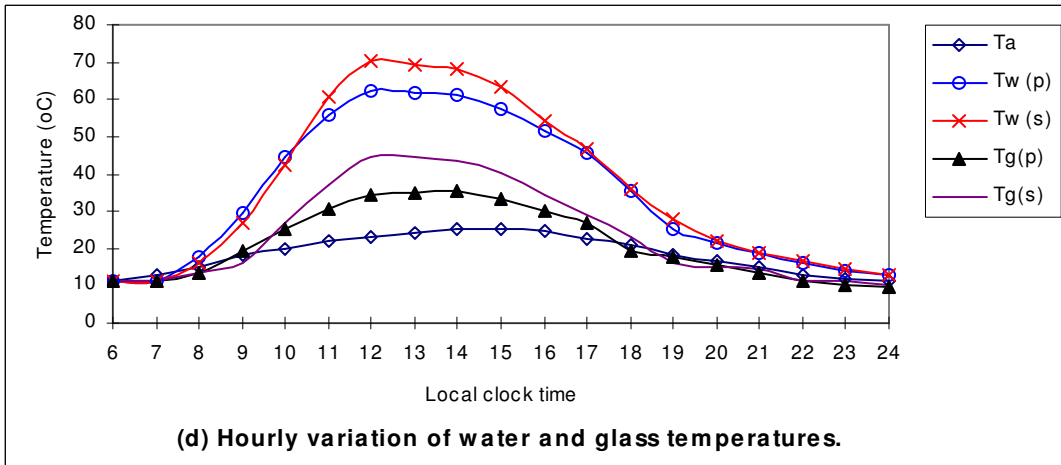
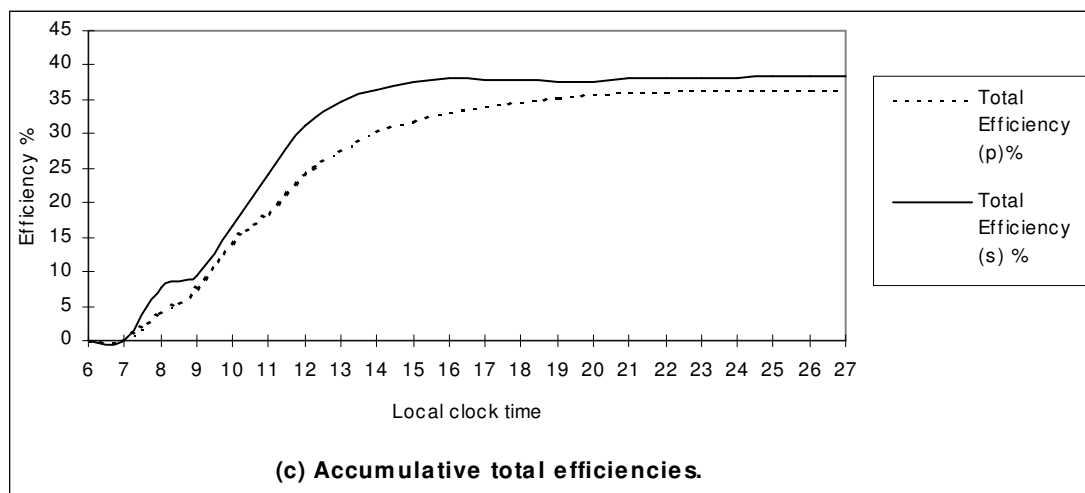
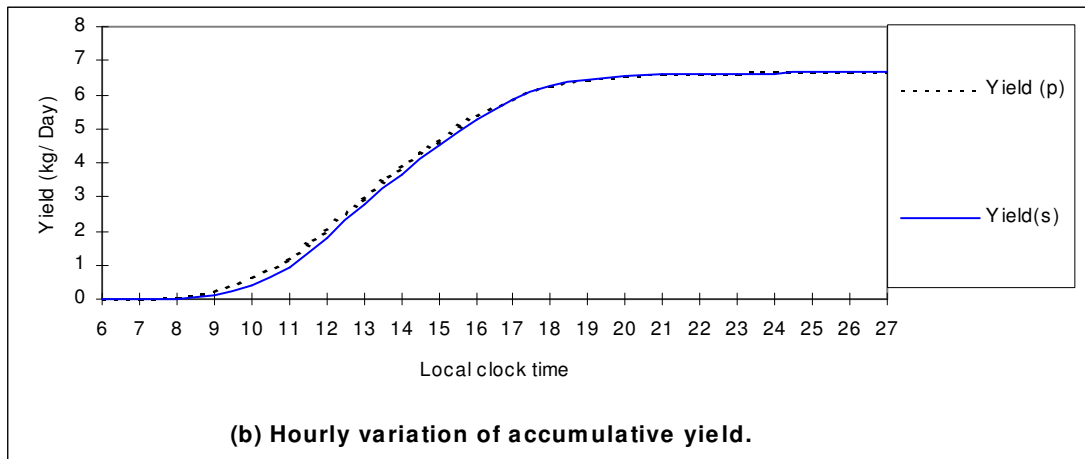
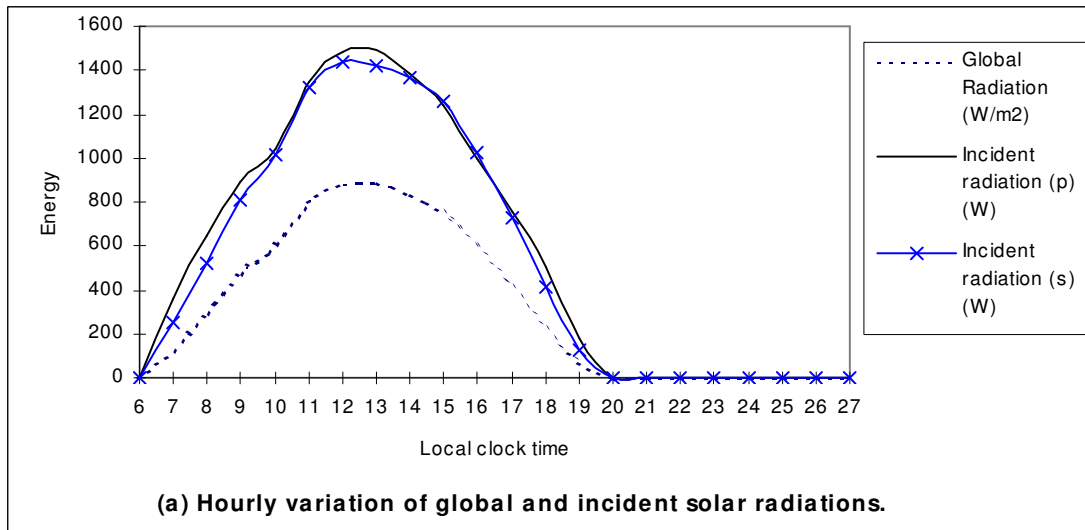


Figure (4) Performance comparison between pyramidal and single slope solar stills, February 1994 ( $A_b = 1.5274 \text{ m}^2$ ).



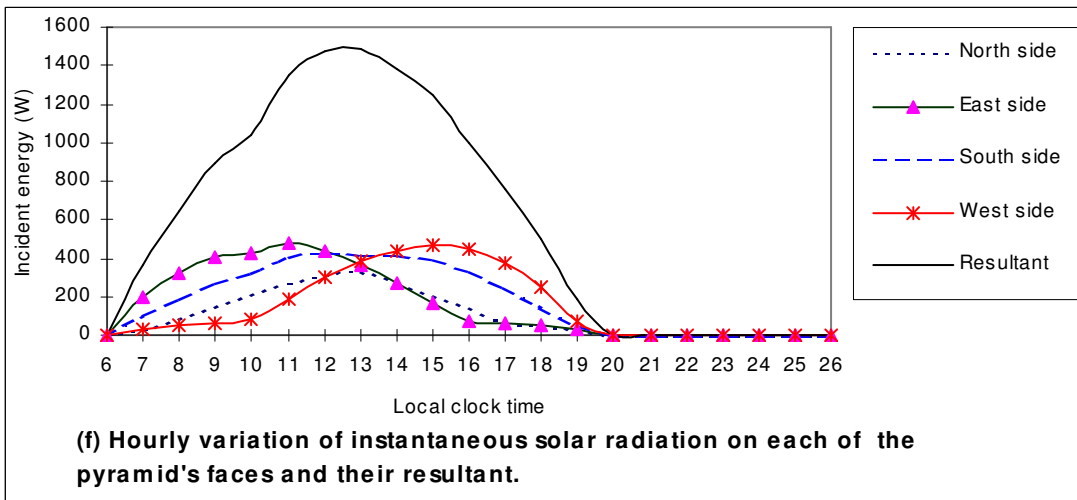
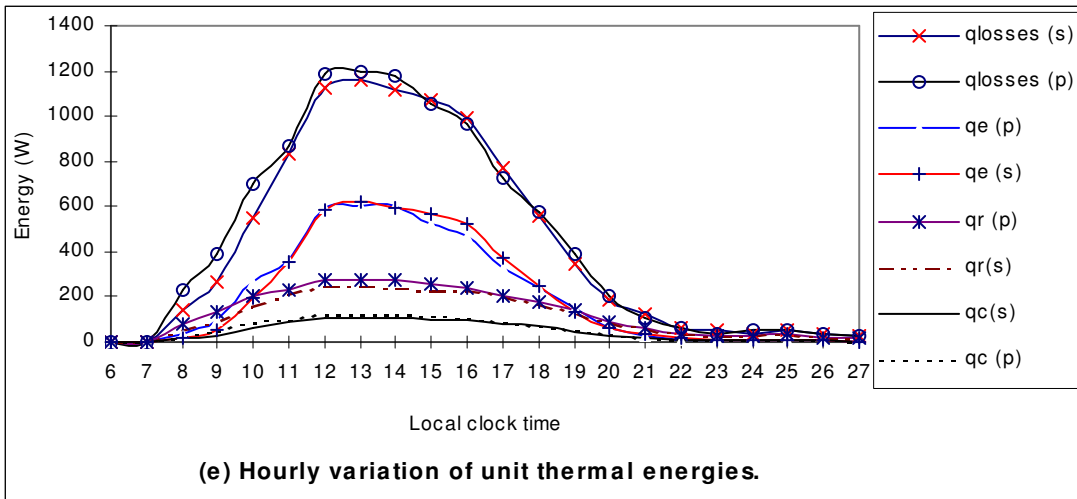
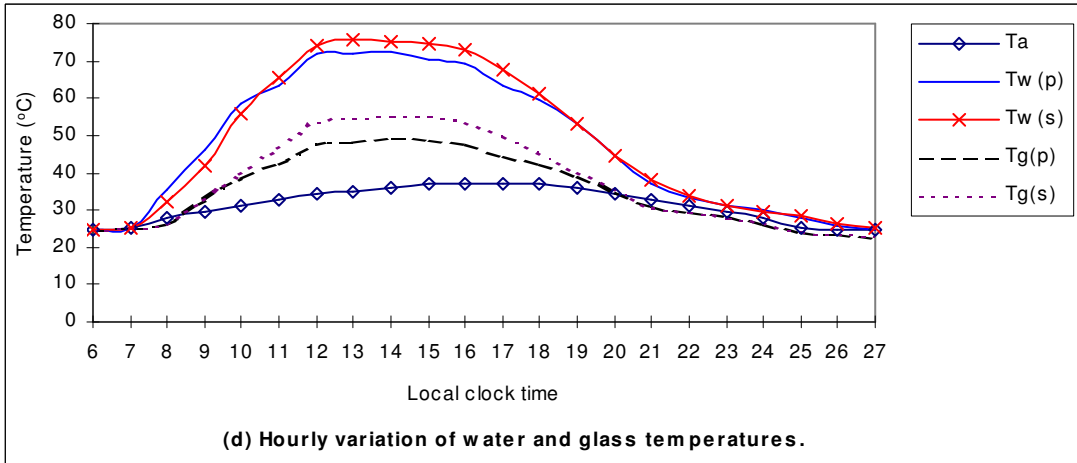
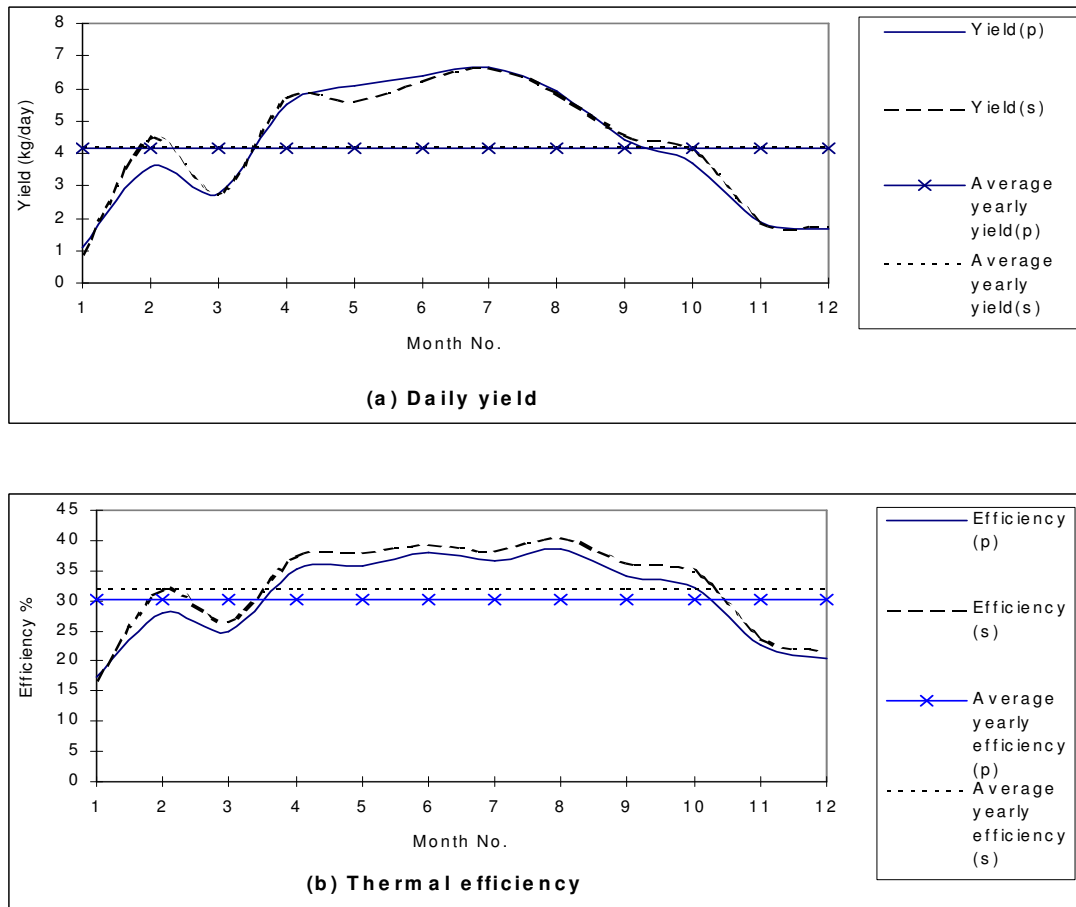
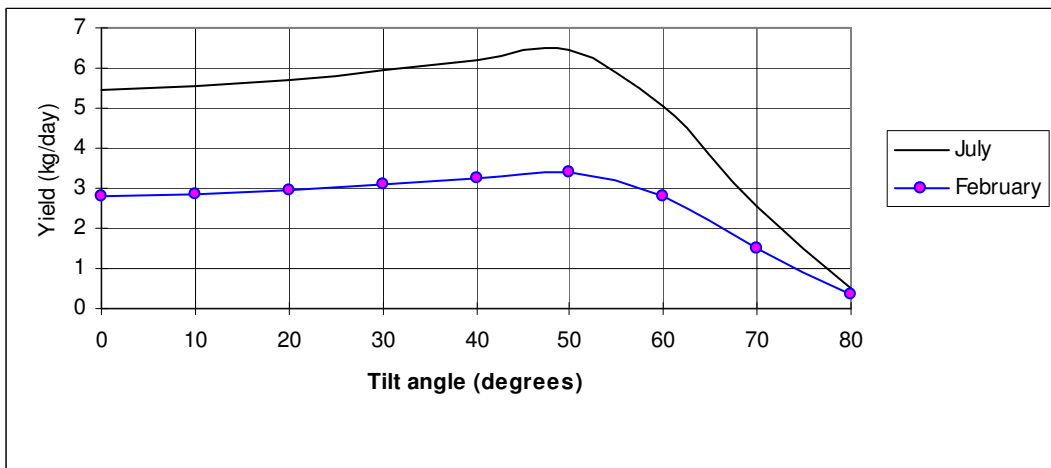


Figure (5) Performance comparison between pyramidal and single slope solar stills, July 1994, ( $A_b = 1.5274 \text{ m}^2$ ).



**Fig. (6) Monthly average yield and efficiency for both pyramidal and single slope stills based on Dunkle relations.**



**Fig. (7) Effect of pyramid's tilt angle on the still performance, in February (winter) and July (summer) 1994.**



**Nomenclature**

A	Area ( $m^2$ )
C	Specific Heat (J/kg K)
h	Film Coefficient of Heat Transfer ( $W/m^2 K$ )
H	Solar Intensity ( $W/m^2$ )
i	Annual Interest Rate (%)
$L_w$	Latent Heat of Evaporation (J/kg)
M	Mass (kg)
m	Mass Rate ( $kg/s.m^2$ )
n	Life Time of the Still
p	Pressure ( $MN/m^2$ )
P	Initial Investment of the Still (L.E. or \$)
Q	Heat Transfer Rate (W)
q	Heat Transfer Flux ( $W/m^2$ )
S	Salvage Value of the System (L.E. or \$)
T	Temperature (C)
t	Time (s)
U	Base Overall Coefficient of Heat Transfer ( $W/m^2 K$ )
v	Wind Velocity (m/s)
V	Volume ( $m^3$ )

**Greek Letters**

A	Solar azimuth angle	$\alpha$	Absorbptivity
$\beta$	Tilt angle	$\delta$	Declination Angle
$\epsilon$	Emissivity	$\phi$	Latitude
$\eta$	Efficiency	$\theta$	Angle

$\rho$	Ground Surface Reflectivity	$\sigma$	Radiation Constant
$\tau$	Transmissivity	$\omega$	Hour Angle
$\gamma$	Surface azimuth angle		

### Subscripts

a	Air	b	Base	c	Convection
e	Evaporative	g	Glass Cover	i	incident
r	radiation	w	Water (basin)	z	Zenith
win	Wind	ga	glass to air	wg	water to glass

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